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Table-Based Numerical Macromodeling for MEMS Devices

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ABSTRACT

In this paper, we present a table-based numerical macromodeling method in which a MEMS device is modeled by a set of numerical ordinary differential equations and model functions are represented by look-up tables of numerical data. Cubic spline interpolation is used to evaluate model functions during simulation. Numerical linear models are also constructed for DC analysis, AC analysis and feedback system design. This method is demonstrated by modeling and simulating a MEMS accelerometer.

Keywords: table-based numerical model; cubic spline interpolation; linear fractional transformation

1 INTRODUCTION

As MEMS devices are studied by measurements or device-level simulations, the results are usually in numerical form. Meanwhile, the efficient system-level simulation demands the use of behavioral macromodels of devices, and the design of feedback control systems requires linear models of system components. Traditionally, macromodels are represented by either equivalent lumped electrical network elements [1] or analytical ordinary differential equations. Thus, great efforts are devoted to extract analytical models from numerical data. The model extraction process is difficult to automate and will introduce systematic model errors.

This paper introduces a numerical modeling method that uses tables of numerical data to describe the relationship between variables. These table-based models are built directly upon data obtained by measurements or device simulations, without detailed knowledge of underlying physics. Therefore, the difficulties and errors associated with extracting analytical models are eliminated and the model extraction can be easily automated. When the table-based models are used in behavioral-level simulations, cubic spline interpolation is used to evaluate the model functions [2]. In addition to the nonlinear numerical model, a set of numerical linear models are constructed. They are used to solve for operating points, to perform AC analysis and to design closed-loop feedback systems.

2 TABLE-BASED NUMERICAL MODELING

A surface micromachined accelerometer will be used throughout this paper to demonstrate our method. This device is fabricated by CMU CMOS-MEMS process [4]. It consists of a micromechanical resonator which translates the acceleration into the displacement of the proof-mass, a capacitive position sensor which converts the displacement into electrical signal, and an electrostatic actuator to provide feedback and compensation. A SEM of the device is shown in Figure 1. The multi-layer lateral parallel-plate capacitor structure that forms both the capacitive sensor and the electrostatic actuator is shown in Figure 3(a).

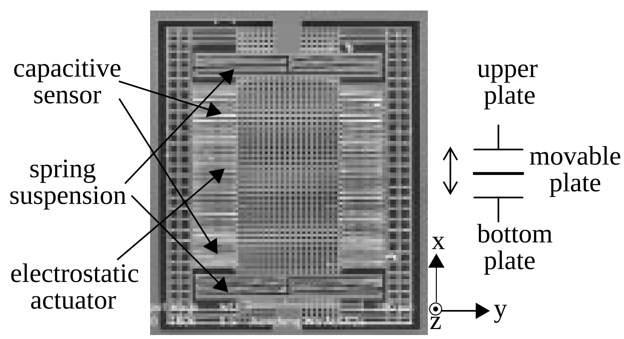

Figure 1: The CMOS-MEMS accelerometer

At behavioral level, this device is described by a set of coupled ordinary differential equations (ODEs),

$$
\begin{aligned}
m_x\frac{d^2x}{dt^2} + b_x\frac{dx}{dt} + f_{spx}(x, z) &= f_x \\
m_z\frac{d^2z}{dt^2} + b_z\frac{dz}{dt} + f_{spz}(x, z) &= f_z \\
f_x &= f_{inx} + e_x(d-x, z)V_p^2 - e_x(d+x, z)V_n^2 \\
f_z &= f_{inz} + e_z(d-x, z)V_p^2 + e_z(d+x, z)V_n^2 \\
V_{out} &= s(x, z)
\end{aligned}
\quad (1)
$$

where m_x and m_z are the effective masses, b_x and b_z are the damping factors, $f_{spx}(x, z)$ and $f_{spz}(x, z)$ are the spring forces, $s(x, z)$ is the sensitivity of the capacitive sensor, and $e_x(d, z)$ and $e_z(d, z)$ describe the electrostatic actuation force per square volt of the actuator. To characterize this device, the mechanical part is simulated by Abaqus, a mechanical simulator using finite element method (FEM), and the capacitive sensor and the electrostatic actuator are simulated by Raphael, a capacitance solver based on boundary element method (BEM). The data acquired by the simulations is plotted in Figure 2 and 3. The elastic forces of the micro-machined spring have good linearity within the operation range. However, nonlinearity is introduced into the total spring force by the parasitic electrostatic spring effects of the capacitive sensor. The simulation also shows that the capacitance of the multi-layer structure deviates significantly from the ideal parallel-plate capacitor approximation because of the fringing field.

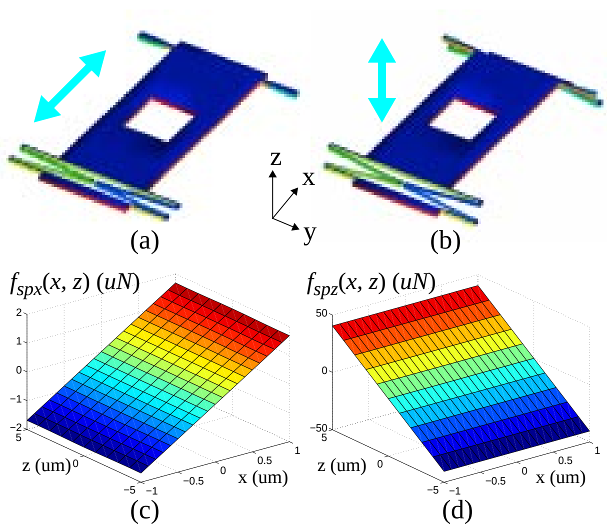

Figure 2: Two main modes of the accelerometer (a, b) and simulated total spring forces with parasitic electrostatic spring effect adjustment (c, d)

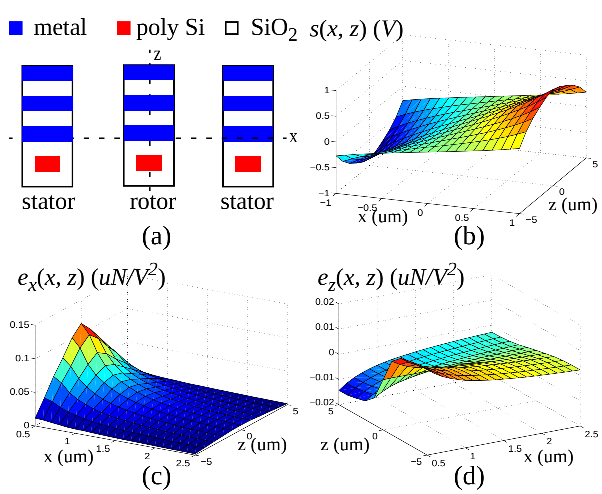

Figure 3: The multi-layer lateral capacitor structure (a), the simulated capacitive sensor sensitivity (b) and the electrostatic force per square volt of the actuator (c, d)

In traditional behavioral macromodeling, the model functions are expressed in analytical form [1]. Therefore, the form of the analytical function must be first determined, often with the help of the knowledge of the device physics. Then, the model parameters are estimated by curve fitting on numerical data. This process often takes iterations with the original function form modified or correction terms added. Except for a few ideal cases, the analytical model extraction introduces global modeling error and is difficult to automate because the forms of the functions must be pre-determined. Although good analytical models provide insight that will help device designers, it is not necessary to have analytical model for simulation purposes.

In the table-based numerical modeling method, the ordinary differential equations in (1) are expressed in numerical form. The functions that describe the relationship between the variables, in our example, f_{spx}, f_{spz}, s, e_x and e_z, are all represented by tables of numerical data obtained by device simulations or experimental characterizations. During simulations, the models are evaluated by performing interpolation on the tables. This approach avoids the difficulties in deriving analytical functions and is much easier to automate. Once the order and the state variables of the ODEs are decided, the model construction is straightforward and requires no human intervention. It also eliminates the systematic global modeling errors if local interpolation is used to evaluate the models. The table-based macro-modeling method is illustrated in Figure 4.

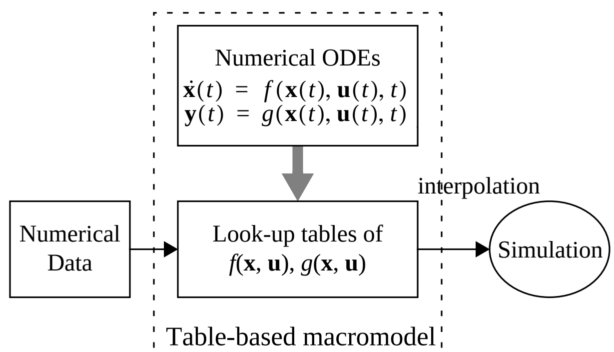

Figure 4: Table-based numerical macromodeling

3 BEHAVIORAL-LEVEL TRANSIENT SIMULATION

In system-level behavioral simulations, the table-based models are evaluated using cubic spline interpolation [2]. The cubic spline interpolation is a 3rd-order local interpolation method which is smooth in the first derivative and continuous in the second derivative. For an one-variable problem, the function value y with respect to the variable value x between x_i and x_{i+1} is computed by a cubic polynomial,

$$
\begin{aligned}
y &= ay_i + by_{i+1} + cy''_i + dy''_{i+1} \\
a &= \frac{x_{i+1} - x}{x_{i+1} - x_i} \\
b &= \frac{x - x_i}{x_{i+1} - x_i} \\
c &= \frac{1}{6}(a^3 - a)(x_{i+1} - x_i)^2 \\
d &= \frac{1}{6}(b^3 - b)(x_{i+1} - x_i)^2
\end{aligned}
\qquad (2)
$$

The cubic spline interpolation is a stable and smooth method in which the interpolation errors are localized and well bounded. This is achieved by using globally determined second derivatives. Therefore, for each model function, a table of second derivatives must be pre-computed and stored. The computation of the cubic spline interpolation is more efficient than evaluating many mathematical functions because of the low order of the interpolation polynomials. One may argue that during simulations, extra effort is needed to locate the interpolation interval. In reality, since most functions vary continuously, the data in two consecutive simulation steps usually fall into the same or the neighboring intervals. Thus, locating the interpolation intervals contributes very little overhead in the simulation time. Figure 5 shows an example of single-variable cubic spline interpolation.

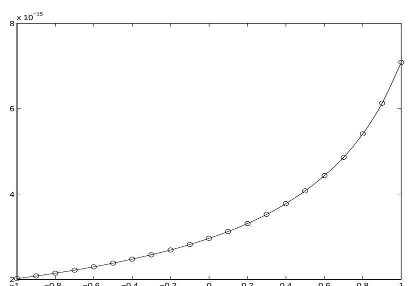

Figure 5: One-variable cubic spline interpolation (line) on a data set (dots)

In this work, the table-based model of the MEMS accelerometer is used to simulate a delta-sigma accelerometry system which includes the MEMS transducer [5]. The block diagram of this system is shown in Figure 6. This system employs closed-feedback control to suppress nonlinearities and sensitivity variations, and uses delta-sigma modulation technique to realize high-resolution analog-to-digital conversion. A multivariate cubic spline interpolation routine is inserted into a customized behavioral simulator to simulate this system. The transient simulation results are shown in Figure 7.

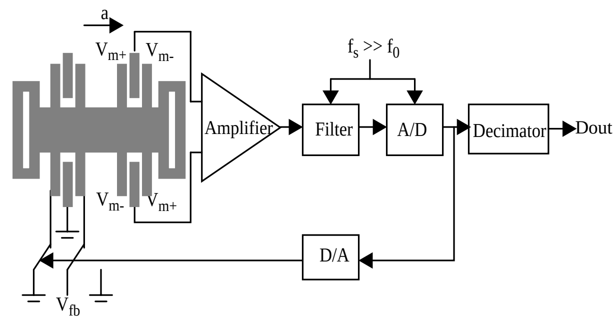

Figure 6: The delta-sigma digital accelerometry system with the MEMS accelerometer

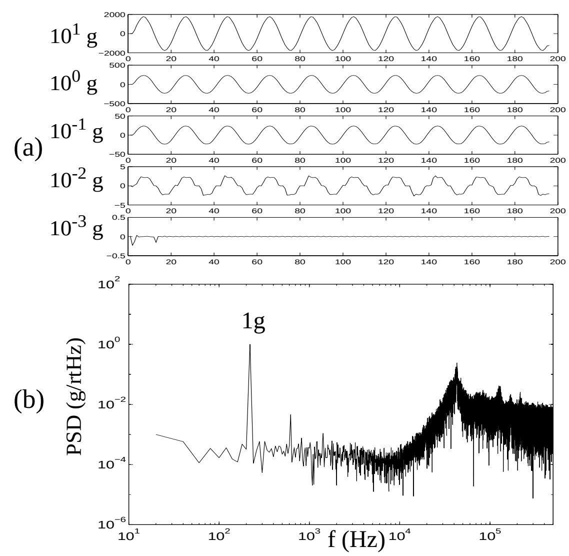

Figure 7: Simulation results: decimated output signal waveforms (a) and quantization noise spectrum (b)

4 NUMERICAL LINEAR MODELS

Linear models are required to solve nonlinear equations to find operating points (DC analysis), to compute frequency-domain responses (AC analysis), and to design closed-loop feedback systems by using various linear feedback control techniques. Based on the table-based numerical modeling method, a set of numerical linear models could be computed directly by numerical differentiation on the tabulated data and be represented also by tables. As the device exhibits nonlinear behaviors, the linear model parameters vary with the operating points. The other sources of the linear model parameter variations are the manufacturing variations and variations induced by surrounding environment, both are large in micro-fabricated devices.

For MEMS devices, high quality (Q) factor is desirable because that reduces the thermal-mechanical noise. To avoid ringing, it is necessary to put a high-Q underdamped device in a feedback loop to stabilize it. However, simulations suggest the closed-loop system performance degrades at high Q because the system tends to go unstable. Designing robust feedback systems with high-Q components is challenging. And the large variations inherently possessed by MEMS devices make this task even more difficult. This situation demands a quantitative representation of the linear model variations. Linear fractional transformation (LFT) is employed to represent the linear model with parameter variations (perturbations) by a multi-input-multi-output (MIMO) system [3]. The LFT is explained in Figure 8. The LFT model of the accelerometer is shown in Figure 9. And the variational AC responses of the device computed by the LFT model is shown in Figure 10. The LFT model enables the use of modern robust control techniques which take model perturbations into considerations [3].

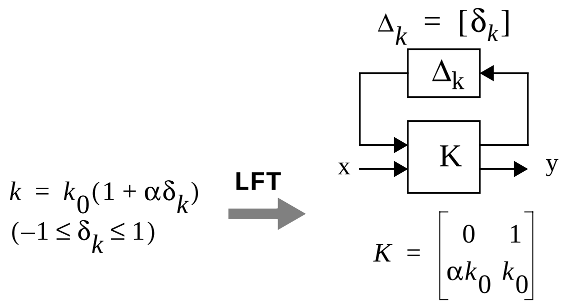

Figure 8: The linear fractional transformation (LFT)

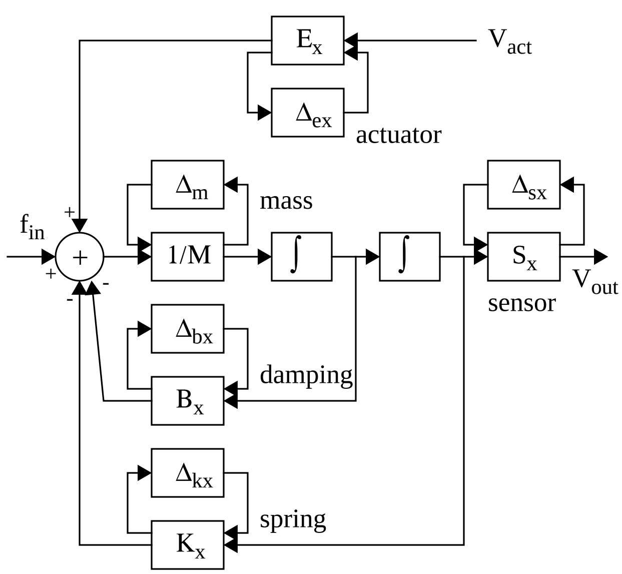

Figure 9: The LFT model of the MEMS accelerometer (Only the sensing direction is shown for simplicity)

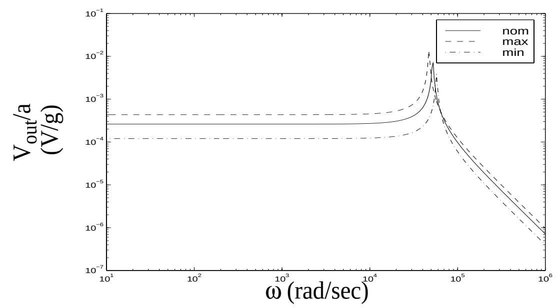

Figure 10: The variational frequency response of the accelerometer computed by the LFT model

5 CONCLUSIONS

We have demonstrated a table-based numerical macromodeling method. The table-based numerical model consists of five integrated components: the numerical ordinary differential equations; the tables of model functions; the tables of the first derivatives representing the linear model; the tables of second derivatives used for the cubic spline interpolation; and the linear state-space LFT model that describes the linear model variations. This method enables us to construct models directly from experimental characterizations and device simulations, and to automate the model extraction process. The table-based model could be used in transient, AC and DC simulations, as well as in the design of closed-loop feedback systems.

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