Semidefinite Programming for Power System State Estimation

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Motivation

- **Based on real-time measurements, Static State Estimation serves as the foundation for monitoring and controlling the power grid.**

❖ *Classical method:* For AC power system, state estimation is usually formalized mathematically as a Weighted Least Square or Weighted Least Absolute Value problem, and solved by Newton's method.

❖ *Problem:* Highly sensitive to the initial point, as it is essentially a local search algorithm.

❖ *New approach:* Employ Semidefinite Programming (SDP) to effectively obtain a good initial state.

Preliminaries of State Estimation

- **Goal:**
  - To determine the most likely state of the system based on the quantities that are measured. [1]
- **Model:** $z = h(x) + u$
  - h : Nonlinear functions, relating state and measurements
  - State(x): Voltage magnitudes and phase angles
  - Measurements (z) and noise (u)
- **Optimization:**

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}} \quad J_p(\mathbf{x}) \\ &\text{subject to} \\ &P' = P(\mathbf{x}) + \mathbf{u}_p \qquad Q' = Q(\mathbf{x}) + \mathbf{u}_q \\ &P'_f = P_f(\mathbf{x}) + \mathbf{u}_{pf} \quad Q'_f = Q_f(\mathbf{x}) + \mathbf{u}_{qf} \\ &P'_t = P_t(\mathbf{x}) + \mathbf{u}_{pt} \quad Q'_t = Q_t(\mathbf{x}) + \mathbf{u}_{qt} \\ &|V_k|' = |V_k| + \mathbf{u}_v \end{aligned}$$

⇕

$$\min_{\mathbf{x}} J_p(\mathbf{x}) = \left(\sum_{i=1}^{m} \left| \frac{z_i - h_i(\mathbf{x})}{\sigma_i} \right|^p \right)^{\frac{1}{p}}$$

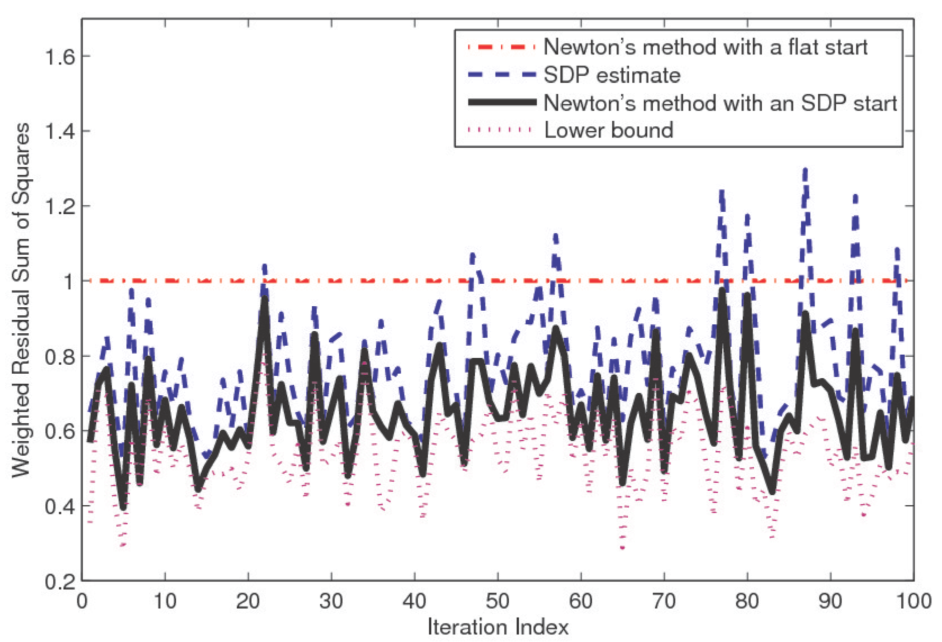
⇕

$$\min_{W} J_p(W) = \left(\sum_{i=1}^{m} \left| \frac{z_i - tr(A_i W)}{\sigma_i} \right|^p \right)^{\frac{1}{p}}$$
subject to $\quad W \succeq 0, \quad \text{rank}(W) = 1.$

Numerical Result

- **Simulation for IEEE 14 bus (SDP initial guess v.s. Flat start)**
  - The state: 14 voltage magnitudes and 14 phase angles
  - The measurements:
    - Voltage magnitudes
    - Active and reactive power flow on the lines
    - Active and reactive power injection at buses
  - 100 random sample sets
- **Objective Comparison**

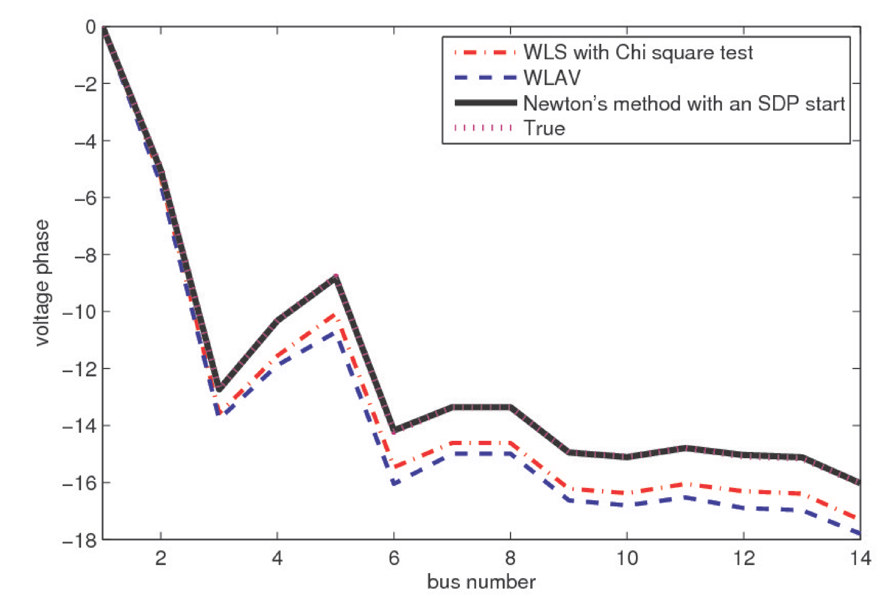
$$\text{WRSS} = \sum_{i=1}^{m} \left(\frac{z_i - tr(A_i W)}{\sigma_i} \right)^2$$



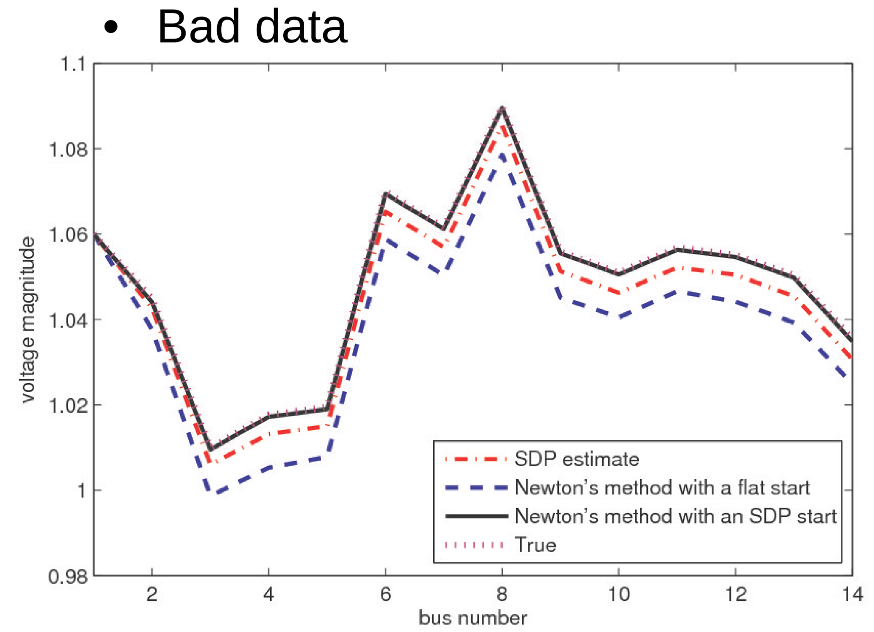
- Figure. Performance comparison for the 14 bus case.

(A lower bound is also computed from relaxed problem's objective)

- **State Comparison**
  - Gaussian Noise



  - Bad data



New Approach

- Reformulate the state in another form. [3]
- Relax the non-convex constraint to convexify the problem.
- Use the "relaxed" estimate as a new initial guess for Newton's method

$$U = (Re(\mathbf{x})^T, Im(\mathbf{x})^T)^T \;\overset{\text{help}}{\dashrightarrow}\; P_k = Re\{V_k^H I_k\}$$
$$= Re\{V^H e_k e_k^T Y V\}$$
$$\triangleq Re\{V^H Y_k V\}$$
$$\widetilde{Y}_k = \frac{1}{2}\begin{pmatrix} Re(Y_k + Y_k^T) & Im(Y_k^T - Y_k) \\ Im(Y_k - Y_k^T) & Re(Y_k^T + Y_k) \end{pmatrix} \dashrightarrow = U^T \widetilde{Y}_k U$$
$$= tr(\widetilde{Y}_k U U^T)$$
$$W = UU^T \;\overset{\text{help}}{\dashrightarrow}\; = tr(\widetilde{Y}_k W)$$

- **Constraint relaxation:**

$$\min_{W} J_p(W) = \left(\sum_{i=1}^{m} \left| \frac{z_i - tr(A_i W)}{\sigma_i} \right|^p \right)^{\frac{1}{p}}$$
subject to $\quad W \succeq 0.$

⇓

$$\hat{W} = \sum_{i=1}^{p} \lambda_i g_i g_i^T$$

- **State Recovery:**

$$\hat{\mathbf{x}} = \hat{U}_{1:n} + j\hat{U}_{n+1:2n}$$

- **Use it as initial guess for Newton's method**

- **Reference**
  - [1] A. Abur and A. G. Exposito, "Power System State Estimation: Theory and Implementation." Marcel Dekker Inc, 2004.
  - [2] A. Monticelli, State Estimation in Electric Power Systems, A Generalized Approach., 1999.
  - [3] J. Lavaei and S. H. Low, "Zero duality gap in optimal power flow problem," Technical Report, California Institute of Technology, vol. PWRD-1, no. 3, pp. 355–360, Aug. 2010.

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