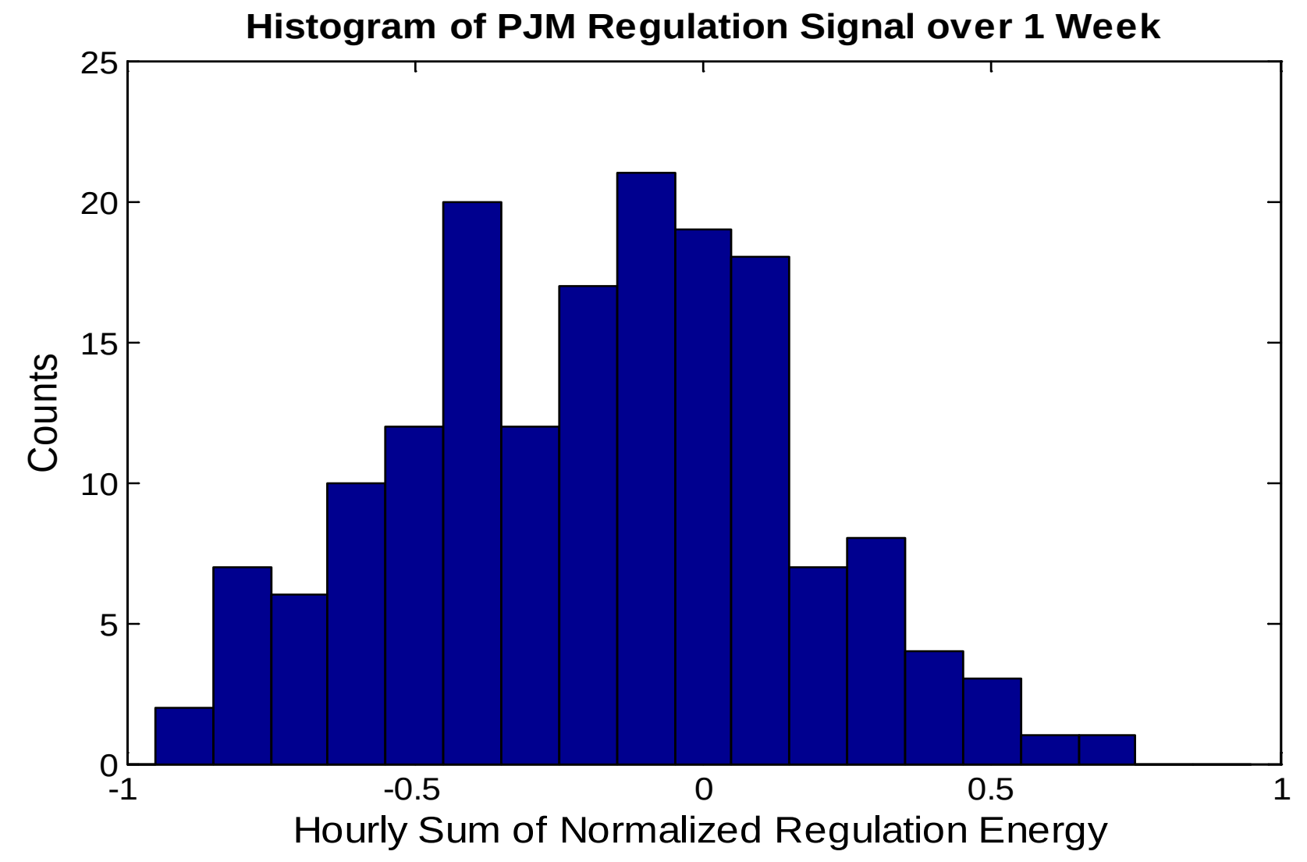
Carnegie Mellon
Electric Energy Systems Group
Electrical & Computer ENGINEERING

Convex Hull Stochastic Dynamic Programming Applied to Electric Vehicle Charging and Frequency Regulation Bids

Jonathan Donadee and Marija Ilić
jdonadee@andrew.cmu.edu, milic@ece.cmu.edu
Electric Energy Systems Group, Electrical and Computer Engineering

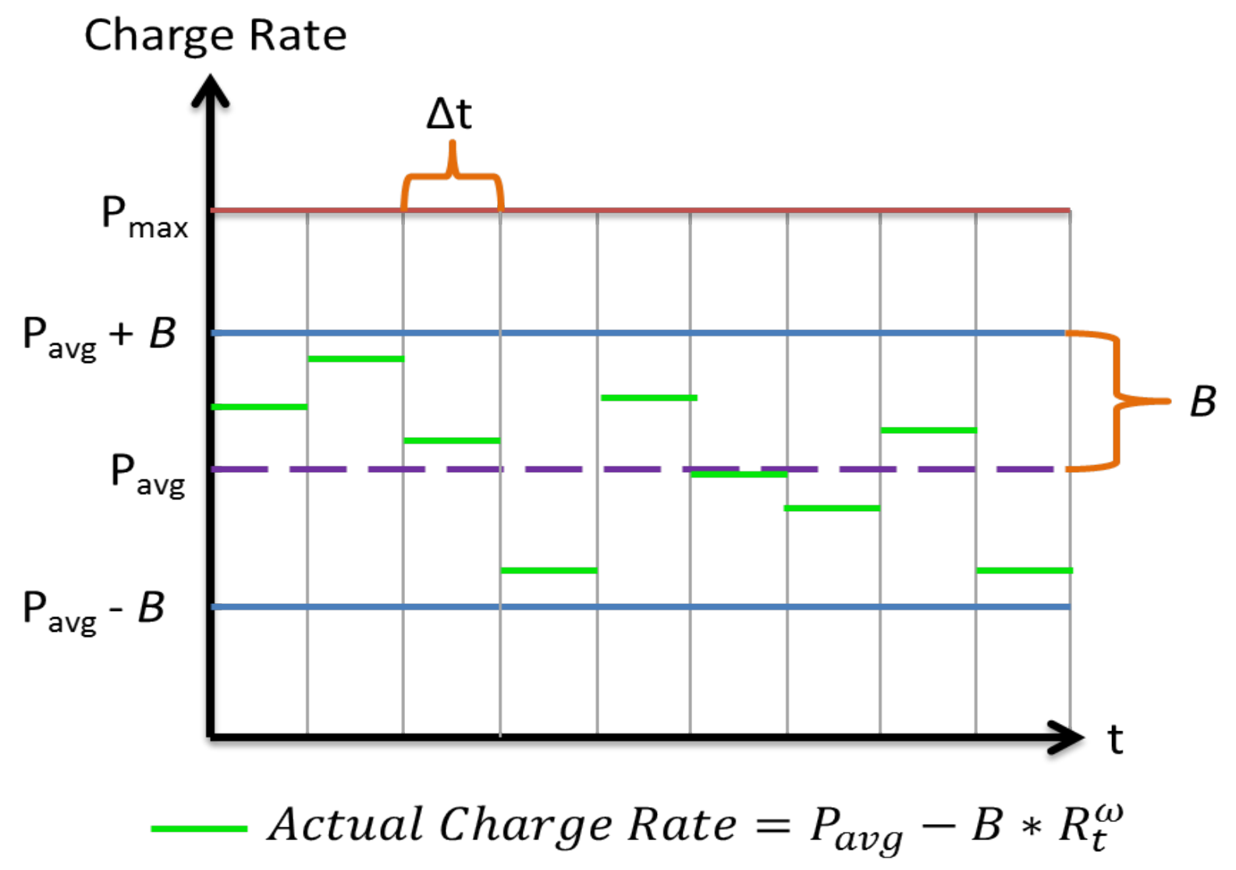
Background and Contributions

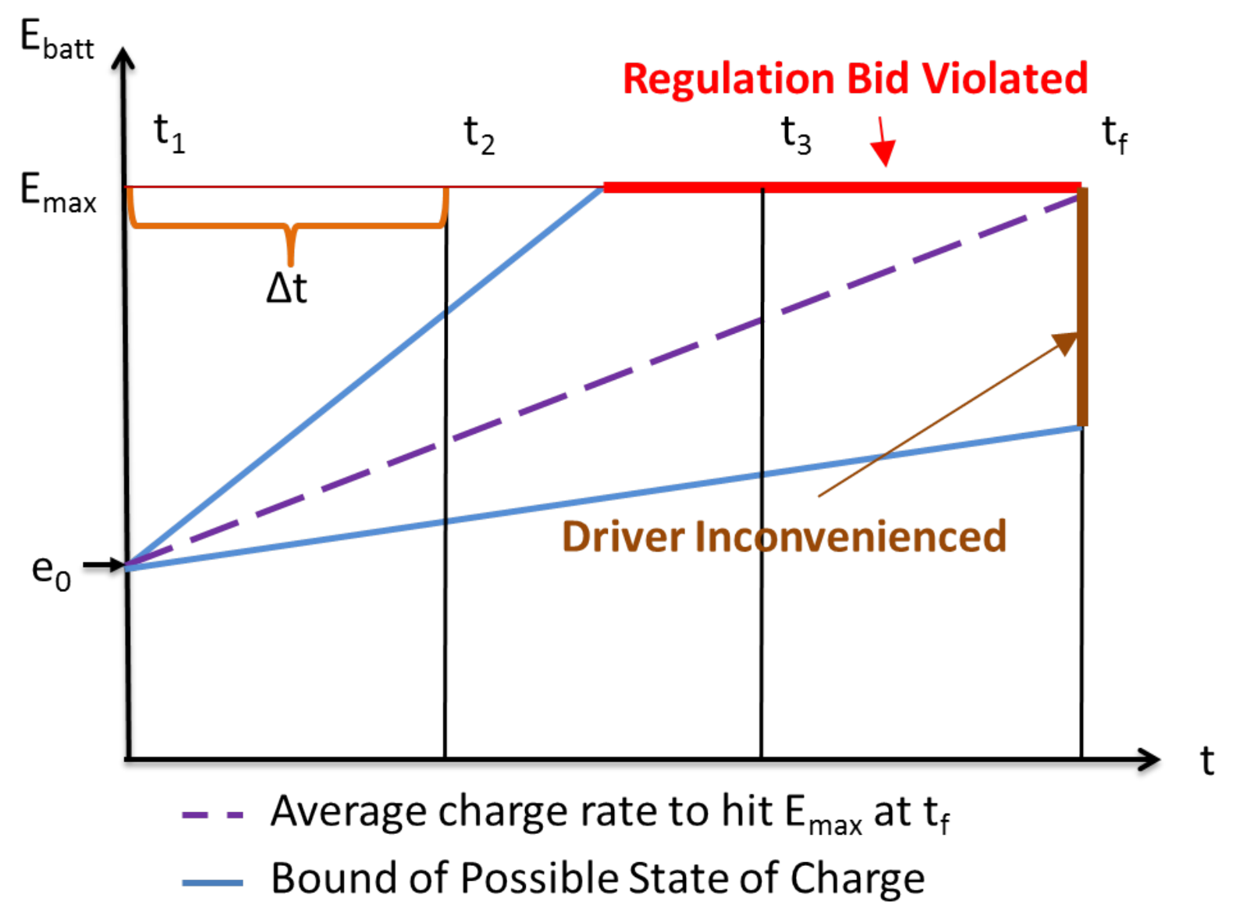
- **Smart Charging of Electric Vehicles allows demand side participation in Energy and Ancillary Services Markets**
  - Intermittent renewable generation requires more ancillary services
  - EV owners want to minimize charging costs
  - EVs can provide Secondary Frequency Regulation by adjusting EV charge rate to follow control signal
- **Existing Deterministic Optimization Methods Ignore Regulation Signal Effect on State of Charge**
  - Assume integrated signal energy is zero over one hour, or use expected value
    - Bad Assumption



- **A Stochastic Method is Needed to Consider the Distribution of the Regulation Signal**
- **Contributions:**
  - **First** tractable stochastic model of EVs providing regulation
  - **First** to model pro-rated contract penalties

Providing Regulation by Charging Only





- P_{avg}- average charge rate
- B – Regulation Bid (identical up and down)
- $B \leq P_{avg}$ and $B \leq P_{max} - P_{avg}$

- **State of charge takes a random walk**
  - Battery might not be charged by desired time:
    - Inconvenience cost can be given by EV owner ($/hr late)
  - Battery might reach capacity before regulation contract expires, breaking contract:
    - pro-rated penalty is a function of time, not energy
    - Binary indicator variables ($\boldsymbol{I_t}$) indicate if contract was violated in each sub-hourly timestep
- **Must solve for optimal average charge rate and regulation bids**

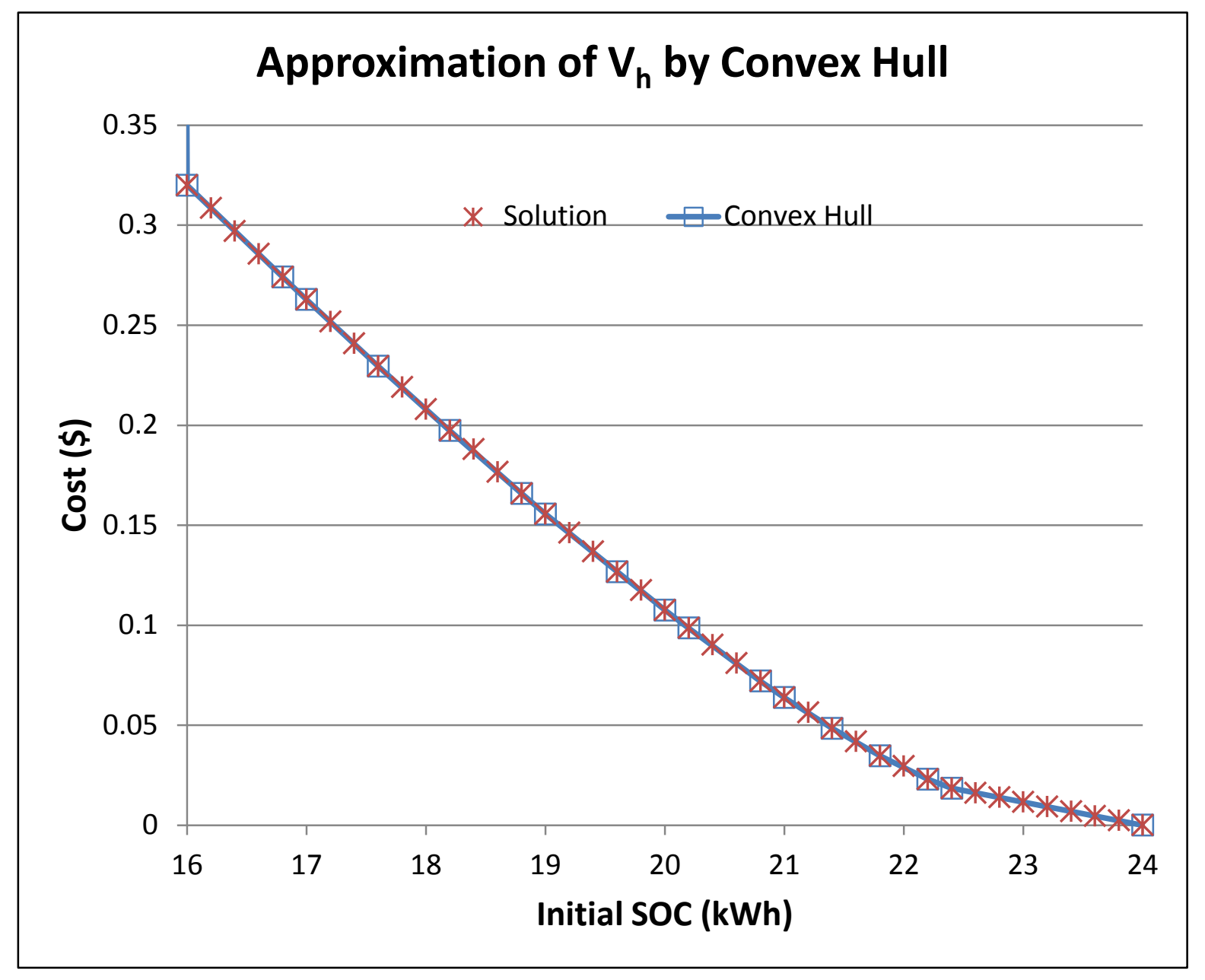
Solution Algorithm

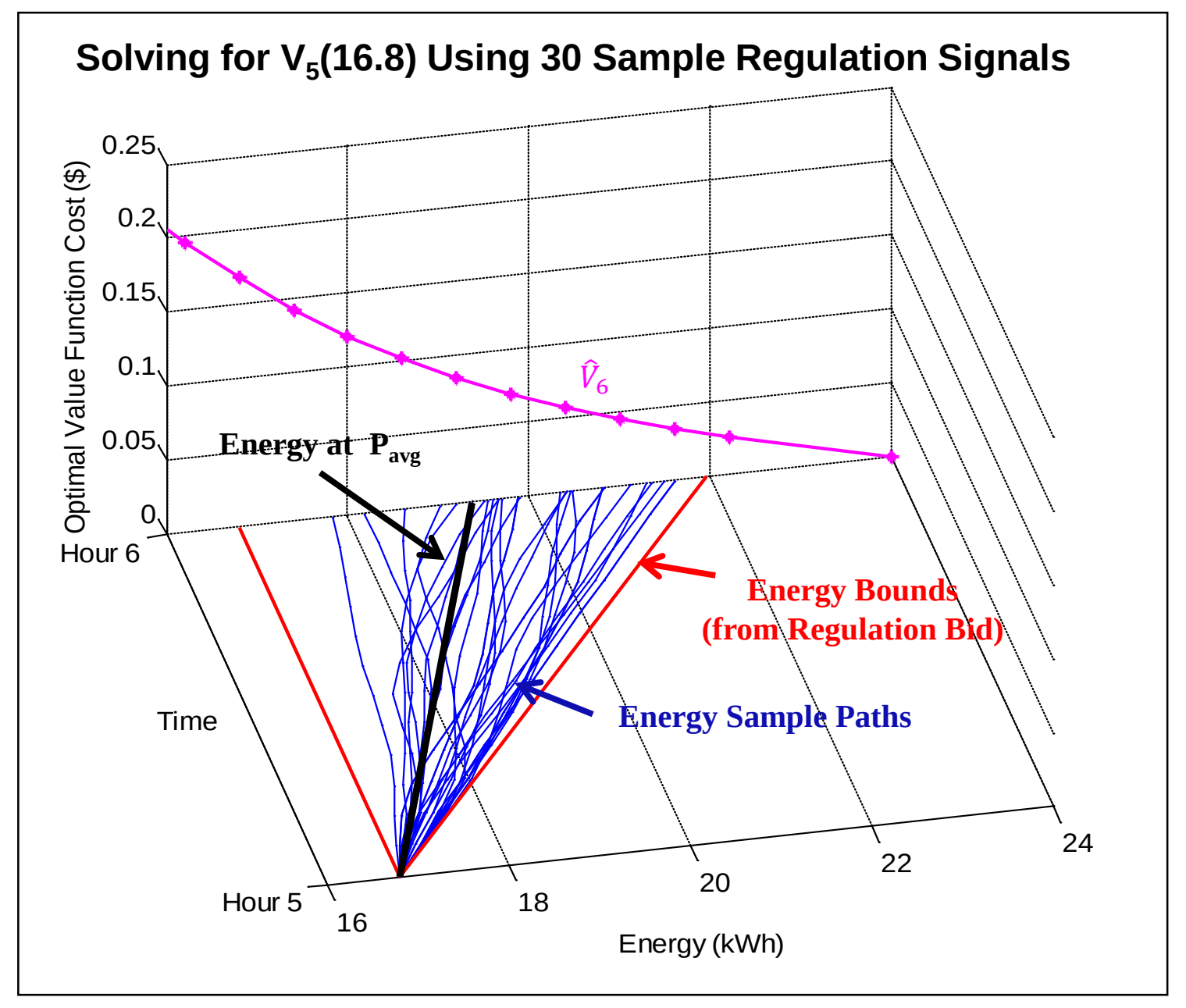
- **Convex Hull Stochastic Dynamic Programming**
  - **Use a piecewise-linear, function to approximate the optimal value function**
    - avoids using more integer variables in backwards recursion
  - **Optimal Value Function – $V_h(E_h)$**
    - Def: Expected cost of making optimal decisions from stage ***h*** through the end of stage ***H*** given that the state of charge is $\boldsymbol{E_h}$ at the start of hour ***h***

1. ***Discretize the feasible state space at the final decision time, $E_{batt}(H)$, into N points***
2. ***For i= 1, …, N solve***

$$V_H\left(E_{batt\,i,H}\right) = \min_{P_{i,H},B_{i,H}} \mathbb{E}_{\omega}\left[C_H\left(E_{batt\,i,H}, P_{i,H}, B_{i,H}, R_H^{\omega}\right) + V_{H+1}\left(E_{batt\,i,H+1}\right)\right]$$

3. ***Approximate $V_H(\cdot)$ with $\hat{V}_H(\cdot)$, a piecewise-linear function on the convex hull of the points $V_H\left(E_{batt\,i,H}\right)$***





3. ***For h= H-1, H-2, …, 1***

   For i = 1, …, N solve

$$V_h\left(E_{batt\,i,h}\right) = \min_{P_{i,h},B_{i,h}} \mathbb{E}_{\omega}\left[C_h\left(E_{batt\,i,h}, P_{i,h}, B_{i,h}, R_h^{\omega}\right) + V_{h+1}\left(E_{batt\,i,h+1}\right)\right]$$

- $\boldsymbol{V_h}$ is a two stage deterministic equivalent stochastic MILP with 30 sample regulation signals, R_H^{ω}

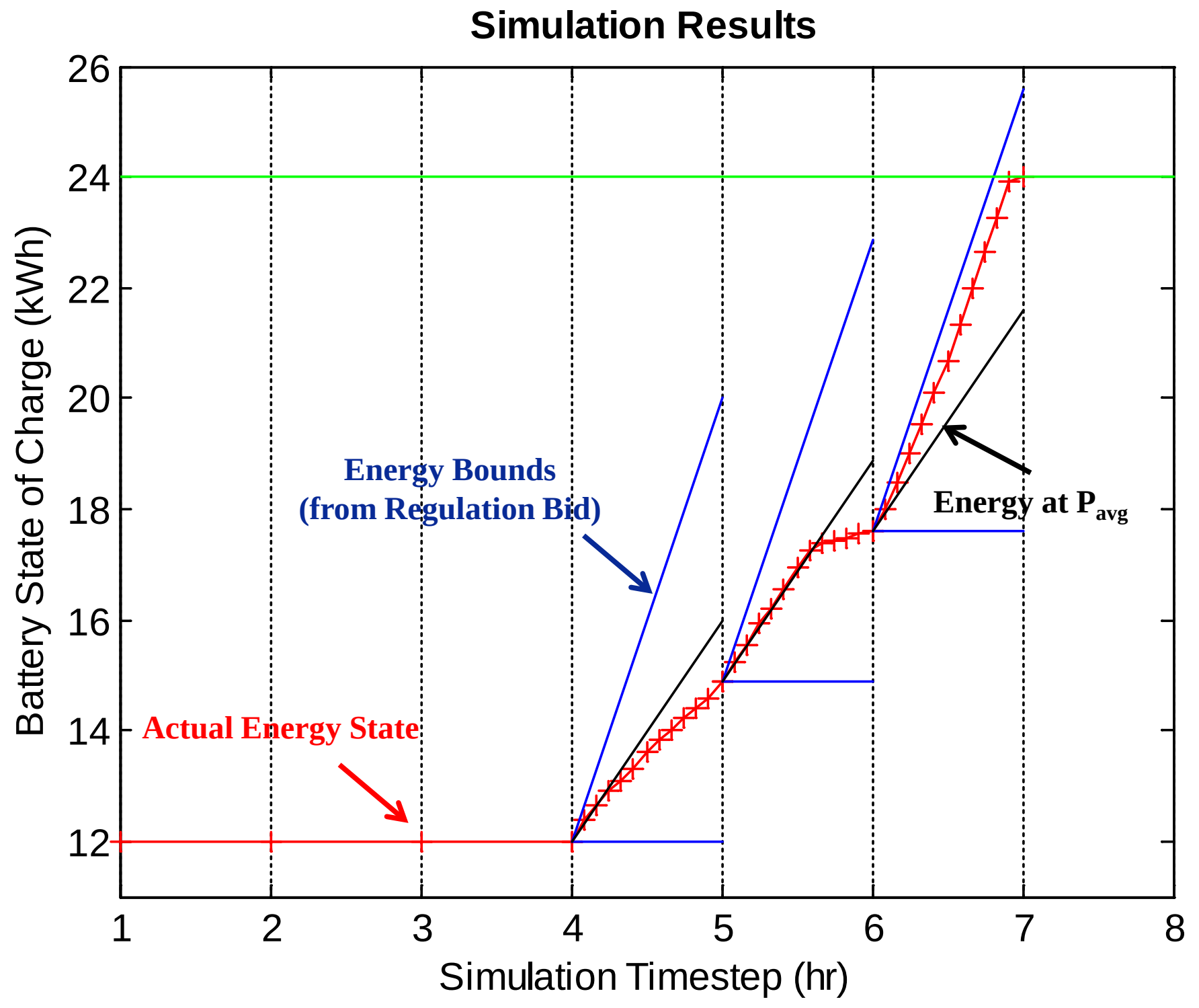
A. $\boldsymbol{C_h}$ -Stage ***h*** costs include
   i. Energy purchase costs and regulation revenues
   ii. Adjustments to energy purchase cost caused by following the regulation signal or reaching E_{max}
   iii. Pro-rated penalty for violating regulation contract for each sample

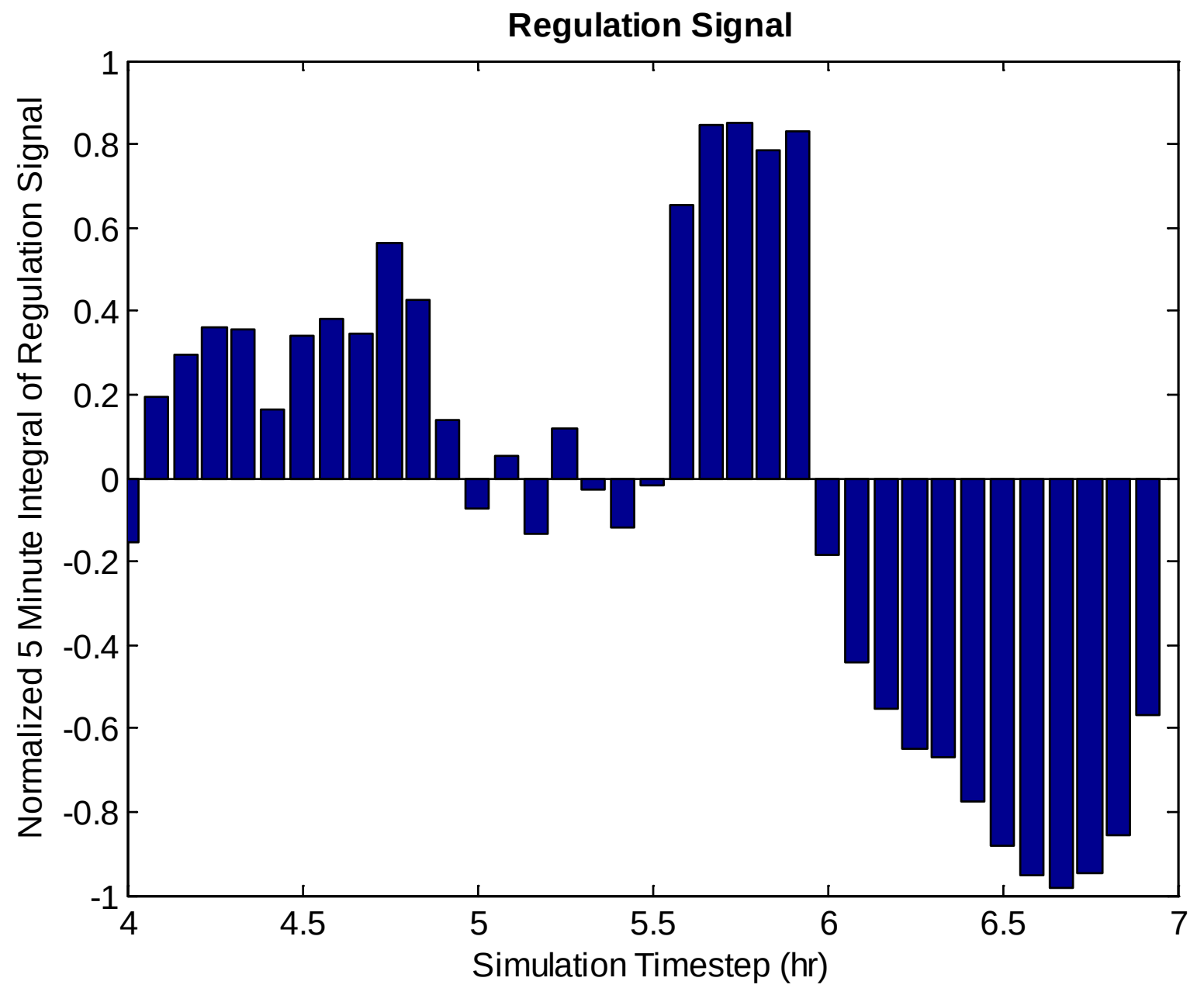
$$\text{Penalty}_j = \text{Q}\, c_r \Delta t\ \text{B} * \sum_t I_t^{\omega j}$$

B. $\boldsymbol{V_{H+1}}$- cost function of the battery state of charge at scheduled vehicle unplug time
   A. Includes an Inconvenience penalty for the remaining time to reach E_{max} by charging at P_{max}

Simulation

- **Setup**
  - Data:1 vehicle , P_{max}=8kW, E_0=12 kWh, Emax=24kWh, PJM DA prices (12/1/2011), Plug in at midnight, Unplug at 7am
  - 21 point state space discretization
- **Problem Size: 1593 variables, 390 binaries, 3,784 equations**
  - Solved 133 times in algorithm
- **Solver: GAMS w/ XPRESS on Intel 6 core 3.2Ghz cpu**
- **Results**
  - total simulation time ~3min
  - ~75% of pts on the convex hull





Optimal Decisions

Hour	1	2	3	4	5	6	7
B	0	0	0	4	4	4	0
P_{avg}	0	0	0	4	4	4	0

Conclusions and Future Work

- **Conclusions**
  - Driver inconvenience is almost always avoided with penalty of $20/hr
    - P_{avg} is biased to finish early in last hour
  - Regulation contract is almost always broken in the last hour, sometimes earlier
  - Convex Hull is a good approximation of optimal value function
  - If P_{avg}>4 kWh, then P_{avg}+B=8, else B=P_{avg}
- **Future Work**
  - Understand convexity properties of the value function
  - Uncertain energy and regulation prices, adds more states
  - Compare with other methodologies
  - Multiple vehicle bid aggregation
  - Apply method to other storage devices (Stationary Batteries, Flywheels)