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Low-Order Squeeze Film Model for Simulation of MEMS Devices

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ABSTRACT

A low-order behavioral squeeze film model of oscillating planar microstructures with critical dimensions in the range of a few microns is presented. Simulation results from this model are within 3% of those obtained from high-fidelity 3D numerical simulations using non-trivial boundary conditions. This low-order model has been added to the NODAS MEMS schematic library for system level design of MEMS. The results of system-level simulation of a CMOS micromechanical bandpass filter using this damping model are presented.

Keywords: Squeeze film damping, low-order model, lumped parameter model.

1 INTRODUCTION

MEMS devices are often characterized by structures that are a few microns in size, separated by micron-sized gaps. At these sizes, air damping dominates over other dissipation mechanisms at atmospheric pressure. As the number of such gaps contributing to damping can be very large, low-order models capable of accurately simulating the dynamics of the damping mechanism are needed.

In the past, researchers have presented both analytical and behavioral models for squeeze film damping. Starr performed an extensive study of squeeze film damping [1]. Blech derived an analytical solution to the linearized Reynolds equation for the case of circular and rectangular plates under the assumption of isothermal and compressible conditions [2]. An equivalent circuit model composed of parallel branches of series-connected R-L elements was proposed by Veijola *et al*. [3]. This model has been shown to be accurate for displacements as high as 90% of the nominal gap [4, 5]. The accuracy of the equivalent circuit model degrades for plates of dimensions lower than 60 μm. As such dimensions are common in many MEMS devices, we present physics-based modifications to the model to extend its accuracy to squeeze films of a few microns size. Since many MEMS devices are designed to operate in their linear region of operation, we focus on improving the accuracy for few micron-sized plates oscillating with amplitudes that are small compared to the nominal gap (< 10% of the gap).

The use of trivial boundary conditions (plate edges fixed at ambient pressure) in numerical simulations underestimates the damping force [6, 7]. In this paper, we use non-trivial boundary conditions to account for the edge effects during numerical simulations, by allowing the simulator to solve for pressure at the plate edges.

Section 2 of the paper presents the lumped parameter model for squeeze film damping. Handling of the boundary conditions in numerical simulations and the improvement of the lumped parameter model are discussed in section 3. Section 4 presents the simulation results and shows the improved accuracy of the behavioral model. Finally, section 5 lists the conclusions that can be drawn from this study.

2 LUMPED PARAMETER MODEL

The lumped parameter model presented here is based on Blech's squeeze film model for rectangular plates [2]. The model contains parallel branches of series-connected damper and spring elements (Figure 1). The expressions of the damper and spring elements are

$$k_{mn} = \frac{6lwP_a}{(mn)^2 \pi^4 (g+z)} \tag{1}$$

$$B_{mn} = \frac{768(lw)^3 \eta_{eff}}{(mn)^2 (m^2 l^2 + n^2 w^2) \pi^6 (g+z)^3} \tag{2}$$

where *m, n* are odd integers, *w* is the width and *l* the length of the squeeze film, *g* is the nominal gap which is equal to the thickness of the squeeze film, P_a is the ambient pressure, η_{eff} is the effective viscosity of the air, and *z* is the displacement of the oscillating top plate which squeezes the air film.

The lumped-parameter model has been implemented in NODAS. NODAS is a circuit-level methodology for hierarchical design and nodal simulation of MEMS developed at Carnegie Mellon University [8].

The B-k values are related to the component values of the R-L equivalent circuit model presented in [3] as $B_{mn} = 1/R_{mn}$ and $k_{mn} = 1/L_{mn}$. The velocity (across variable) of the top plate was mapped to the voltage across the R-L branches and the damping force (through variable) was mapped to the current.

3 MODEL ENHANCEMENTS

In this section, we first motivate the need to modify the model for improving the overall accuracy by way of

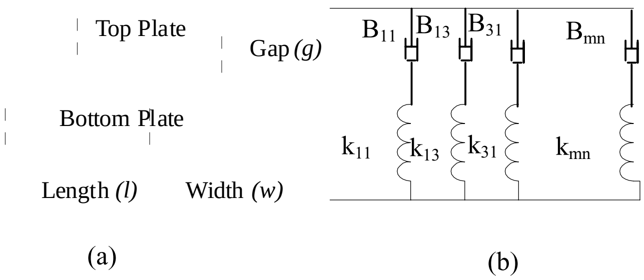

Figure 1 (a): Squeeze film between two parallel plates
(b): Spring-damper behavioral model

numerical simulation results. Later in the section, we describe the implemented modifications

3.1 Numerical Simulations

In the past, 3D numerical simulations have been performed with the pressure at the edges of the squeeze film set to the ambient pressure [6, 7]. We overcome the underestimated gauge pressure due to the use of trivial boundary conditions by extending the control volume beyond the plate edges. We fix the pressure (at ambient pressure) at the extended control volume boundaries. This is done by placing additional blocks of air surrounding the squeeze film on all sides and allowing the pressure to settle at ambient pressure away from the squeeze film edges. The lightly shaded region in Figure 2 shows the control volume surrounding a plate oscillating above the substrate. Air blocks on the top of the plate are included to model a realistic MEMS structure. The faces on the top surface are defined as outlets at ambient pressure. In this situation, the solver is not constrained by unrealistic boundary conditions. Though this simulation requires more computer resources in terms of memory and CPU time, it is needed for a more accurate characterization of the behavioral model and for better understanding of the resulting air flow. Figure 3 shows the pressure distribution across the plate for the system with extended boundary conditions in which air film of thickness 2 µm and size 20 µm on a side is being squeezed by the top plate oscillating sinusoidally with an amplitude of 0.1 µm and frequency of 1 kHz. Unlike the case of trivial boundary conditions which fixes the gauge pressure to zero at the plate edges, the non-trivial boundary conditions show non-zero pressure at the plate edges.

3.2 Edge and Finite Size Effects

The lumped parameter model inherits Blech's original assumption of trivial boundary conditions that set the gauge pressure to zero right at the plate edges. As we've just seen, the gauge pressure is zero *only at some distance* from the plate edges. One approach to handling the edge effects is to increase the plate size in the behavioral model by δL to match the numerical simulation results with non-trivial

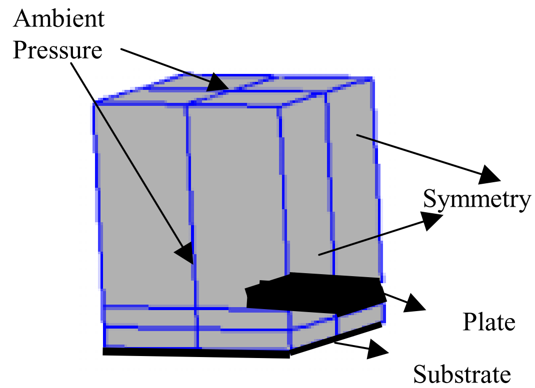

Figure 2: Control volume around an oscillating plate

boundary conditions. With the extension of the plate dimensions, the effective width and the length become $w_{eff} = w+\delta L(w)$ and $l_{eff} = l+\delta L(l)$. These effective dimensions are used in equations 1 and 2. δL is likely to depend on the geometry of the squeeze film and the oscillation frequency. To screen the important factors, 3D numerical simulations with frequencies in the ranges of 100 Hz to 10 kHz, gap sizes from 1.5 µm to 4 µm and plate sizes in the range of 10 µm to 1 mm were performed and the values of plate extensions (δL) needed to match the behavioral and numerical results determined. These variable-screening experiments indicated a strong dependence of δL on the gap and the plate size. Simulations were performed to determine the dependence of δL on these variables. The values of $\delta L/g$ that gave the closest match between the behavioral and the numerical simulations are determined for plate sizes in the range of 10 µm to 100 µm (Figure 4). By a linear fit minimizing the sum of squared errors, $\delta L(d)=g(0.8792+0.01d)$ where d is a variable

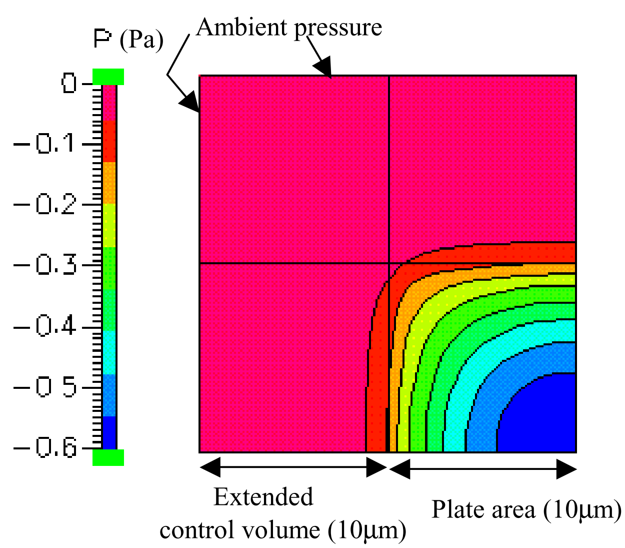

Figure 3: Pressure distribution on the surface of oscillating plate (Quarter plate shown)

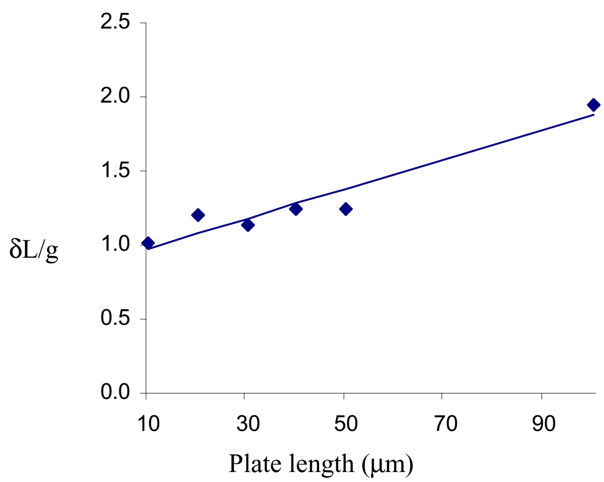

Figure 4: δL/gap values that best match the behavioral and numerical results for different plate sizes

representing *l* or *w* in microns. The relation indicates that δL is increasing with the plate size. This is as expected because larger plates squeeze more air and cause greater pressure perturbation at the plate edges. The pressure settles to ambient pressure further away from the plate edges.

Zhang *et al.* suggested scaling of damping force in case of laterally moving structures to model the edge and the finite-size effects [9]. These fringing effects were more striking in case of plates of smaller dimensions as suggested by the high scaling factor in smaller plates like comb fingers.

4 SIMULATION RESULTS

4.1 Characterization of the Model

Simulation results from the old and the enhanced behavioral models for plate sizes varying from 10 µm to 100 µm are compared to the FEA results. Figure 5 plots the error in the old and the new behavioral models. The new model is *always* more accurate than the old. The error of the old model increases drastically for plate sizes less than 60 µm on a side, and is still about 10% for large plate sizes due to the trivial boundary conditions assumed in its derivation. The enhanced plate-extended model matches the FEA results to within 3% for all the plate sizes in the range of 10 µm to 100 µm.

As a specific example, we present the results obtained by transient analysis using behavioral models for a 20 µm square plate. Figure 6 plots the damping force determined by FEA and the behavior models. Upon extending the plate dimensions, the accuracy of the behavioral model improves from 37% to 3% of the FEA.

4.2 NODAS System Level Simulation

Figure 7 (a) shows the NODAS schematic of a micromechanical resonator. Three such resonators are connected by O-springs to form a bandpass filter [10]. The resonator has lateral comb drive with 5.5 µm thick and 37 µm long fingers (Figure 7 (b)). The behavioral model is used for determining the damping on the rotor of the comb drive. The plot of output voltage of the bandpass filter is shown in Figure 7 (c). The use of enhanced squeeze film model over the old model decreases the output voltage by 8 dB. The dynamic behavior of a system depends on the damping model used. This underscores the significance of an accurate squeeze film damping model for plates as small as the comb fingers.

5 CONCLUSIONS

A lumped parameter squeeze film model containing spring and damper elements is presented. This model incorporates the edge and the finite size effects that tend to dominate MEMS scaled dynamics. The accuracy of the model is within 3% of the FEA for plate sizes from 10 µm to 100 µm. The model can be used in hierarchical behavioral simulation of MEMS devices.

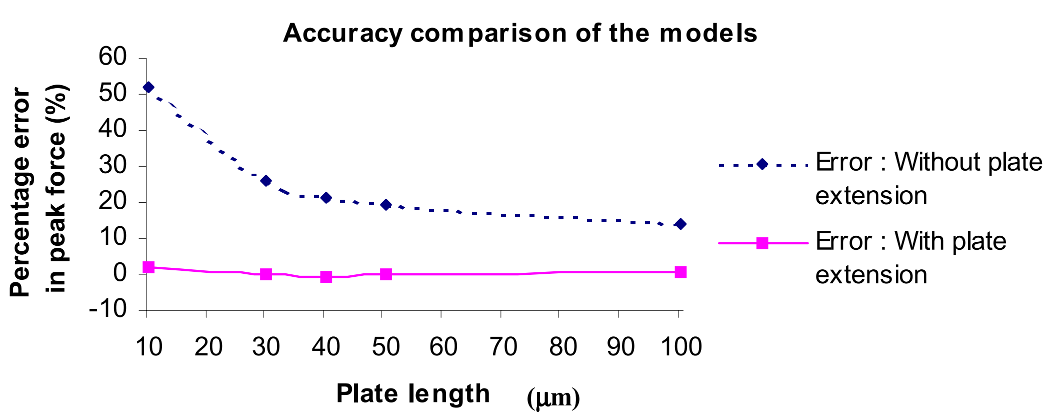

Figure 5: Accuracy of the modified model as compared to the original one for different plate sizes

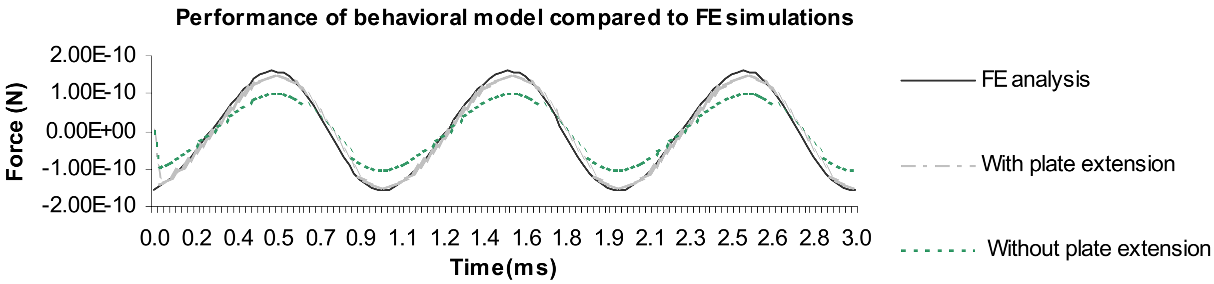

Figure 6: Plots showing the damping force on a 20µm square plate

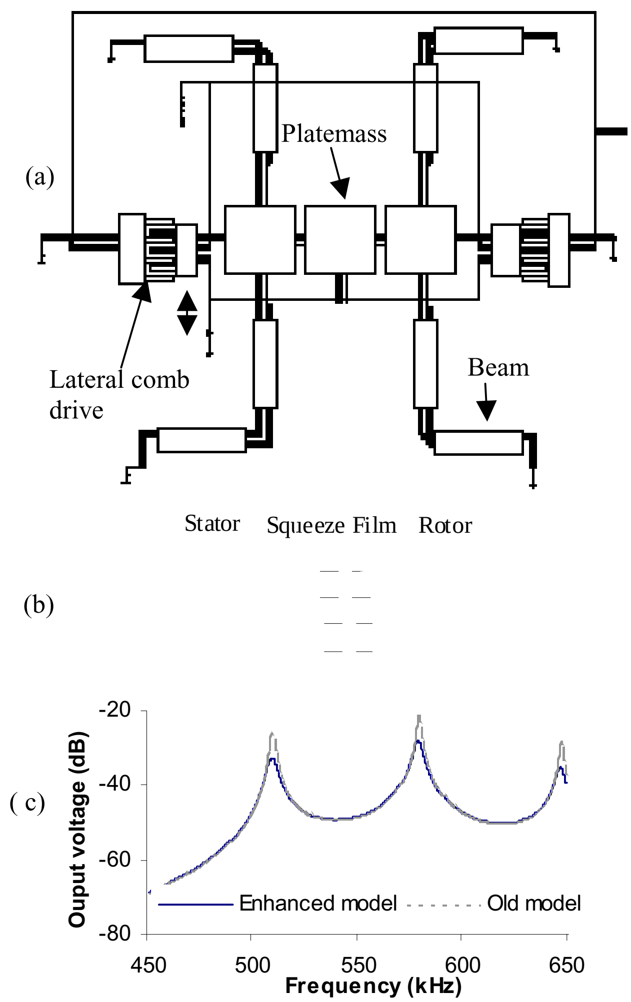

Figure 7 (a): Schematic of microresonator
(b): Squeeze film between comb fingers
(c): Output voltage of a bandpass filter

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