Cluster Computing: A Tool for Looking Before Leaping

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Electricity Transmission in Deregulated Markets: Challenges, Opportunities, and Necessary R&D Agenda

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Abstract:

Much power system behavior goes unstudied because it lies outside the conventional software environment of the industry. Large cascading outages, for example, are rare events and are not considered in reliability investigations. Experience in using a cluster to investigate cascading outages will be reported in which 160,000 major disturbances were produced for a 3000 bus system. Cluster applications for investigation of seams problems and examining the implication of different policies will be presented. The low cost (\$5.2 million) and the speed of Virginia Tech's System X (the third fastest computer in the world as of November 2003) made up of 1100 Mac G5 computers suggests that we can now examine more versions of the future than previously believed possible.

Introduction

There are a number of emerging problems such as:

- Simulation of rare events such as cascading outages,
- Development of wide area measurement and control schemes possibly involving layers of computer agents,
- economic and system problems involving the seams between large ISOs, and
- co-optimization of energy and reserve markets involving large numbers of optimal power flows

that seem to be natural applications of parallel computing. In each case large numbers of studies of realistic systems should be preformed before implementing the suggested changes. The industry, even at the research level, has been somewhat reluctant to embrace cluster computing as a solution to these and other similar problems. Recent developments in affordable terascale computing may be sufficiently impressive to force us to reconsider. This paper will examine a few of the possibilities.

In October 2003 Virginia Tech announced System X, a cluster of 1,100 Power Mac G5s. The original System X was benchmarked at 10.28 teraflops. In November of 2003 it was rated by TOP500 Super Computing Sites as the third fastest supercomputer in the world and the fastest at any academic institution. The price was \$5.2 million approximately one-fifth to one-tenth of the world's fastest machines. In October 2004 Virginia Tech

announced that its rebuilt System X is now operating at 12.25 teraflops. The new system replaced Power Mac G5 desktop computers with Apple's new Xserve G5 in January. The Xserve G5, the most powerful Xserve yet, delivers over 18 gigaflops of peak double-precision processing power per system and features the same revolutionary PowerPC G5, 64-bit processor used in Virginia Tech's original cluster of 1,100 Power Mac G5s. The additional cost to rebuild System X was about \$600,000, and it included 50 additional nodes. In the next section a simulation of major disturbances performed on a cluster in 2000 will be reviewed and the time it would take today on System X given. In subsequent sections some other cluster applications will be described.

Case Studies of Power System Blackouts

Over a long interval, more than 70% of the major disturbances involved relaying systems: not necessarily as the initiating event but contributing to the cascading nature of the event [1,2]. The 1965 Northeast Blackout was initiated by a relay tripping on load current, a number of relays failed to trip in the 1977 NYC (New York City) Blackout, and there were incorrect relay operations in the multiple disturbances in the western US in the Summer of 1996.

The list of causes for the incorrect relay operations in the NERC reports were: Maintenance (42%), Application (29%) (The wrong relay or relaying philosophy used), Setting (10%), and others. An example of the maintenance issue is the fact that one of the relays involved in the 1977 NYC blackout had been maintained a few weeks before the event. The last act in the maintenance involved pushing a contact that was bent in the test and inadvertently left damaged. Many of the incorrect relay operations can be characterized as "hidden failures" in that, since relays are only called upon in unusual circumstances, there can be something wrong in the relay that is hidden until there are faults or heavy loads near the relay. Defects that are so serious that the relay would misoperate immediately when it was returned to service after maintenance are not "hidden". Unfortunately hidden failures seem to account for a number of relay operations involved in major cascading outages. It has been observed that other large systems also have protective systems. The congestion detection and handling mechanism in the TCP protocol could also be thought of as a "traffic breaker" in the Internet. Financial systems actually have "circuit breakers". The human immune system is a very sophisticated protective system, and most of the symptoms of a cold are the results of this protection system. Large chemical plants have protection systems which also occasionally misoperate.

In spite of its importance, the impact of protection system malfunctions on overall system reliability has not been well studied. A part of the problem is that major disturbances are rare. While individual blackouts have been studied in great detail there have been no simulation tools to simulate large numbers of disturbances. For example, there has been no qualitative evaluation of the effect on system reliability of digital relay self-checking and monitoring. The NERC database for the last 15 years has only about 300 events. In addition to studying the few actual blackouts we would like to examine many cascading outages leading to blackouts and create a larger database of simulated disturbances

It must be recognized that the flows in the remaining lines are governed by Kirchhoff's laws, i.e., there is no local control of the current (hence power) flowing in the lines. Once a line is removed by the circuit breakers, new flows will take over. There are many types of hidden failures depending on the relaying scheme (there are a large number of schemes). A disturbance can be described by a sample path made up of a sequence of hidden failure trips and correct trips due to overloads with resulting load and generation shedding.

The probability of a hidden failure, p, is taken as being independent for the sole purpose of above illustration. More generally, each line will have a different flow-dependent probability of tripping incorrectly [3-6]. The line is more likely to trip because of a hidden failure if the line is heavily loaded. The probabilities are small enough that multiple hidden failure trips at one branch almost never happen. The sample path is approximately one-dimensional spreading in the system like a crack rather than two dimensionally like a forest fire.

Rare-Event Simulation Techniques

One way to simulate rare events is to use the importance sampling technique. In importance sampling, basically, rather than using the actual probabilities, the simulation uses altered probabilities (usually much bigger than actual ones) so that the rare events occur more frequently [7,8]. The resulting statistics are then modified appropriately. Although importance sampling can significantly speed up the rare-event simulations, it still spends most of the computation resources in generating repeated samples to maintain the unbiased probability estimation. Another algorithm, referred to as Heuristic Random Search [5,6], was devised to search the important sample paths more efficiently. Each disturbance corresponds to a sample path made up of hidden failure trips and trips due to overloads. This algorithm concentrated on tracking only important sample paths and reducing the repetitions in the simulated samples as much as possible. One measurement of the importance of a sample path is the product of blackout size and probability. Bigger number indicates more significant disturbance. The algorithm used a mixing of random walk and greedy search based on the observation that power system disturbances spread in a one-dimensional fashion, i.e., more than one hidden failures are seldom triggered at the same time. The algorithm was a DFS (Depth First Search)-like search. Basically the search is Depth-First, but at each branch point the sub-path with higher probability will be explored first. And the search will stop if the abovementioned measurement is too small (the actual number is system-dependent). Once it stops, no matter whether or not it reaches the leaf of the "blackout" tree, the search will restart from the root and try to find another "unvisited" important sample path. The procedure is repeated until the rightmost of the "blackout" tree is reached (suppose the search starts from the leftmost).

Modified Hidden Failure Mechanism

Hidden failure mechanism is essential to the simulation of power system blackouts. The previous hidden failure model [3,4] can be improved with a small modification. Suppose

a line is exposed multiple times during a cascading event. One would expect that, if relay misoperation occurs, it would be more likely to occur on the first exposure than in subsequent exposures. However, the previous version of the model allowed relay misoperation with equal probability on all the line exposures. The improved model reduces or simply zeros the probability of misoperation after the first exposure.

Simulation Procedure

The simulation procedure of power system cascading disturbances is summarized here, details can be found in [3-6]. Briefly, the simulation begins by randomly choosing an initial line trip. This action exposes all lines connected to the ends of the initial line and also may overload lines. If one line flow exceeds its preset limit then the line is tripped. Otherwise, the hidden failure mechanism is applied to let the chosen exposed line trip. After each line trip, the line flow is recalculated and checked for violations in line limits. The process is repeated until the cascading event stops. As a final step, an optimal distribution of generation and load is calculated. The above simulation is repeated over an ensemble of randomly selected transmission lines as the initiating fault locations.

A specific study was made [5,6] of a New York Power Pool (NYPP) 3000-bus system including a sequence of full AC load flows simulating cascading disturbances based on a model of hidden failures in malfunctioning relays. The system included load shedding, transmission line limits, generator's VAR limit, remote controlled buses, phase shift transformers, and switched shunt elements. Heuristic Random Search and distributed computing were used to achieve computational efficiency. The simulation was performed on Cornell Velocity Cluster and employed 60 Pentium III 500MHz processors for 10 hours. More than 160,000 sample paths were generated and reduced to about 40,000 with the largest expected power loss. A new load flow was performed at each step along the sample path resulting in more than a million total load flows for the 3000 bus system. Results showed that hidden failures in relays at some locations were more prone to triggering cascading disturbances than elsewhere. The number of times a given line appeared in the 40,000 sample paths or the expected energy loss in the sample paths with that line being involved can be used as performance indices. It was possible to identify locations where limited resources should be expended to reduce the probability of hidden failures. Since actual hidden failure probabilities are not known the improvement was taken to be a reduction in hidden failure probability by 50%. The optimization found the ten locations where a reduction in hidden failure probability by 50% would have the greatest impact.

Comparision with System X

The archives at http://www.top500.org/ give the Top 500 list from 1993 to the present with data about the machines on the list. The cluster used for the previous study was rated at 47.39 GFlops for Rmax in the Linpack benchmark. There were 256 processors in the cluster although we used only 60 in the result given above. The time required to find the 160,000 blackout sample paths using all the processors in System X would be 32.9 seconds rather than the 10 hours in 2000.

Wide Area Monitoring and Control

Various wide area monitoring and control schemes involving advanced communication and computer agents have been discussed. In this context, an agent is a self-contained piece of software that has the properties of autonomy and interaction. A possible research area is the use of agent-based backup protection system. Several levels of agents have been proposed to make a more resilient power system. In order to make this distributed systems work effectively, the relays must be capable of autonomously interacting with each other. An agent makes decisions without the direct intervention of outside entities. This flexibility and autonomy adds reliability to the protection system because any given agent-based relay can continue to work properly despite failures in other parts of the protection system. It is certainly necessary to explore the expected communication traffic patterns in order to make agents more intelligent and robust towards network conditions. By linking the most widely used computer network simulator NS2 to an accurate power systems simulation engine EMTDC/PSCAD, a family of new power grid protection and monitoring algorithms can be investigated. For at least one protection scheme simulation reveals a surprising issue: in TCP-based power systems communication networks, relay protection algorithms may malfunction if TCP's congestion-control mechanisms are triggered (e.g. because of competing network traffic) [9-11]. Using a federation of NS2 with a power systems simulation engine such as EMTDC or PSCAD detailed simulation of the power system and the Utility Intranet can be studied. Cluster computing makes it possible to investigate many scenarios simultaneously.

Energy, Reserves, and Seams

The term "seam" has come into common use recently in the restructuring electric industry to refer to a boundary between neighboring control areas or power markets. Seams issues may be characterized as the difficulties of conducting power transactions across the boundary due to differences in operating rules and market designs, as well as differences in business practices [12]. They include diverse matters such as different bidding rules, different pricing mechanisms, inconsistent transaction submittal times, or different operating procedures. Even apparently small differences in rules can create seams problems. In 2002, the Northeastern Independent Market Operators Coordinating Committee [13] was formed among three ISOs (NYISO, ISO-NE, and IMO) to work towards solutions of a host of seams and market standardization issues.

Three Northeastern ISOs have struggled to coordinate their rules to lower trading barriers but have only achieved limited success after several years [14]. It requires the coordination of net exchange between neighboring ISOs and involves the coordination of the energy flow and payments between ISOs as well as the coordination of ancillary service requirements and characterizations needed for grid management. Market standardization issues are beyond the scope of this paper. Instead, the interchange coordination is our focus here, assuming no trading barriers exist.

Most of the existing LMP based markets currently utilize proxy bus mechanisms to represent and value inter-regional exchanges. Simply put, the proxy bus models the location at which *marginal* changes in generation are assumed to occur in response to

changes in inter-regional transactions. Nevertheless, as has been pointed out [15], a proxy bus system can fail to produce the optimal level of net interchange and may jeopardize the overall efficiency of the market if the number and location of the proxy buses are not appropriately chosen.

In essence, the ultimate goal of coordinating interchange between regions is trying to achieve the overall system optimum while preserving independent optimal dispatch for each of the connected regions. This has much in common with decomposition approaches for solving large-scale optimization problems. Kim and Baldick pioneered the related work for the large-scale distributed OPF [16-18], in which the overall OPF problem was decomposed into several regions through an iterative update on constraint Lagrange multipliers. Although their focus was on the implementations for the parallel OPF computing, their results have implications for market coordination. Another relevant work is [19], in which Hogan et al. proposed a similar decomposition approach while in the market setting to tackle the transmission loading relief problem across multiple regions. Even though their work is not directly towards seams issues, their experience gives reason for optimism for resolving seams coordination via the same regional decomposition approach.

An ultimate goal would be to coordinate energy as well as ancillary services, in particular, spinning reserves and VAR support, across the seams. The proposed market has a joint market structure based on a co-optimization that can simultaneously optimize energy and ancillary services. The same decomposition principle as in [16-19] is applied to coordinate the inter-regional exchanges. The LMP for energy is derived in the cooptimization setting.

Optimization Framework and Decomposition Schemes

The coordination of energy and ancillary services is based on a co-optimization (CO-OPT) framework first introduced in [20-21]. A quick review of the formulation for the overall system is provided in the following section. Then the decomposition approach is conceptually illustrated step by step through simple examples. The regional decomposition for a multi-area OPF is introduced first, followed by the system decomposition approach for multiple system cases in a single-region CO-OPT setting, and finally we present the way to decompose the combined system — multi-area multi-system CO-OPT.

The CO-OPT framework [20-21] is utilized to optimize energy and spinning reserves simultaneously. In brief, the CO-OPT is to minimize the total expected cost over the predefined base case and credible contingencies,

$$\min_{P, R} \sum_{k=0}^{K} p_k \left\{ \sum_{i=1}^{I} \left[C_{P_i}(P_{ik}) + C_{R_i}(R_{ik}) \right] \right\}$$
 (1)

where there are K predefined contingencies $(k = 0, 1, \dots, K)$, 0 indicates the base case (intact system), p_k is the probability of the k^{th} contingency, P_{ik}/Q_{ik} are the real/reactive power output of generator i in the k^{th} contingency, R_{ik} is the spinning reserve carried by

generator i in the k^{th} contingency, θ_{jk} is the voltage angle of bus j in the k^{th} contingency, V_{jk} is the voltage magnitude of bus j in the k^{th} contingency, S_{lk} is the power flow of line l in the k^{th} contingency, P_l^{\min} , P_l^{\max} is the minimum and maximum real power capacity for generator i, Q_i^{\min} , Q_i^{\max} is the minimum and maximum reactive power capacity for generator i, R_i^{\max} is the maximum reserve for generator i, V_j^{\min} , V_j^{\max} is the voltage magnitude limits for bus j, S_l^{\max} is the power flow limit for line l, $C_{P_i}(P_{ik})$ is the energy cost for operating generator i at output level P_{ik} in the k^{th} contingency, and $C_{R_i}(R_{ik})$ is the reserve cost for generator i carrying R_{ik} spinning reserve in the k^{th} contingency,

The minimization is subject to network and system constraints enforced by each of the base case and contingencies. These constraints include nodal power balancing constraints,

$$F_{ik}(\theta, V, P, Q) = 0, \qquad j = 1, \dots, J \qquad k = 0, \dots, K$$
(2)

line power flow constraints (detailed formulations for (2) and (3) are referred to [11]),

$$|S_{lk}| \le S_l^{\text{max}}, \qquad l = 1, \dots, L \quad k = 0, \dots, K$$
 (3)

voltage limits

$$V_j^{\min} \le V_{jk} \le V_j^{\max}, \quad j = 1, \dots, J \qquad k = 0, \dots, K$$

$$\tag{4}$$

generation limits

$$P_i^{\min} \le P_{ik} \le P_i^{\max}$$

$$Q_i^{\min} \le Q_{ik} \le Q_i^{\max}, \quad i = 1, \dots, I \quad k = 0, \dots, K$$
(5)

spinning reserve ramping limits

$$0 \le R_{ik} \le R_i^{\text{max}}, \qquad i = 1, \dots, I \quad k = 0, \dots, K$$
(6)

and unit capacity limits

$$P_{ik} + R_{ik} \le P_i^{\text{max}}, \qquad i = 1, \dots, I \quad k = 0, \dots, K$$

$$(7)$$

The concept of *Total Unit Committed Capacity (TUCC)* is introduced to build connections between the base case and contingencies. In particular, the TUCC of unit i in the kth contingency is defined as

$$G_{ik} = P_{ik} + R_{ik}$$
, $i = 1, \dots, I$ $k = 0, \dots, K$ (8)

As indicated in [20], the TUCC for any generator i is required to be the same over all K+1 cases, thereby is denoted as G_i^{max} , and the following holds,

$$G_{ik_1} = G_{ik_2} = G_i^{\text{max}}, \quad i = 1, \dots, I \quad k_1, k_2 = 0, \dots, K$$
 (9)

Regional Distributed OPF

The purpose is to carefully decompose the overall OPF problem into geographical regions by introducing "dummy" variables at the border buses that mimic the effects of the external part of the system, and introducing constraints that they be equal at adjacent regions. By solving the optimal power flows for each region and coordinating them through iterative updates on the constrained Lagrange multipliers, the algorithm is shown to converge to a solution of the full OPF problem [16-18].

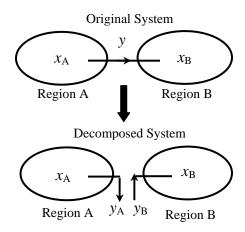


Figure 1. Example of regional OPF decomposition

To illustrate the regional decomposition, consider Fig.1 where a power system consists of two regions (region A and region B) connected by one single tie-line. The variables within each region are denoted by x_A and x_B respectively, which are real and reactive power-flows through the buses and the voltages and phase angles at the buses. At the border, for the tie-line between the regions, a "dummy" bus is created and the associated variables for this bus are donated by y.

The approach relies on decomposing the overall problem into regions by duplicating the border variables and imposing coupling constraints between the two copies. Hence, the "dummy" variables associated with region A are y_A while those associated with region B are y_B , and the coupling constraint is $y_A = y_B$.

Among the four duplicated *boundary variables* (voltage magnitudes, phase angles, real and reactive powers), the duplication of phase angles deserves more attention. The reason is that, in the overall OPF problem there is only one global reference, however, there are as many references generated as the number of the subsystems in the decomposed

problems. These references have to be assumed the right values against the global reference to achieve convergence. One possible way is that altering each reference at each iterate in a manner such that the average of the phase angles at the border buses of a region equals the corresponding ones in adjacent regions.

<u>Distributed Co-optimization</u>

The above described decomposition scheme applies for the CO-OPT also. Nevertheless, there are no physical "tie-lines" between the base case and contingencies. The "tie" here is the TUCC for each of the generators. Take Fig.2 as an example, for the simple matter, which assumes the CO-OPT is formulated for one single-region power system with the base case and only one contingency case. The non-physical "ties" are represented by the dashed lines in the graph.

The state variables for the base case (case #0) and the contingency case (case #1) are denoted by x_0 and x_1 respectively. The tie variables, TUCC, are denoted by y. And the two copies y_0 and y_1 are assigned to each of the two cases with the coupling constraint $y_0 = y_1$ for the purpose of decomposition.

Hence, the overall CO-OPT formulation and the decomposition iterative scheme for this example can be written as Equations (10) ~ (14) with only subscripts changes. The subscripts '0' and '1' replace the subscripts 'A' and 'B' respectively.

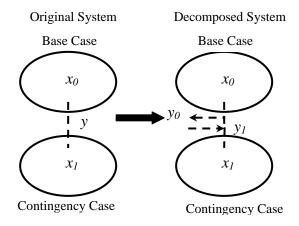


Figure 2. Example of the CO-OPT decomposition

Distributed Co-optimization Across Seams

Our ultimate goal is to do *Distributed Co-optimization Across Seams* (DCAS), by which the energy and ancillary services can be coordinated simultaneously between multiple regions. Apparently, the same decomposition concept works. What we need to do is to bundle up regional distributed OPF and distributed CO-OPT in the multiple-region setting. The iterations are not only over the collection of border "dummy" variables between multiple areas but also over the collection of generator TUCCs between the base case and all predefined contingencies. Fig. 3 depicts the simplest possible situation where

a power system consists of two regions with only one contingency to worry about. The state variables are denoted by x_{0A} , x_{0B} , x_{1A} and x_{1B} for two areas and two cases respectively. y_0 and y_1 represent the physical "ties" between regions, while z_A and z_B represent the non-physical "ties" between cases.

The overall CO-OPT then can be decomposed into four independent sub-problems for each of the two regions in each of the two system conditions. The use of eight processors to compute an OPF for a 1777 bus system as reported in [17]. A speed up of a factor of five was reported. Similar results can be expected for real system. Some number of partitions of the physical systems in the base case will result is a faster OPF. The gain in speed is system and partition dependent and will certainly not be linear in the number of processors. Let us assume 25 processors are involved in the parallel solution of the base case OPF. Then using a 1100 processor cluster we can consider 44 contingencies simultaneously as in Figure 3. More contingencies would mean fewer processors could be devoted to individual OPFs.

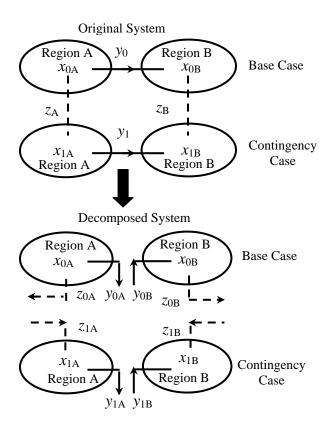


Figure 3. Example of the CO-OPT across seams

Conclusions

We have examined a few situations where simulation and/or optimization that are more computer intensive than is usually found in the study of power systems would be desirable. To produce large numbers of realistic cascading outages in order to determine what parts of the system are most likely to be involved, to examine the actual

communication requirements down to the level of the protocols in a multi-level - multi-agent protection and control scheme, to determine the optimum operation of a combined energy and reserve market across seams are all possible with affordable terascale cluster computation. The increase in performance is considerably beyond that expected by Moore's Law if price is a consideration.

References

- [1] North American Electric Reliability Council Disturbances Analysis Working Group Database. [Online]. Available: http://www.nerc.com/dawg/database.html
- [2] Tamronglak, S.; Horowitz, S.H.; Phadke, A.G.; and Thorp, J.S, "Anatomy of power system blackouts: preventive relaying strategies" IEEE Transactions on Power Delivery, 11, 1996, pp708-715
- [3] Bae, K. and Thorp, J.S, "An importance sampling application: 179 bus WSCC system under voltage based hidden failures and relay misoperations", Proceedings of the 31st Hawaii International Conference on System Sciences, Jan. 1998, pp39-46
- [4] Bae, K. and Thorp, J.S., "A Stochastic Study of Hidden Failures in Power System Protection", Decision Support Systems, 24, 1999, pp 259-268
- [5] Wang, H. and Thorp, J.S., "Enhancing Reliability of Power Protection Systems Economically in the Post-Restructuring Era", Proceedings of the 32nd North American Power Symposium (NAPS), 2000
- [6] Wang, H. and Thorp, J. S., "Optimal Locations for Protection System Enhancement: A Simulation of Cascading Outages", IEEE Transactions on Power Delivery, 16, 2001, pp 528-534
- [7] Bucklew, J.A., Large Deviation Techniques in Decision, Simulation, and Estimation. John Wiley and Sons, Inc. 1990
- [8] Thorp, J.S. and Phadke, A.G. "Anatomy of Power System Disturbances: Importance Sampling", Proceedings of the 12th Power Systems Computational Conference (PSCC), Dresden, Germany, Aug. 1996
- [9] Wang, X.R.; Hopkinson, K.M.; Thorp, J.S.; Giovanini, R.; Birman, K.P.; and Coury, D.V., "Developing an Agent-based Backup Protection System for Transmission Networks". Submitted to the Conference on Power Systems and Communication Systems Infrastructures for the Future, Beijing, China, September 2002
- [10] Coury, D.V.; Thorp, J.S.; Hopkinson, K.M.; and Birman, K.P., "An Agent-based Current Differential Relay for use with a Utility Intranet", IEEE Transactions on Power Delivery, Vol. 17, No 1, 2002, pp 47-53.

- [11] Coury, D.V.; Thorp, J.S.; Hopkinson, K.M.; and Birman, K.P., "Improving the Protection of EHV Teed Feeders Using Local Agents", IEEE Seventh International Conference on Developments in Power System Protection, 9-12 April 2001, Amsterdam, The Netherlands
- [12] Seams Issues High Priority Items, [online]: http:// www.isomou.com/working_groups/business_practices/documents/general/bpwg_matrix.pdf
- [13] IMO of Ontario, ISO-NE and NYISO Complete Planning and Market Development Agreement, [online]: http://www.theimo.com/imoweb/pubs/pressReleases/pr_NYISO-ISONE-IMO_20020717.pdf
- [14] Northeast ISOs Seams Resolution Report: History of Seam Issues Resolution, [online]: http://www.iso-ne.com/FERC/seams/NESeamsDetListing011503.pdf
- [15] S.Harvey, "Proxy Buses, Seams and Markets [Draft]", May, 23, 2003, [online]: http://www.ksg.harvard.edu/hepg/Papers/ Harvey_Proxy.Buses.Seams.Markets_5-23-03.pdf
- [16] B.H. Kim and R. Baldick, "Coarse-Grained Distributed Optimal Power Flow", *IEEE Trans. On Power Systems*, v12 n2, May, 1997, p932-939
- [17] R. Baldick, B.H. Kim, C. Chase, and Y. Luo, "A Fast Distributed Implementation of Optimal Power Flow", *IEEE Trans. On Power Systems*, v14 n3, Aug. 1999, p858-864
- [18] B.H. Kim and R. Baldick, "A Comparison of Distributed Optimal Power Flow Algorithms", *IEEE Trans. On Power Systems*, v15 n2, May, 2000, p599-604
- [19] M.D. Cadwalader, S.M. Harvey, S.L. Pope and W.W. Hogan, "Market Coordination of Transmission Loading Relief Across Multiple Regions", Dec. 1998, [online]: http://ksghome.harvard.edu/~.whogan.cbg.Ksg/extr1298.pdf
- [20] J.Chen, T.D. Mount, J.S. Thorp, R.J. Thomas, "Location-based Scheduling and Pricing for Energy and Reserves: A Responsive Reserve Market Proposal", submitted to special issue on Competitive Electricity Market of Journal of Decision Support Systems
- [21] J.Chen, J.S. Thorp, R.J. Thomas, and T.D. Mount, "Locational Pricing and Scheduling for An Integrated Energy-Reserve Market", 36th Hawaii International Conference on System Science, Hawaii, Jan. 2003