Identification of complex deterministic behavior in power systems

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(hello Carnegie Mellon!)

Complexity in power systems?

- Power systems are networks
- Engineered interdependencies
- Potential for nonlinear dynamics
- Becoming more and more adaptive/ autonomous

Complexity and stability/control

- Complex behavior tends to be highly unpredictable
  - *chaos, and intermittency*
- Such behaviors have been shown to lead to instability and failures in power systems
  - *voltage collapse, angle divergence, etc..*
- Control decisions may be affected by the system being in a "complex state"
  - *centralized vs. decentralized control*
  - *isolating components*

Benefits to a time series based approach..

- **No assumptions made about the system**

  -applicable to all dynamical components

  -do not need a probability distribution function

- **Mathematically simple algorithm, which does not require many data points**

  -fast computation

Why entropy is not enough to measure complexity

- **Entropy is maximal for completely disordered (random) systems**

  -randomness is not complexity

  -high entropy DOES NOT IMPLY complexity

- **In the presence of random noise, entropy increases**

  -the presence of noise does not indicate complexity

Time delay embedding

-For a sequence of measurements

$$X = \{x_1, x_2, \ldots, x_N\}$$

We construct vectors in an *m*-dimensional space, so that the vector components are elements of the sequence *X*, as follows:

$$\vec{X}_n = \langle x_n, x_{n+\tau}, x_{n+2\tau}, \ldots, x_{n+(m-1)\tau} \rangle$$

Where τ is called the ***time delay***.

Time delay embedding

-For instance, if we let τ =1, and *m*=3,

$$\vec{X}_1 = \langle x_1, x_2, x_3 \rangle, \ \ \vec{X}_2 = \langle x_2, x_3, x_4 \rangle, \ldots$$

Whereas, if τ=2, and *m*=3,

$$\vec{X}_1 = \langle x_1, x_3, x_5 \rangle, \ \ \vec{X}_2 = \langle x_2, x_4, x_6 \rangle, \ldots$$

The total number of *m*-dimensional vectors able to be made from N data points is N_0=(N-(m-1)τ)

Embedded data as state space reconstruction

- State space structure is a geometry of behavior

-simple behavior ⇔ simple geometry

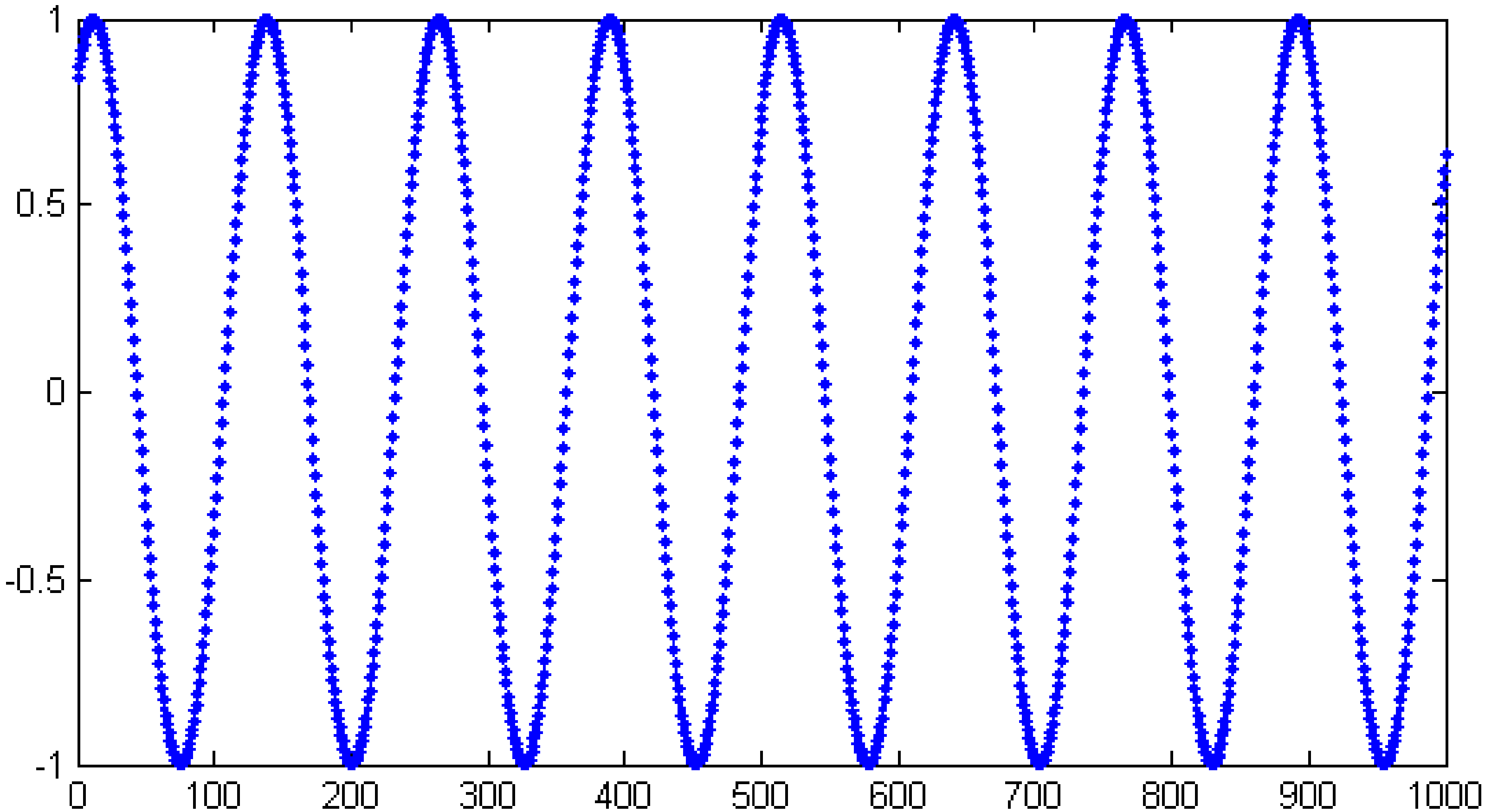
-disordered behavior ⇔ disordered geometry

-”complex behavior” ⇔ “complex geometry”

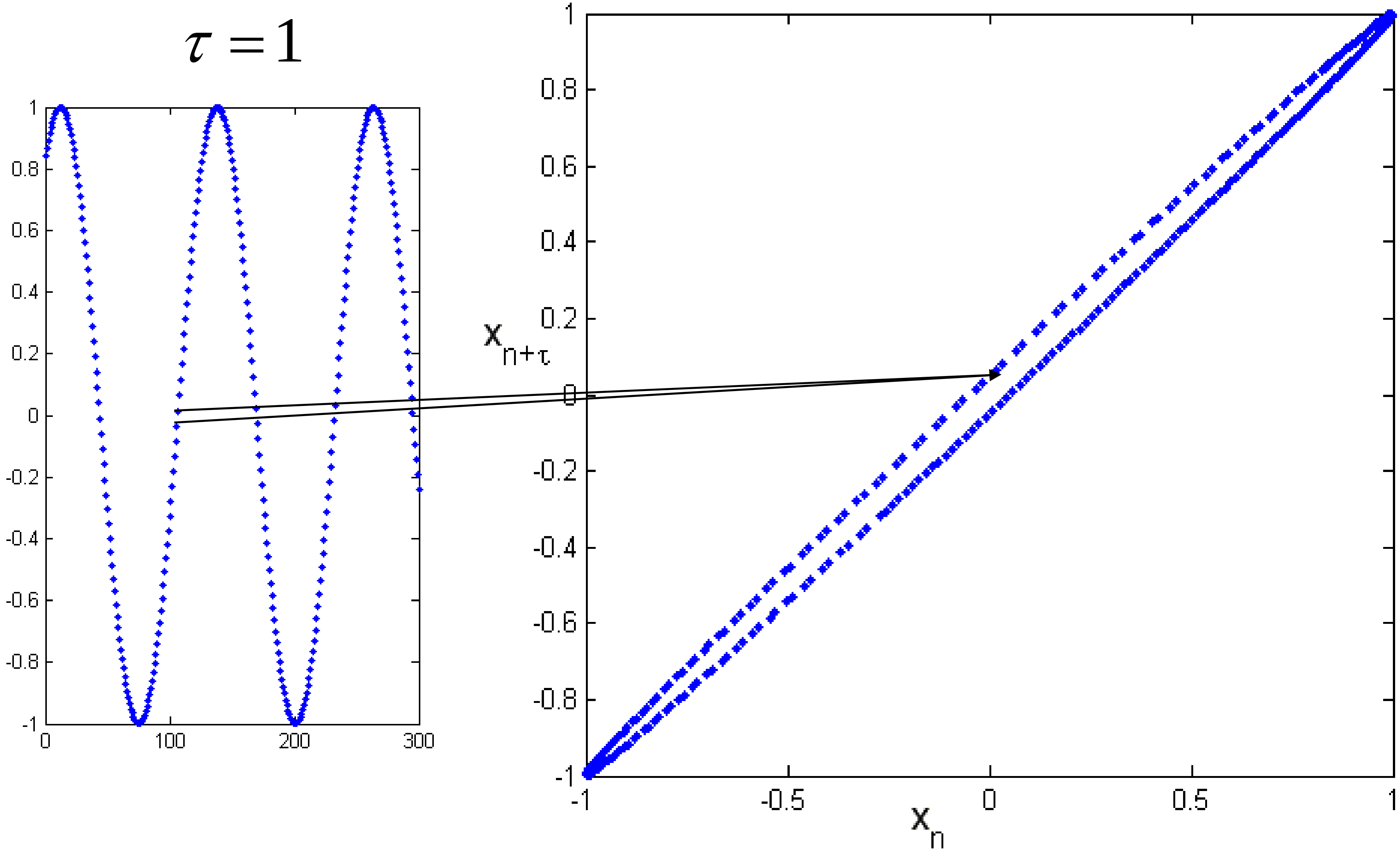
- With this in mind, we wish to quantify the disorder of geometric structures in state space

1st example

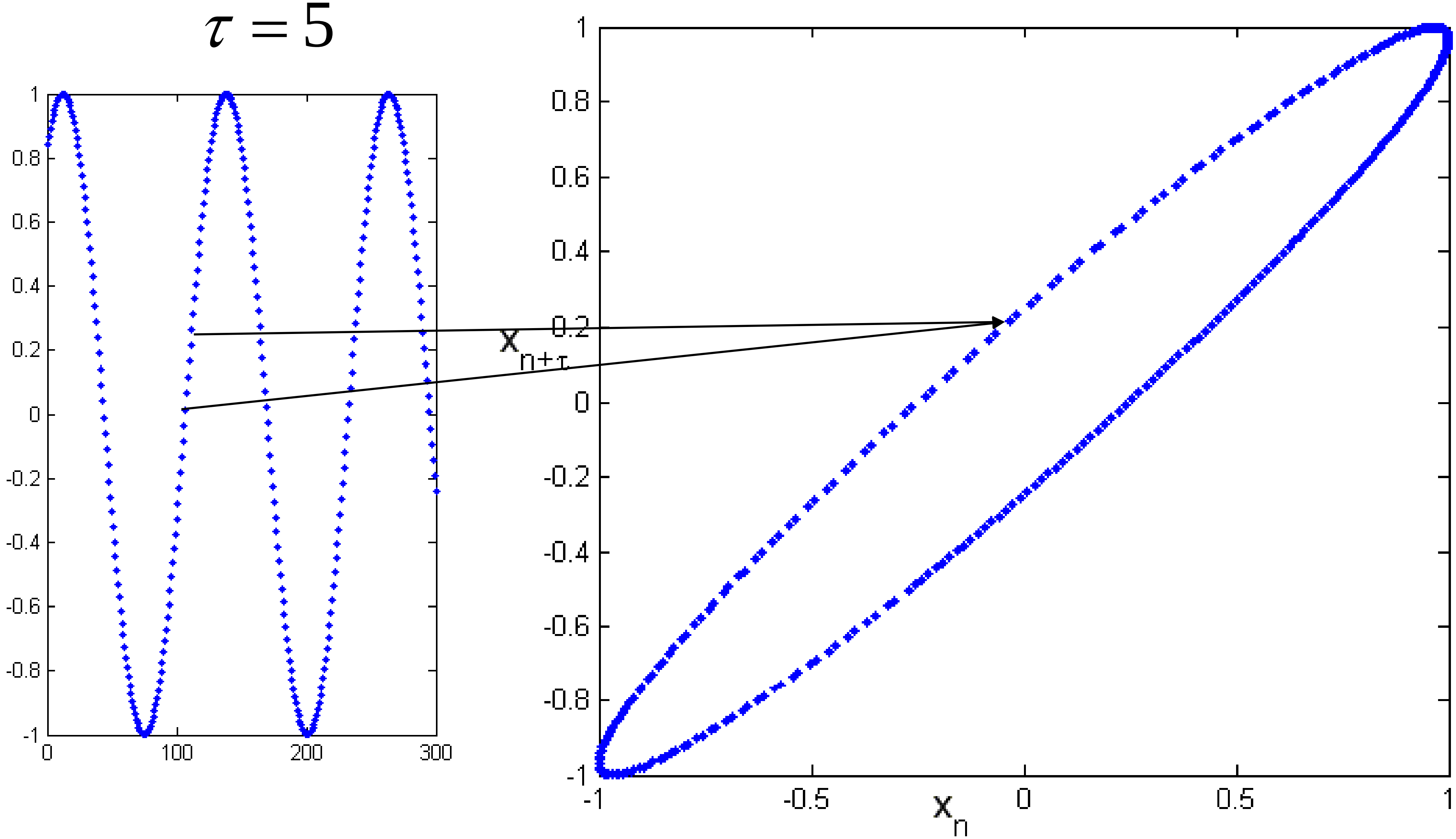
Periodic signal…



$\tau = 1$

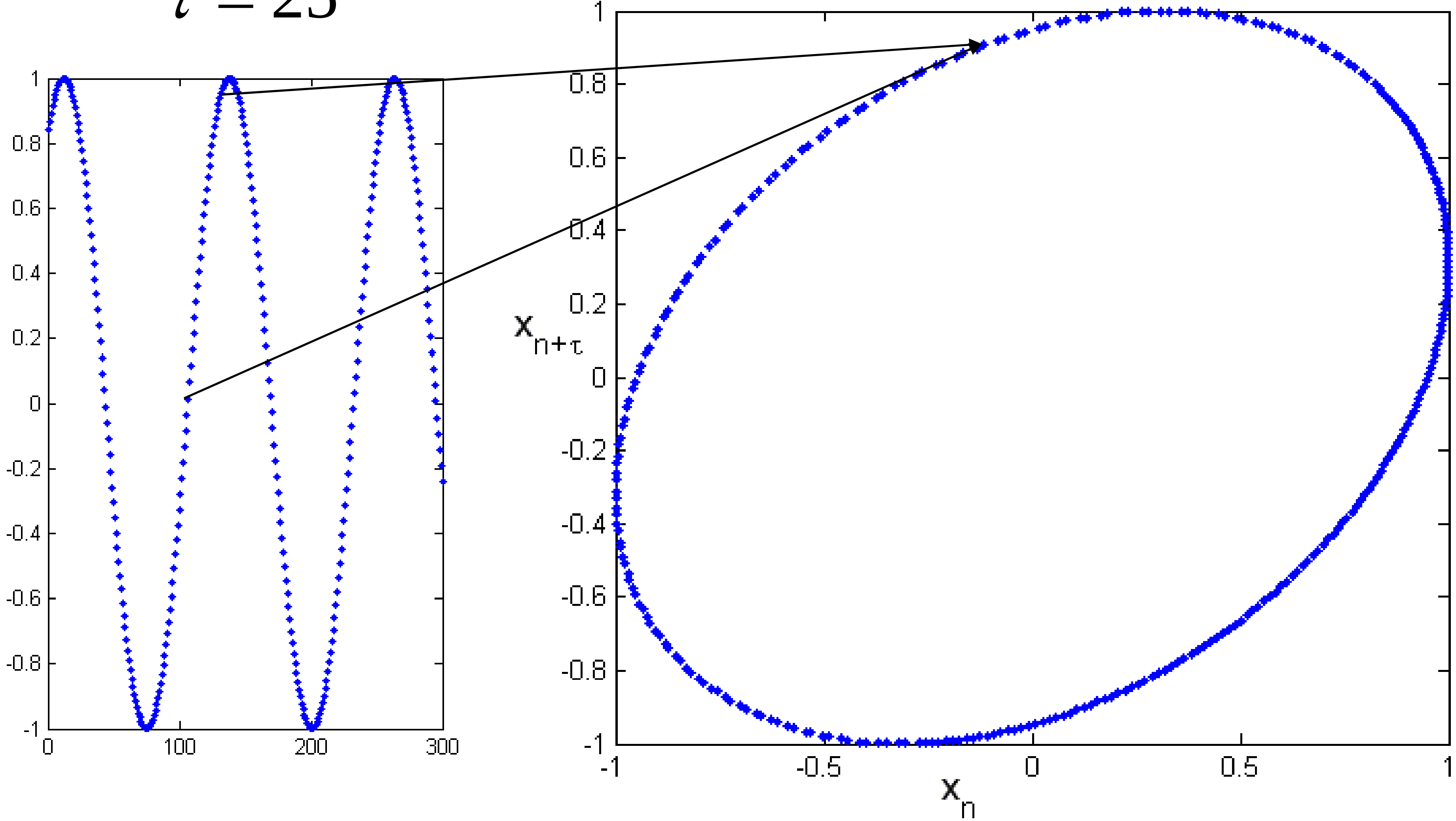


$\tau = 5$

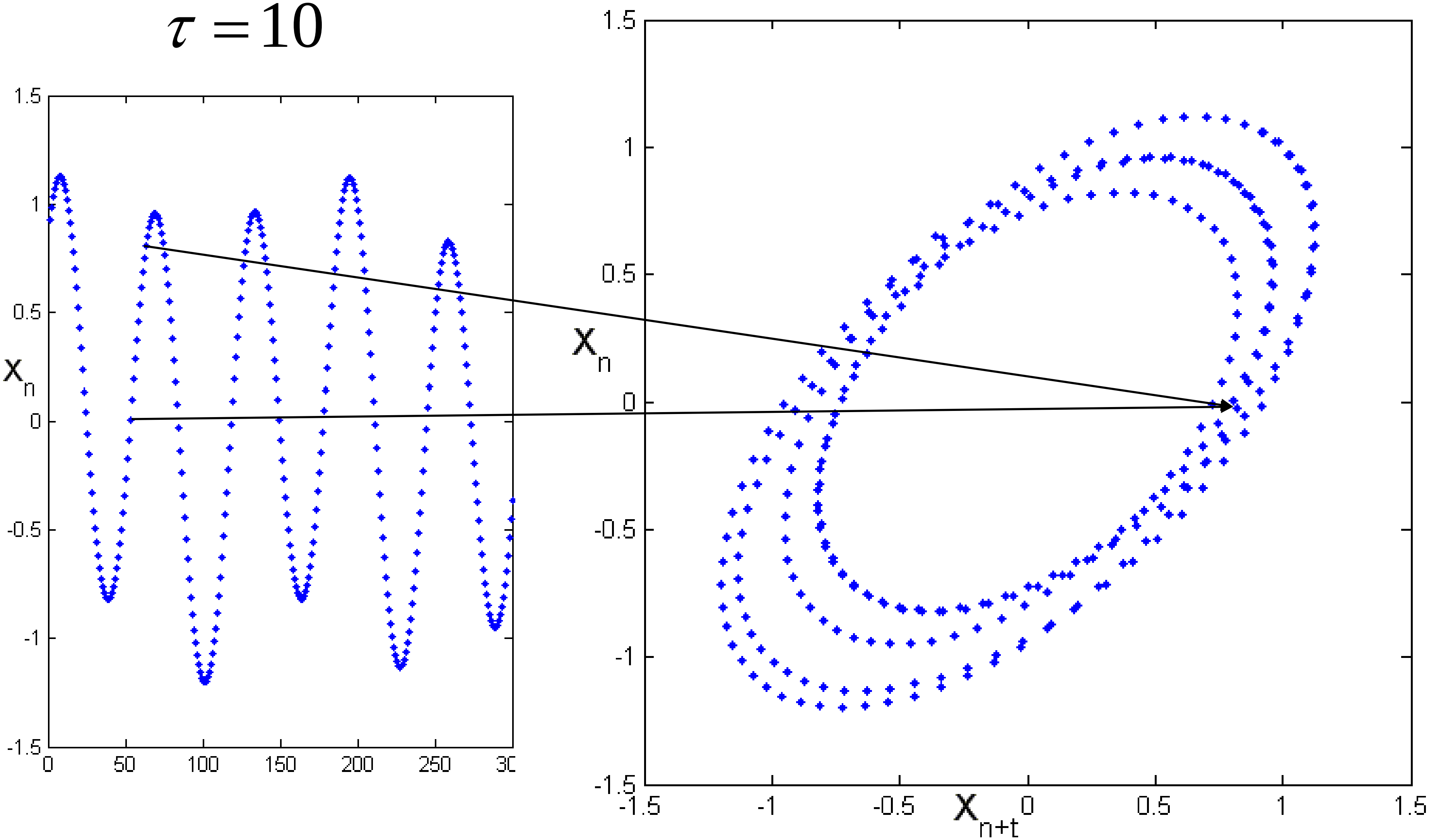


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
100
200
300
x n+τ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1
x n

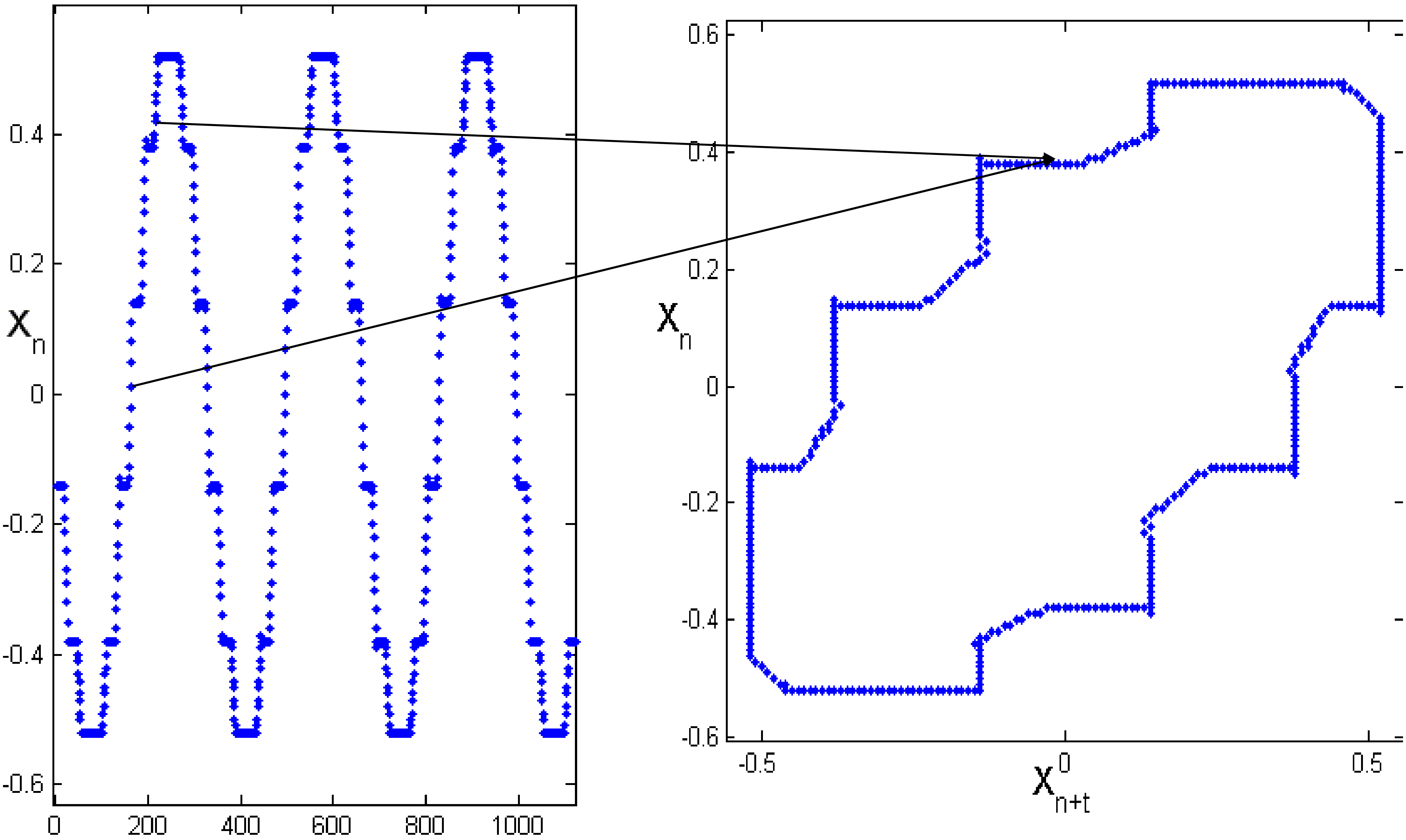
$\tau = 25$



$\tau = 10$


1.5
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
30
x_n
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
X_n
X_n+t

$\tau = 50$



0.4
0.2
X n
0
-0.2
-0.4
-0.6
0
200
400
600
800
1000
0.6
0.4
0.2
X n
0
-0.2
-0.4
-0.6
-0.5
0
0.5
X n+t

Consider the sequence generated by solving

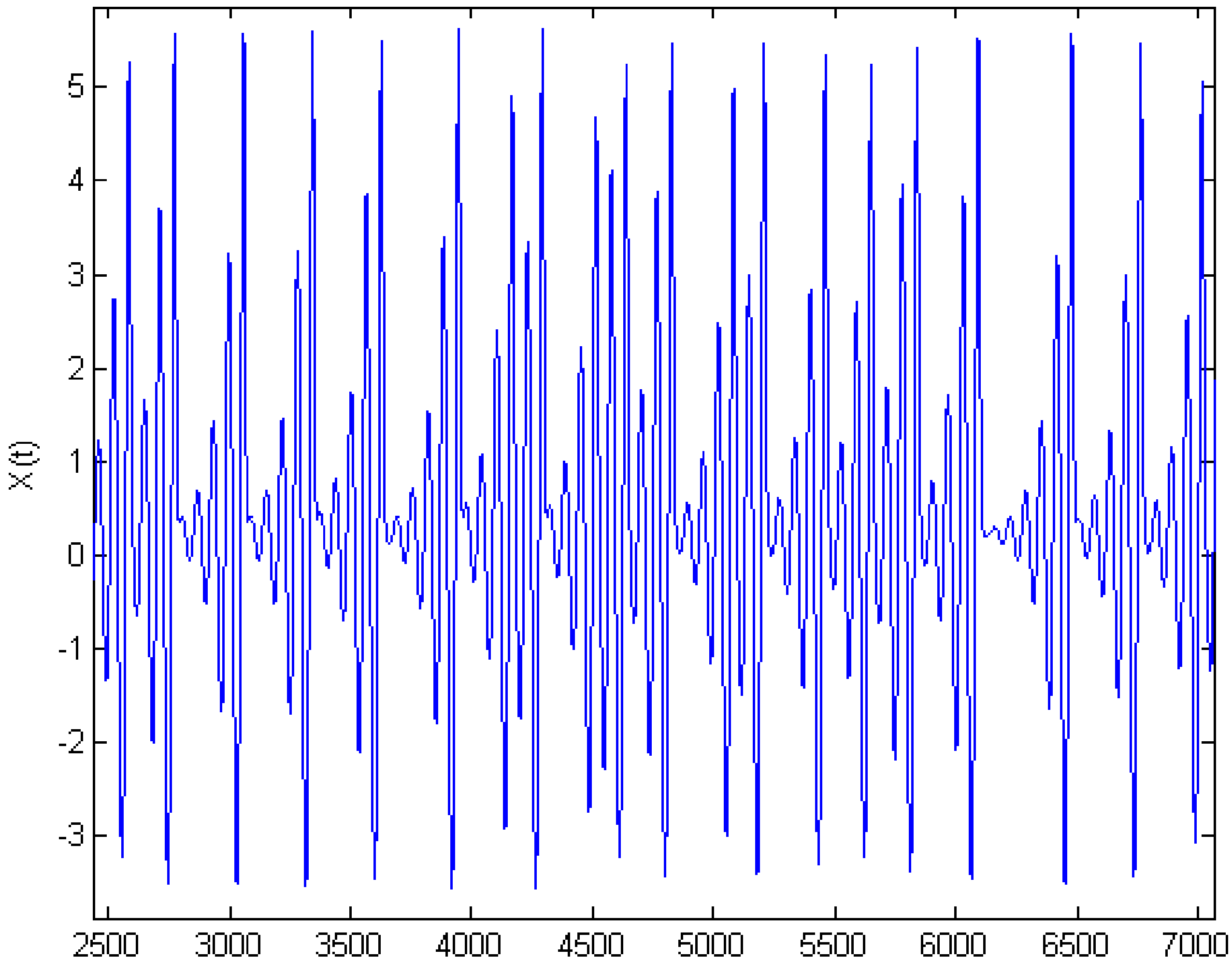
the Rossler system

$$\dot{x} = -y - z$$

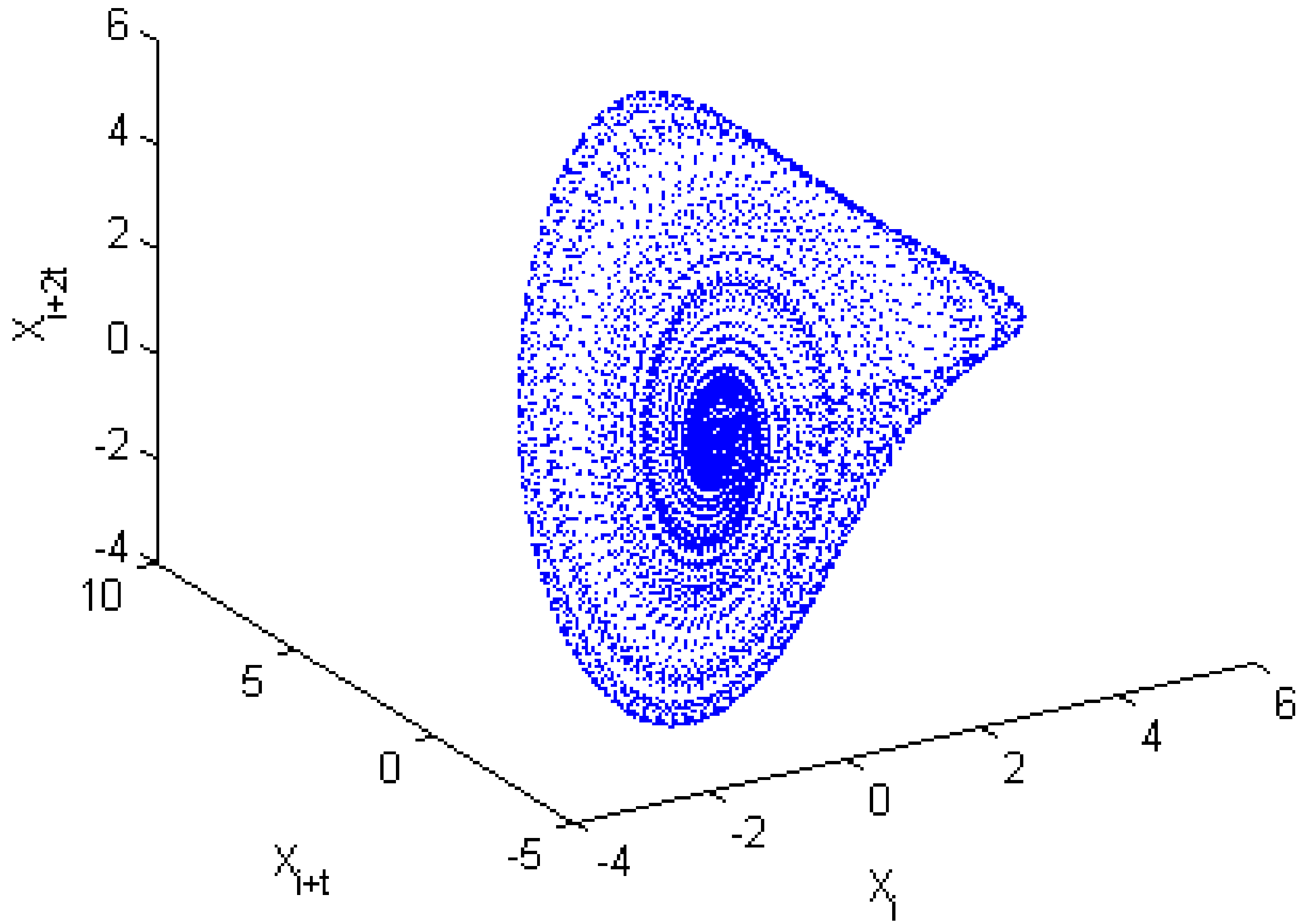
$$\dot{y} = x - ay$$

$$\dot{z} = b + z(x - c)$$

when b and c are fixed..


X(t)
5
4
3
2
1
0
-1
-2
-3
2500
3000
3500
4000
4500
5000
5500
6000
6500
7000

the Rossler attractor



Time delay

- The choice of τ is very important
- A few methods have been given
- We have chosen the computationally easiest, attributed to Kim et. al.

Embedded data as state space reconstruction

- State space structure is a geometry of behavior

-simple behavior $\Leftrightarrow$ simple geometry

-disordered behavior $\Leftrightarrow$ disordered geometry

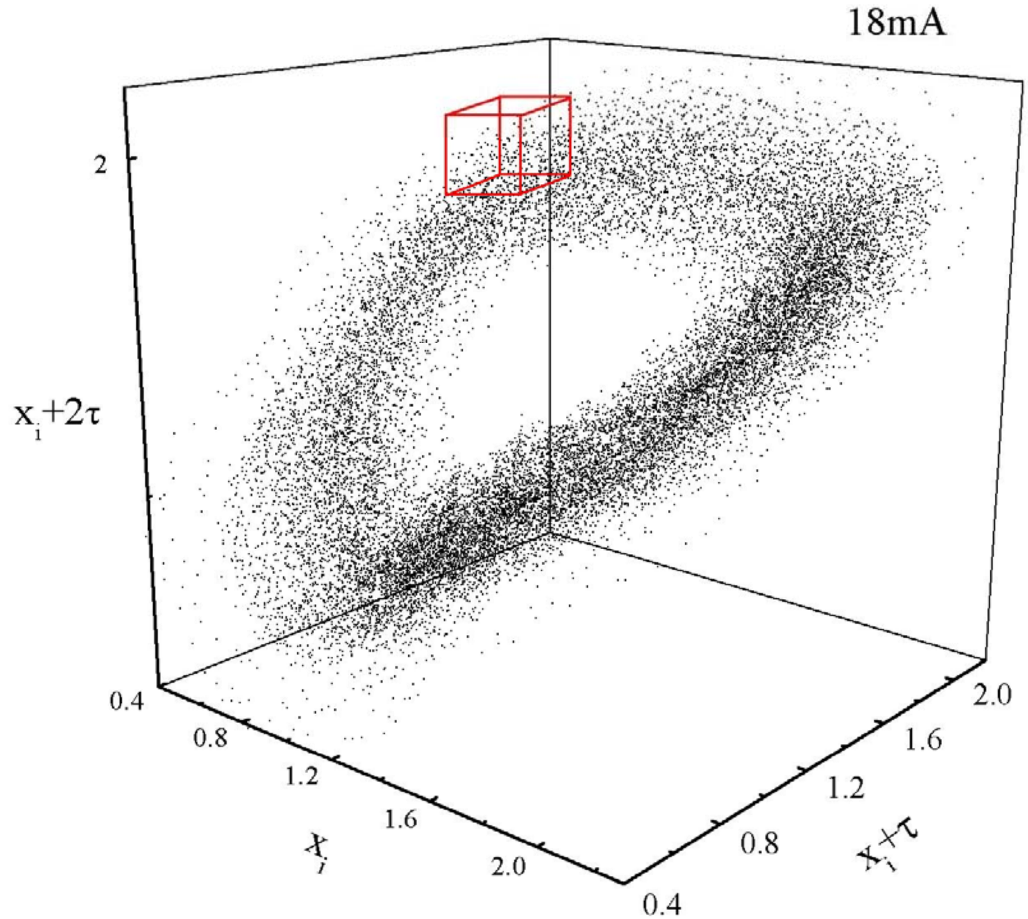
-”complex behavior” $\Leftrightarrow$ “complex geometry”

- With this in mind, we wish to quantify the disorder of geometric structures in state space entropy?

-Consider partitioning the state space into *n* boxes of size δ

-let each box approximate a discrete "state" of the system

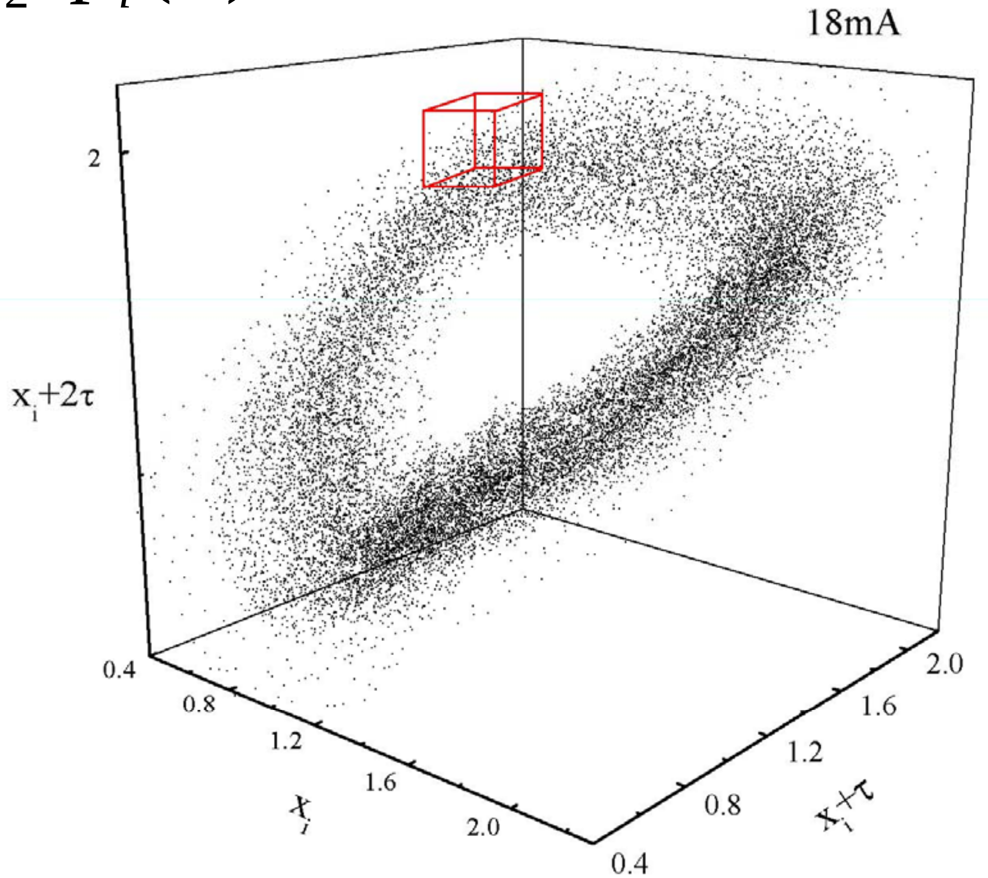
-if a vector falls into the i^{th} box, we say that the system is in state i.



- Let $p_i(\delta)$ be the probability that a point is in the i^{th} box.

-the entropy can be seen as

$$H(\delta) = \sum_i p_i(\delta) \log_2[p_i(\delta)]$$



$$H(\delta) = \sum_i p_i(\delta) \log_2[p_i(\delta)]$$

$$= \sum_i \left(\frac{k_i}{N}\right) \log_2[p_i(\delta)]$$

where k_i is the number of vectors in the i^{th} box

$$= \frac{1}{N_0} \sum_i k_i \log_2[p_i(\delta)]$$

$$= \frac{1}{N_0} \sum_t \log_2[p_{i(t)}(\delta)]$$

where *i(t)* is the index of the box containing the t^{th} reconstructed vector

$$\approx \frac{1}{N_0} \sum_t \log_2[P_t(r)]$$

Where $P_t(r)$ is the probability that a vector is found in a neighborhood of radius r, centered at the t^{th} vector. $r = \delta / 2$, so that the neighborhood has radius δ.

$P_t(r)$ can be calculated relatively easily, by simply counting the number of points in a neighborhood of radius r, centered at the t^{th} vector.

$$\frac{1}{N_0}\sum_t \log_2[P_t(r)] = \frac{1}{N_0}\sum_t \log_2 C_t^m(r)$$

where $$C_t^m(r) = \frac{1}{N}\sum_{s\neq t}\Theta(r - \|x(t) - x(s)\|)$$

counts the number of points in an r-neighborhood of the t^{th} point. *m* indicates the dimension of the vectors.

Uses of entropy

- Quantifies disorder in the system
- Allows further information theoretic quantities to be computed

  -*Conditional entropies*

  -*Mutual information*

Shortcomings of entropy

- Can not identify chaos in a signal
- Sensitive to noise

many further statistics defined..

Eckmann-Ruelle (E-R) entropy

(rate that a system generates information)

- Define $\Phi^m(r) = \frac{1}{N_0} \sum_t \log C_t^m(r)$

Then the E-R entropy is defined as

$$H_{(ER)} = \lim_{r \to 0} \lim_{m \to \infty} \lim_{N_0 \to \infty} [\Phi^m(r) - \Phi^{m+1}(r)]$$

E-R properties

- A value of H_{ER} that is finite, and non-zero suffices to demonstrate the existence of chaos in a signal
- H_{ER} is infinite in the presence of noise
- A proper calculation of H_{ER} requires infinite data, and lots of computation…

ApEn (approximate E-R entropy)

Define $ApEn(m, r) = \Phi^m(r) - \Phi^{m+1}(r)$

ApEn is function of *m* and *r*, with *m,r* prescribed by the original authors as *m*=2 or 3 and r= (.1)SD or (.2)SD,

Where SD is the standard deviation of the original data.

ApEn (approximate E-R entropy)

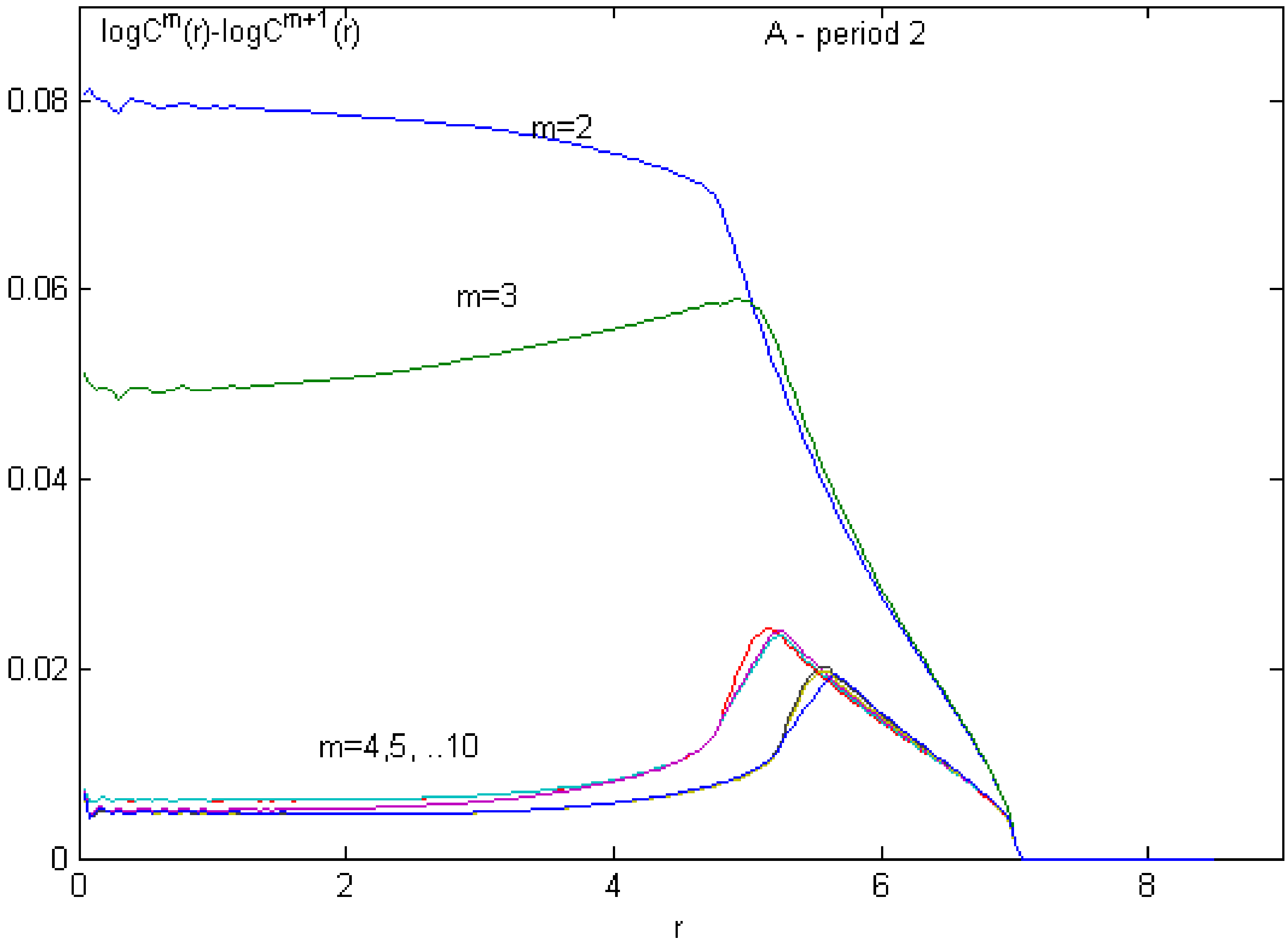
- -formulation lacking appreciation/motivation for embedding dimension, time delay, and choice of r.
- -*despite this*, ApEn is still a useful indicator of irregularity in a signal, which behaves like entropy (maximal on randomness)
- -robust to as little as a few hundred (well sampled) data points

ApEn (approximate E-R entropy)

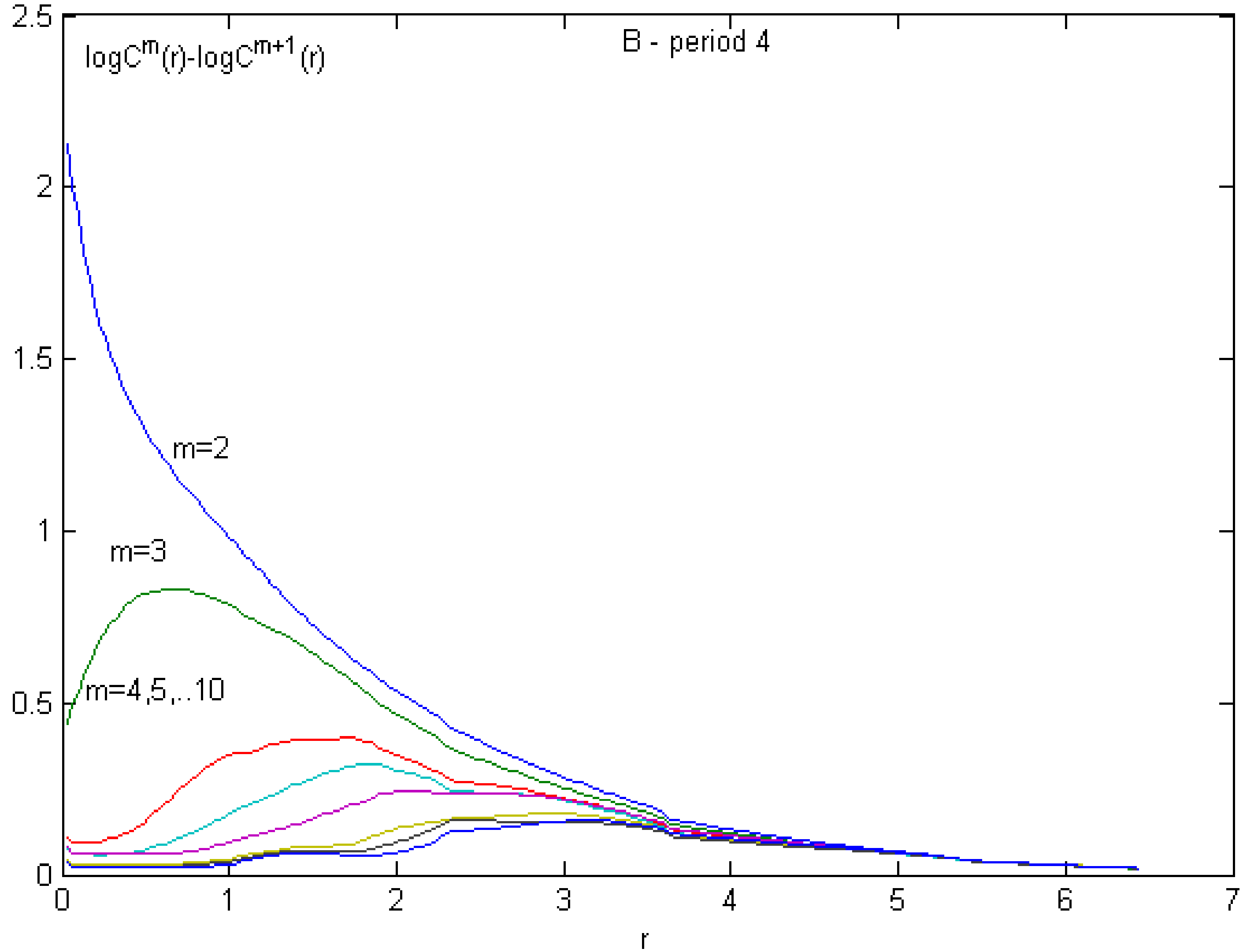
- -in our work, we implement motivated choices of time delay, and embedding dimension
- -furthermore, by examining the ApEn dependence on r, we find that we can modify it so that it takes zero values on random (high entropy) data, as well as periodic (low entropy) data, making it a candidate for a proper measure of complexity
- -we call this modified version ApCx, or approximate complexity

examples

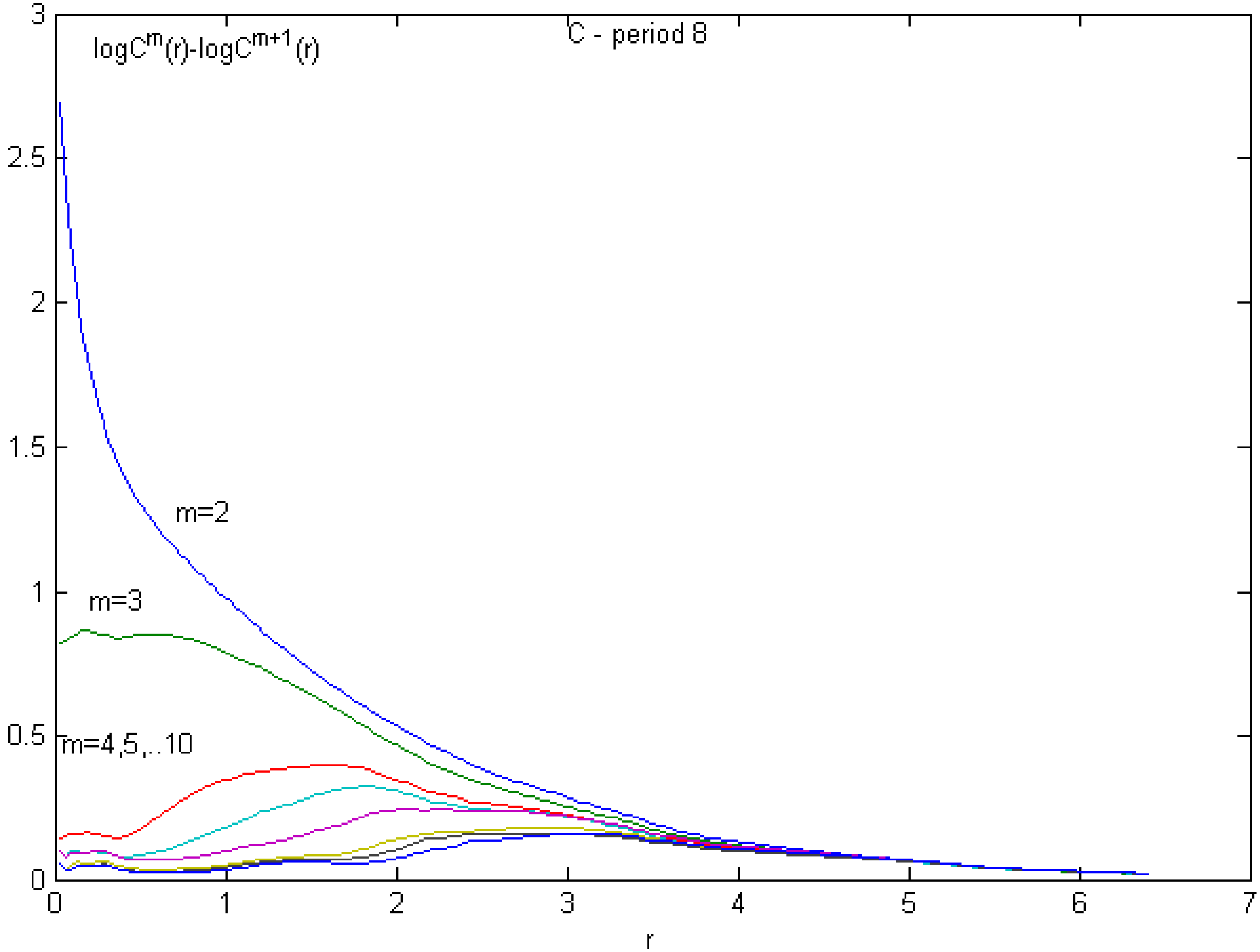
Occurance of bifurcations



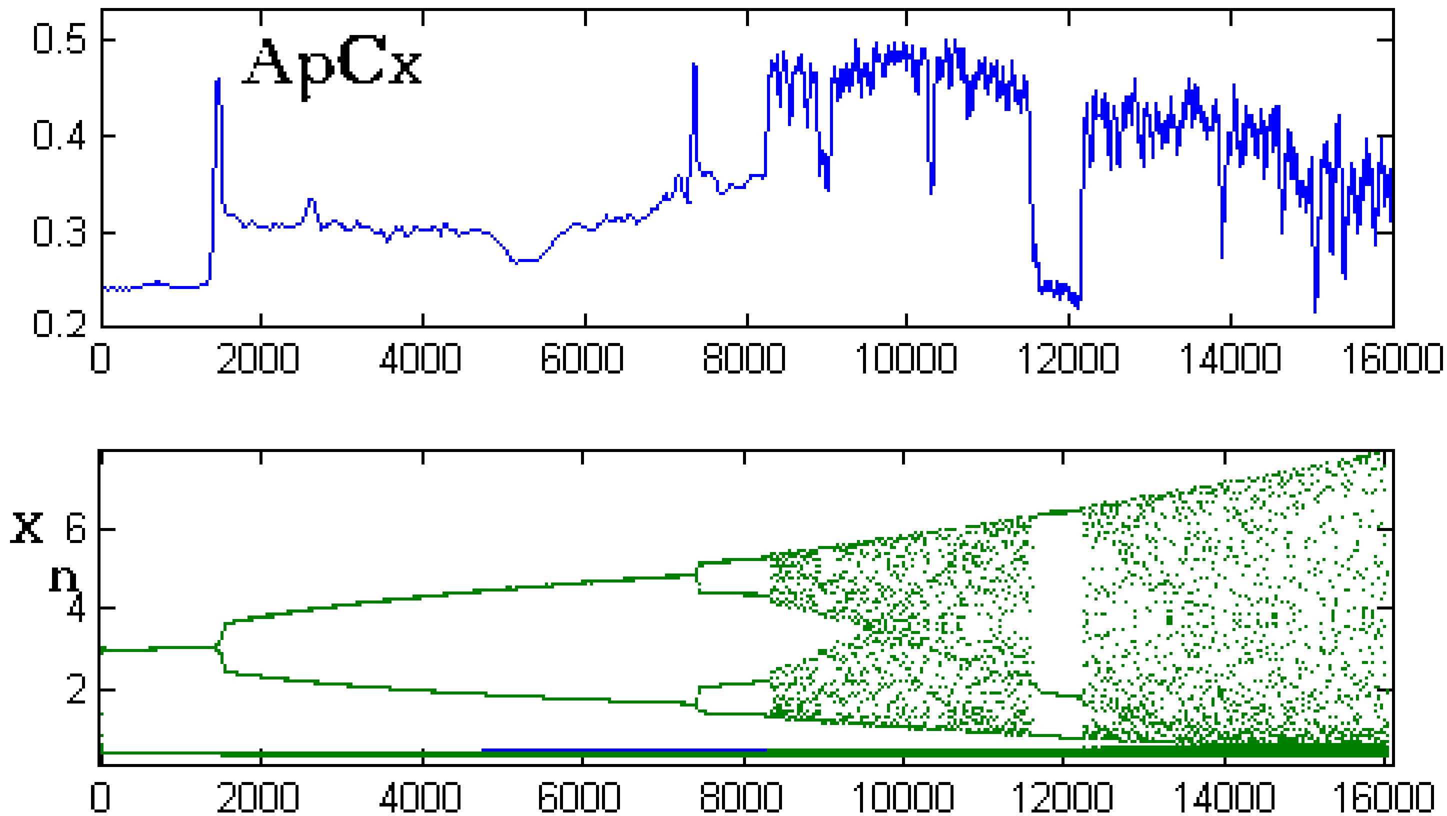
logC^m(r)-logC^(m+1)(r)
A - period 2
0.08
0.06
0.04
0.02
0
m=2
m=3
m=4,5, ..10
0
2
4
6
8
r



logC^m(r)-logC^m+1(r)
B - period 4
m=2
m=3
m=4,5,..10
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
r



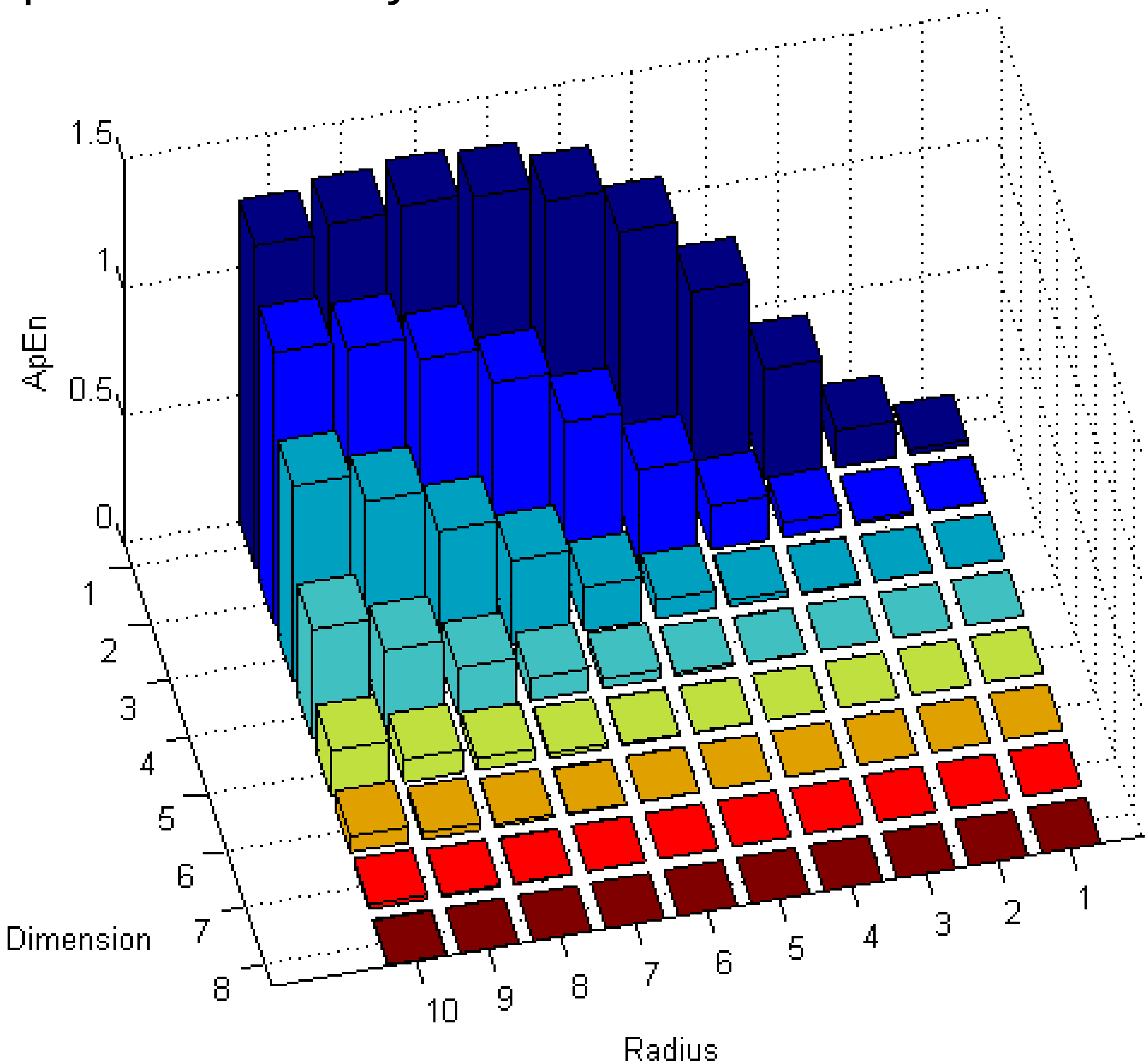
C - period 8
logC^{m}(r)-logC^{m+1}(r)
m=2
m=3
m=4,5,..10
r
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
7

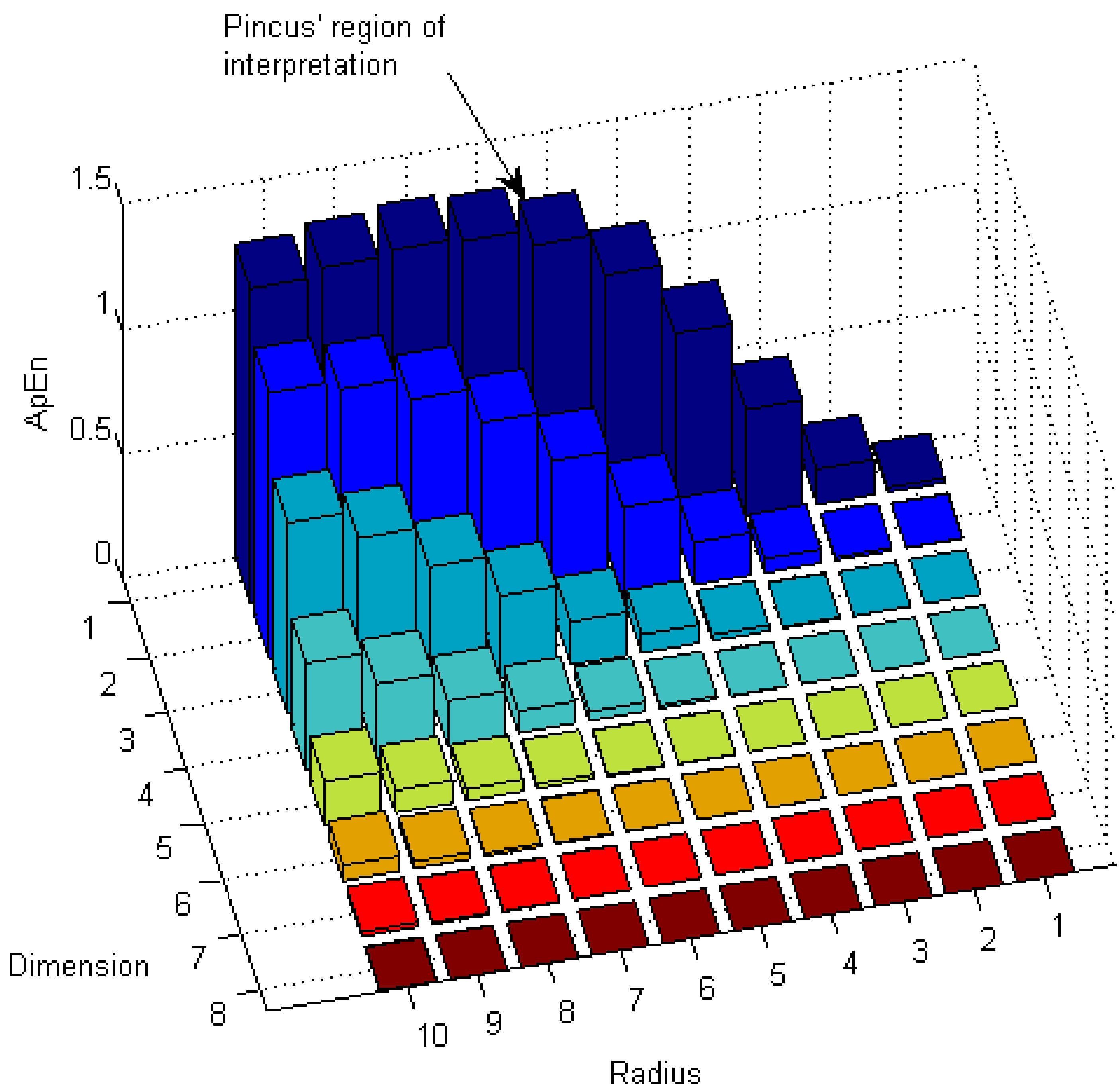

ApCx
0.5
0.4
0.3
0.2
0
2000
4000
6000
8000
10000
12000
14000
16000
x
n
6
4
2
0
2000
4000
6000
8000
10000
12000
14000
16000

examples

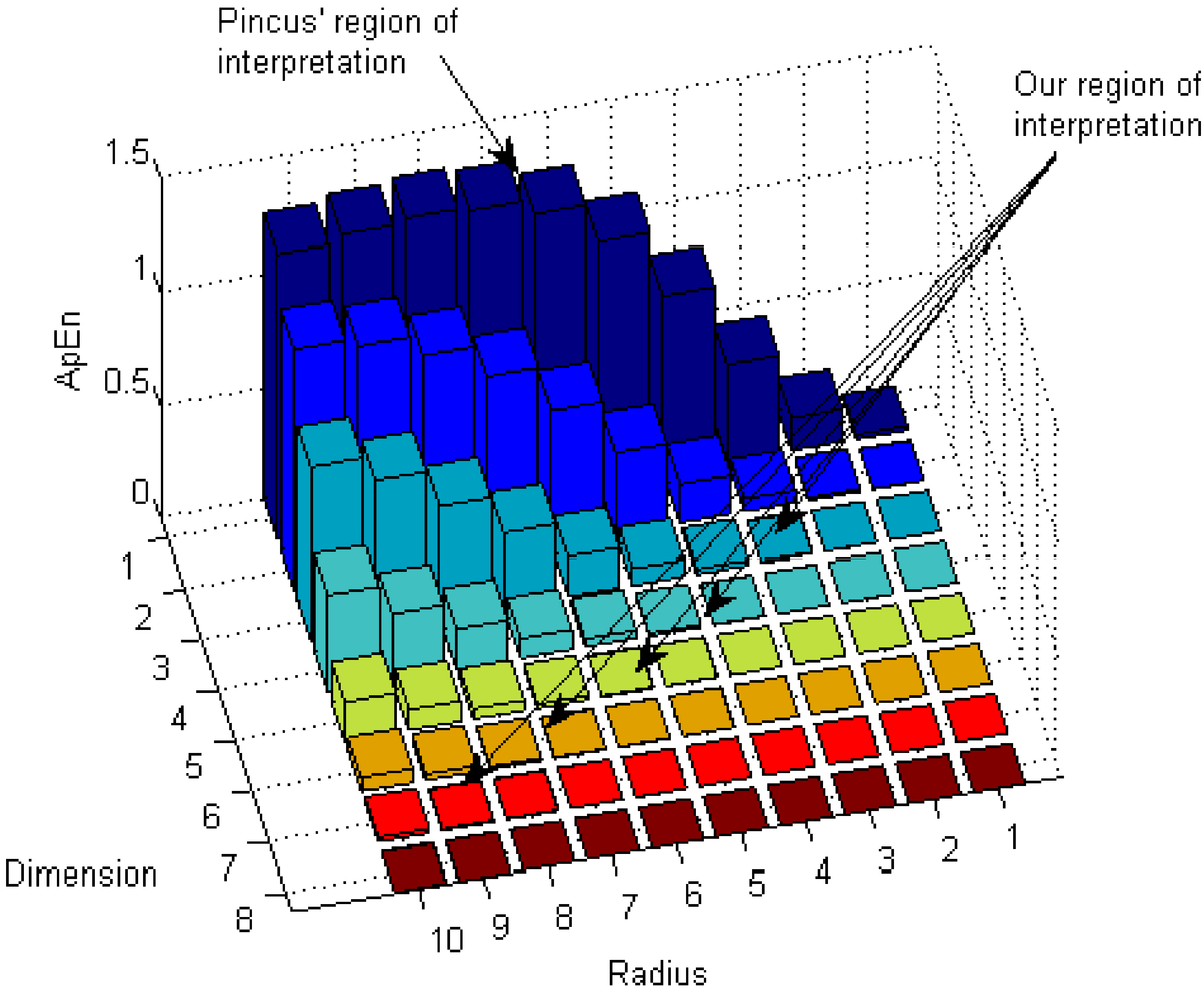
noise

ApEn for normally distributed random time series

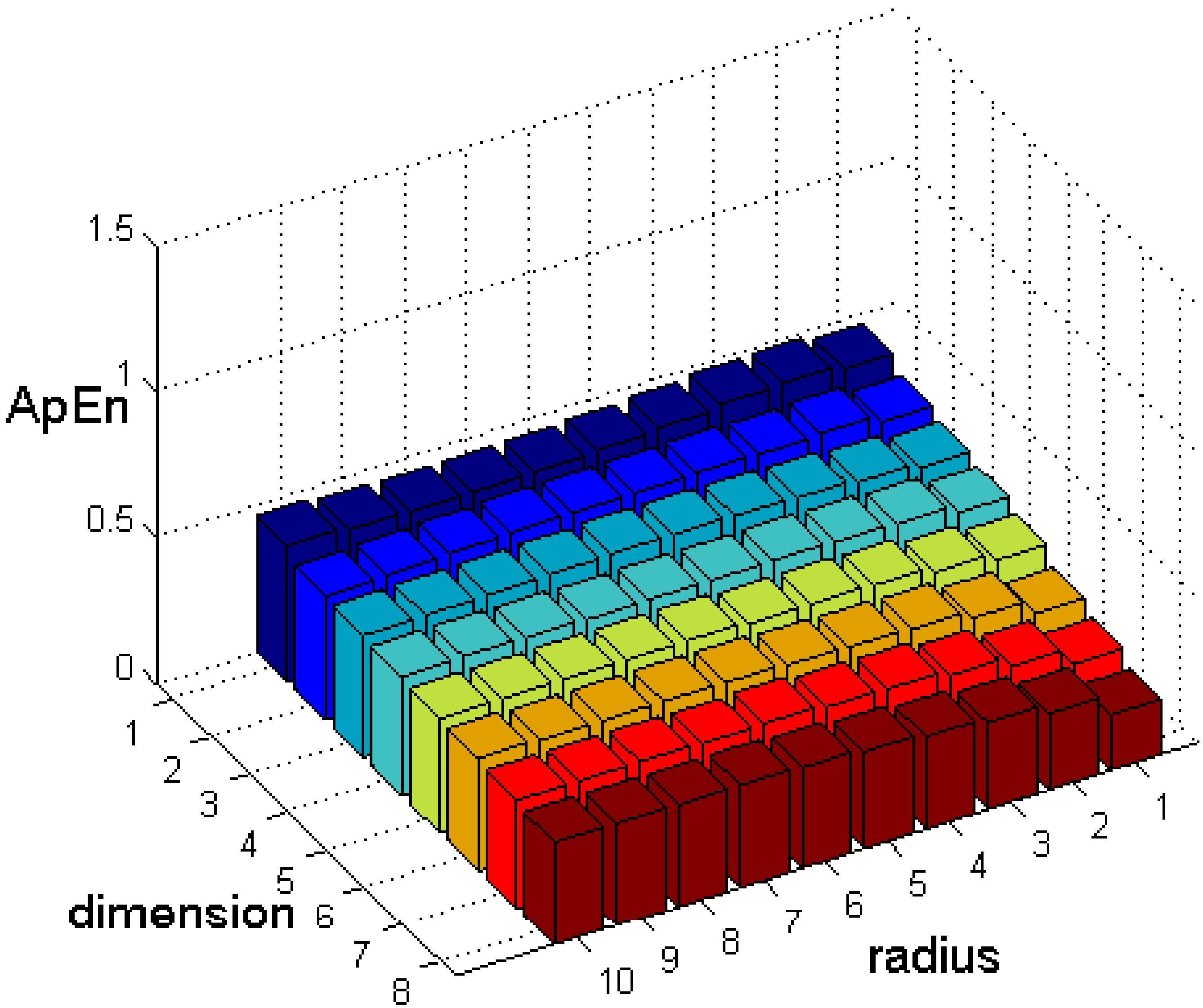


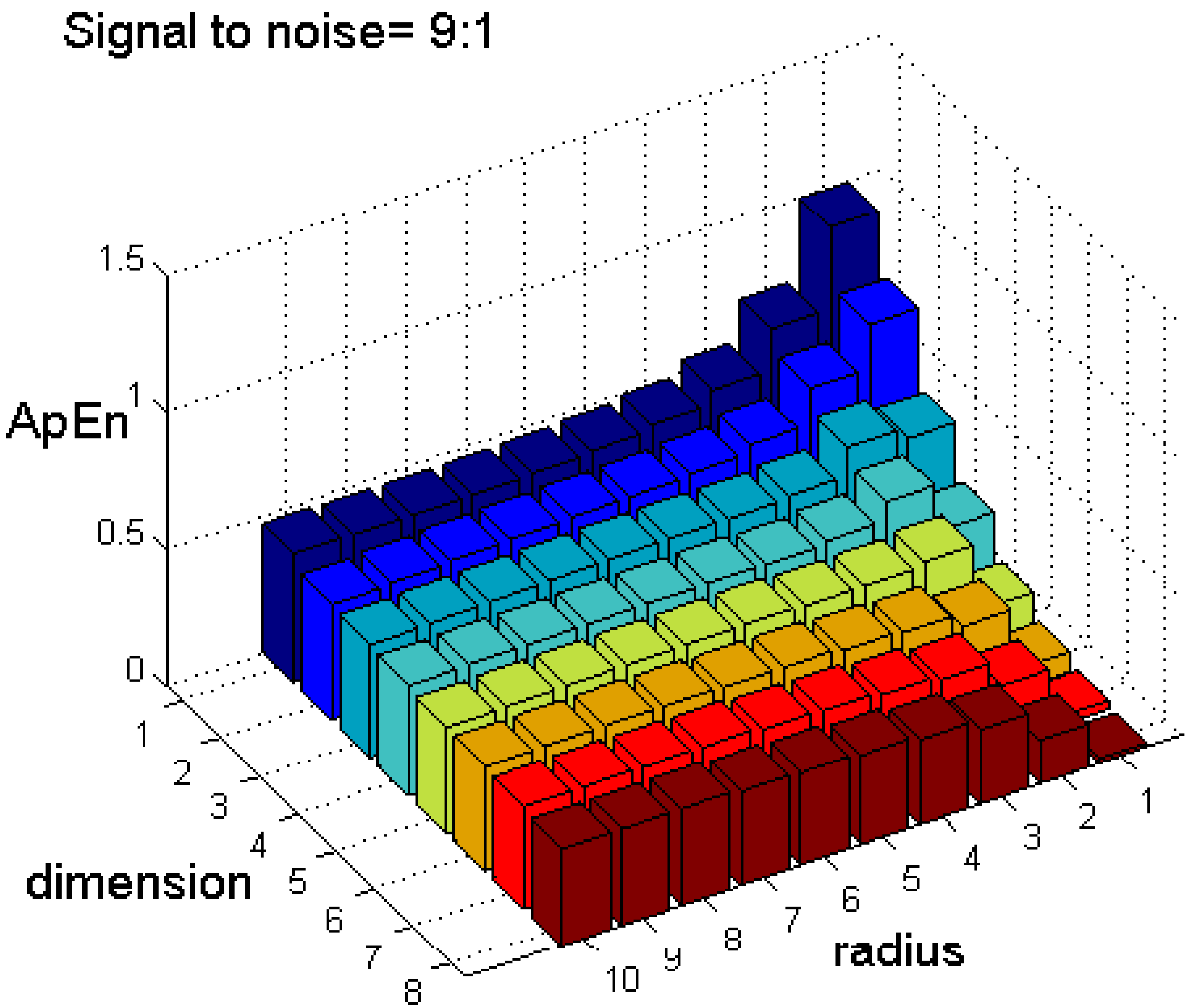


Pincus' region of interpretation
ApEn
1.5
1
0.5
0
Dimension
1
2
3
4
5
6
7
8
Radius
10
9
8
7
6
5
4
3
2
1

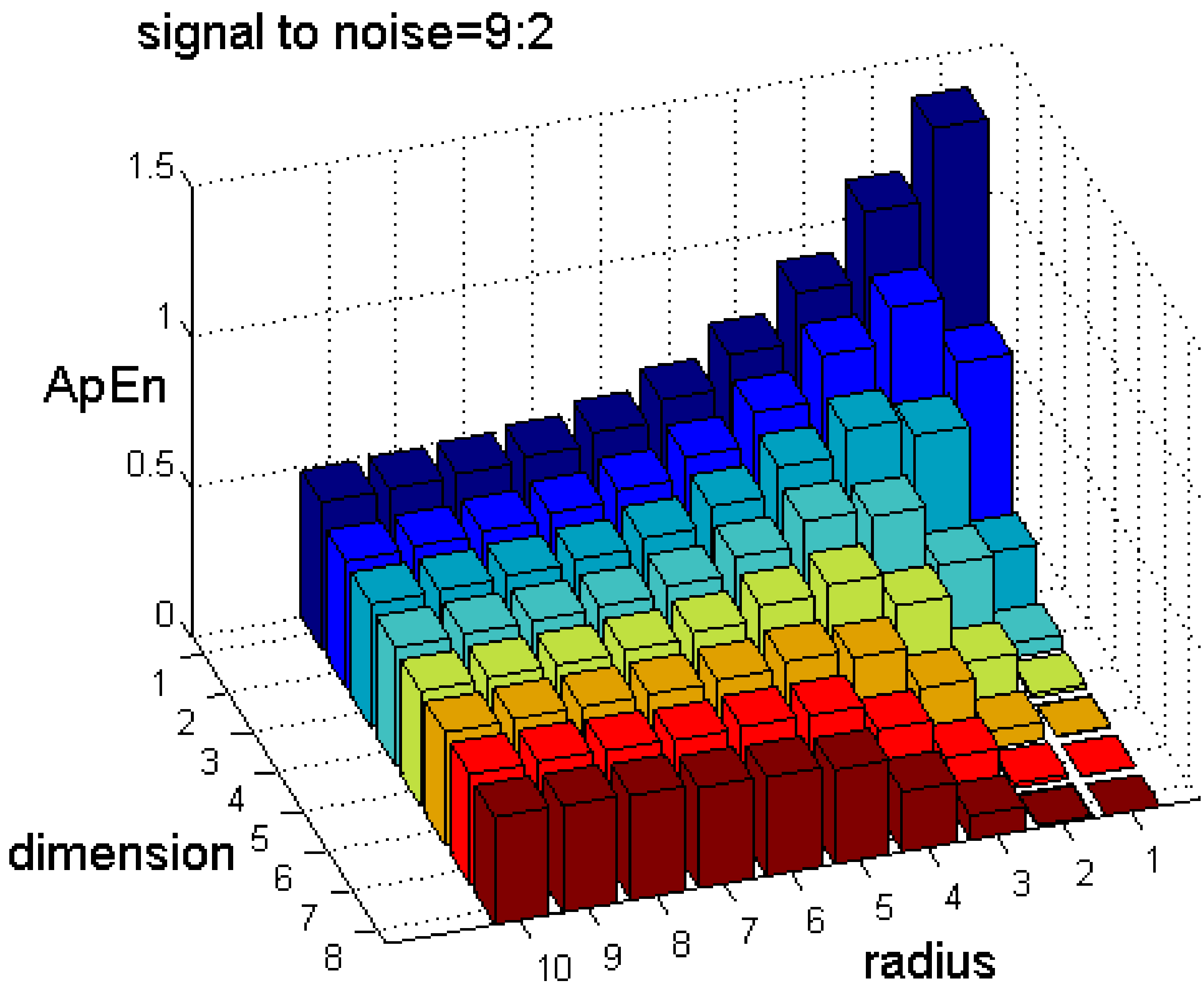

Pincus' region of interpretation
Our region of interpretation
ApEn
1.5
1
0.5
0
Dimension
1
2
3
4
5
6
7
8
Radius
10
9
8
7
6
5
4
3
2
1

ApEn for chaotic henon map

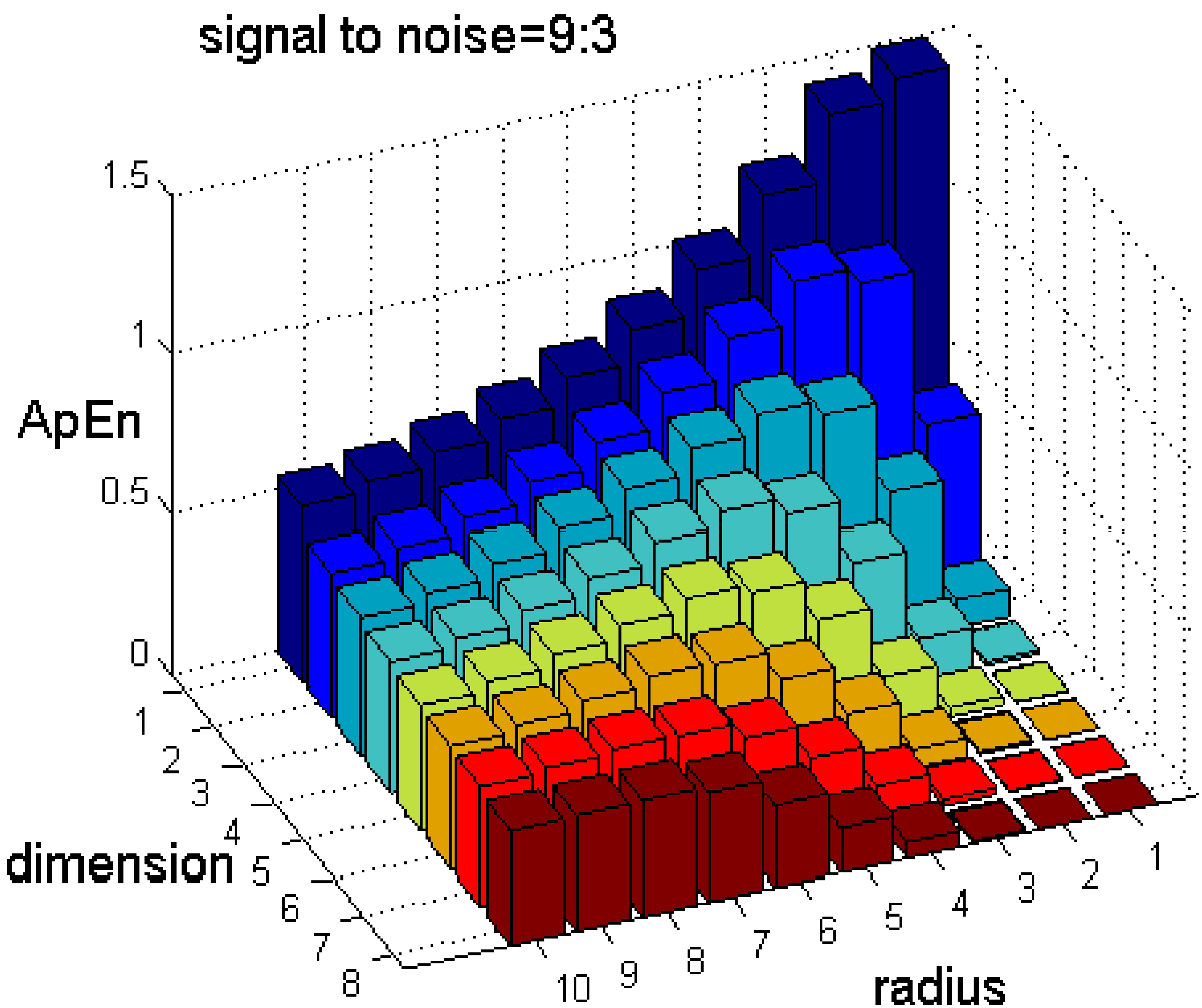




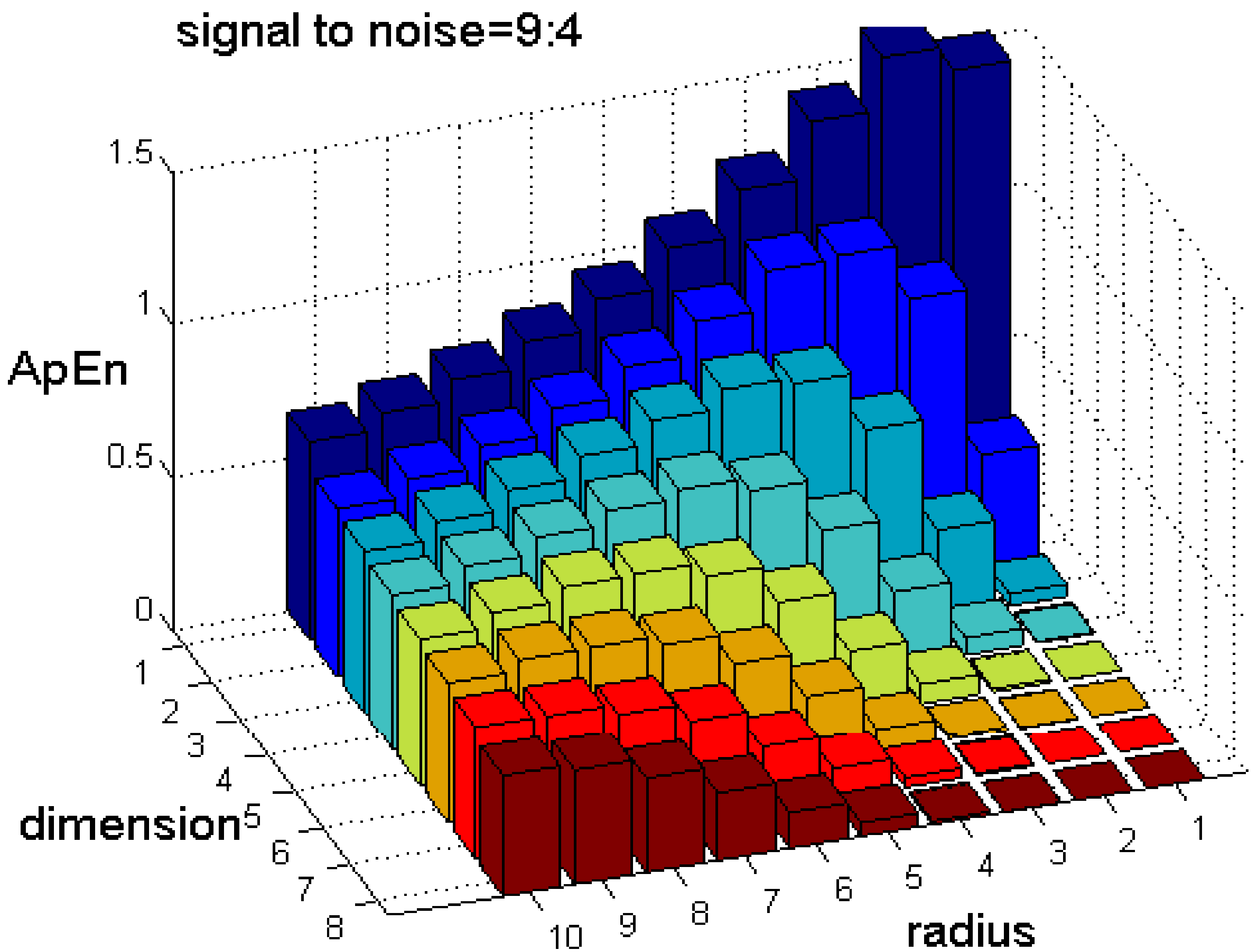
Signal to noise= 9:1
ApEn
1.5
1
0.5
0
dimension
1
2
3
4
5
6
7
8
radius
10
9
8
7
6
5
4
3
2
1

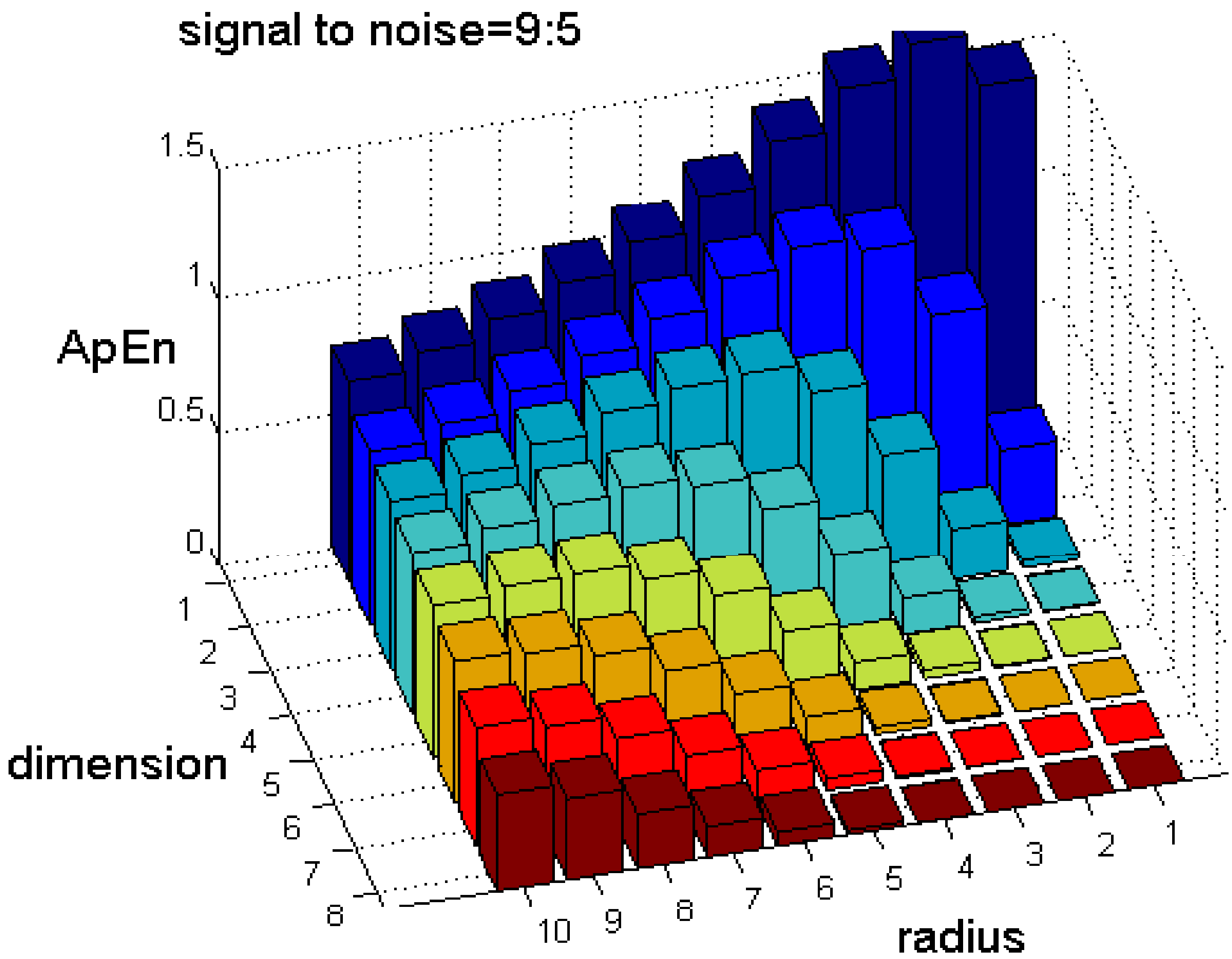


signal to noise=9:2
ApEn
1.5
1
0.5
0
dimension
1
2
3
4
5
6
7
8
radius
10
9
8
7
6
5
4
3
2
1

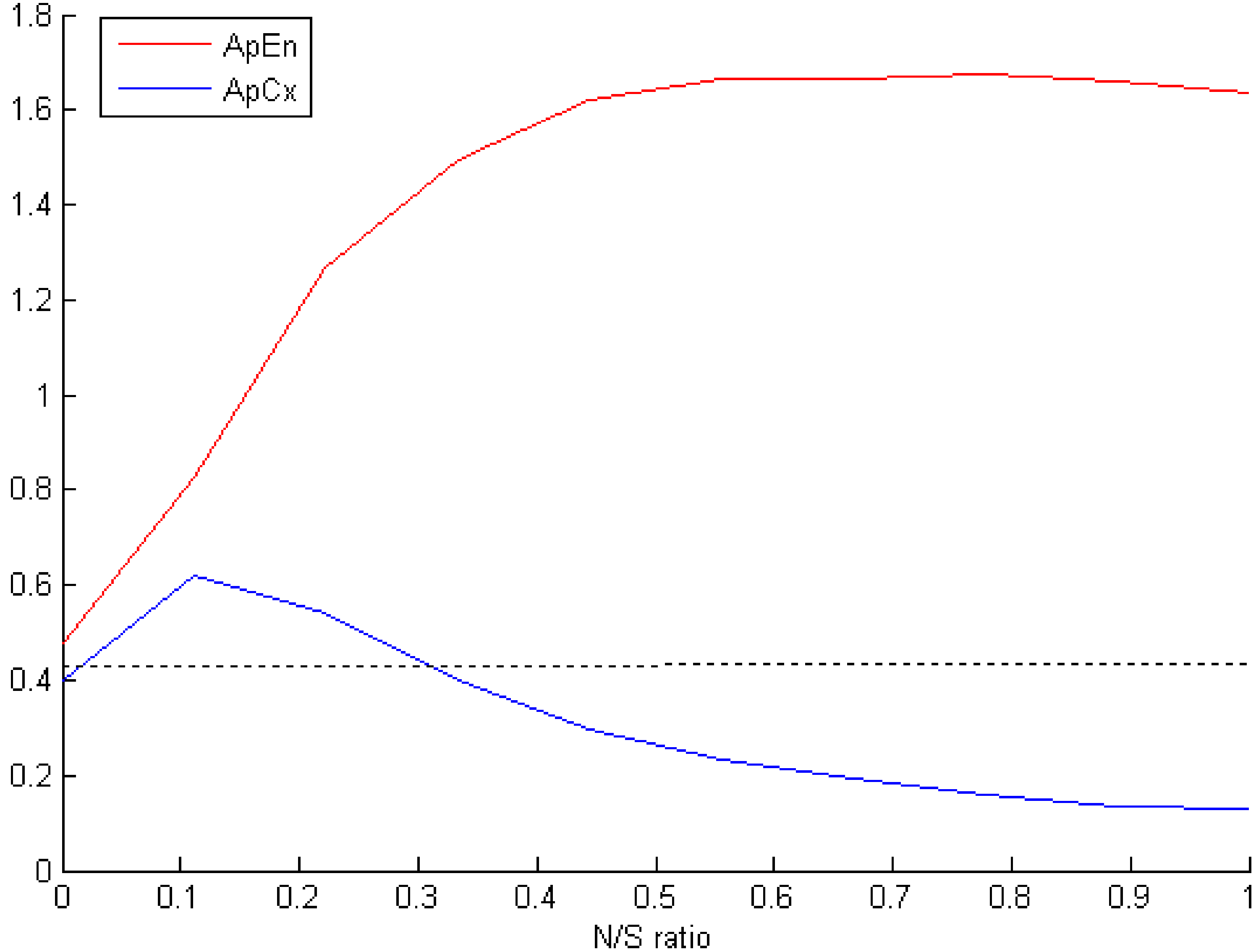


signal to noise=9:3
ApEn
1.5
1
0.5
0
dimension
1
2
3
4
5
6
7
8
radius
10
9
8
7
6
5
4
3
2
1

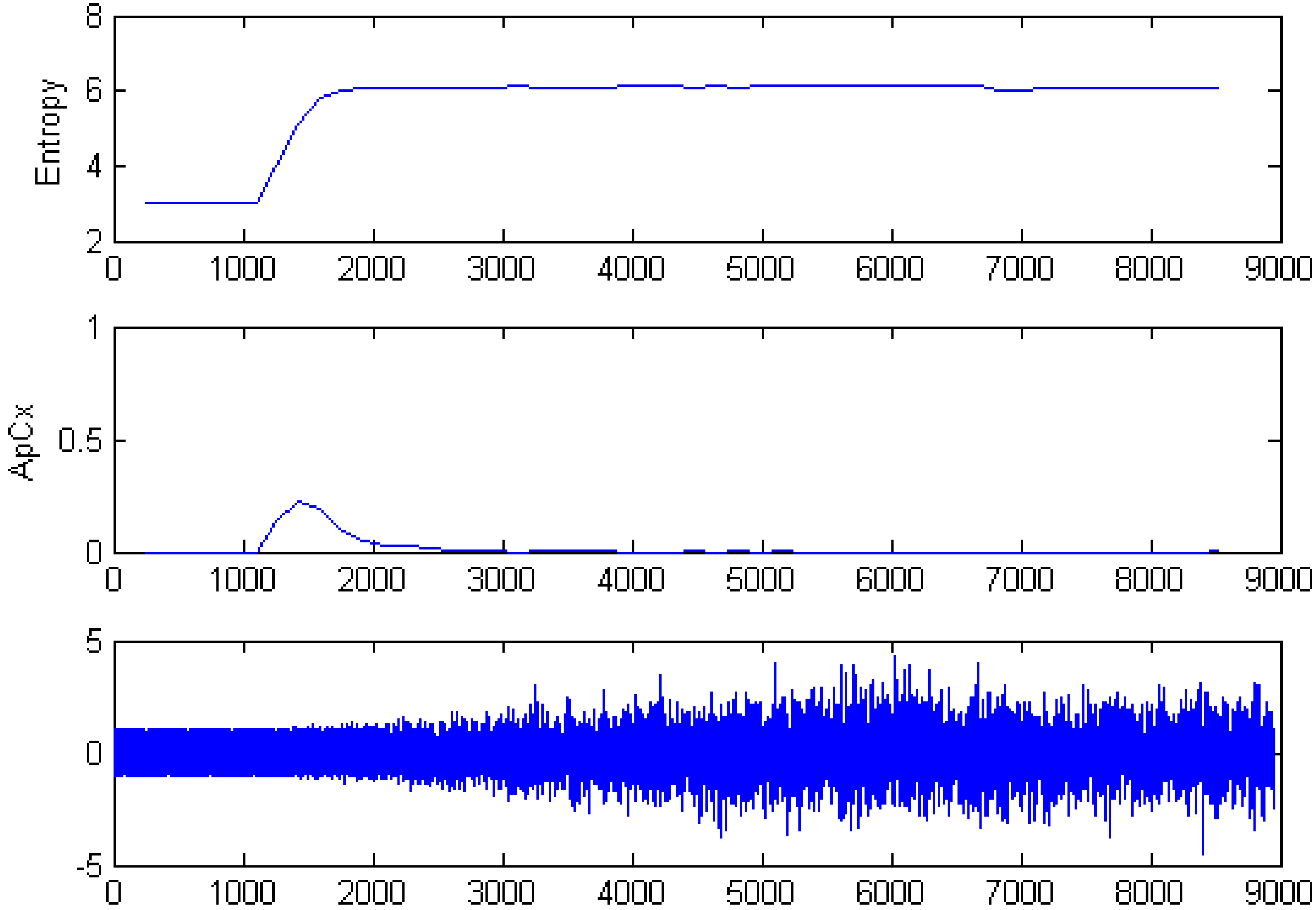

signal to noise=9:4
ApEn
1.5
1
0.5
0
dimension
1
2
3
4
5
6
7
8
radius
10
9
8
7
6
5
4
3
2
1


signal to noise=9:5
ApEn
1.5
1
0.5
0
dimension
1
2
3
4
5
6
7
8
radius
10
9
8
7
6
5
4
3
2
1

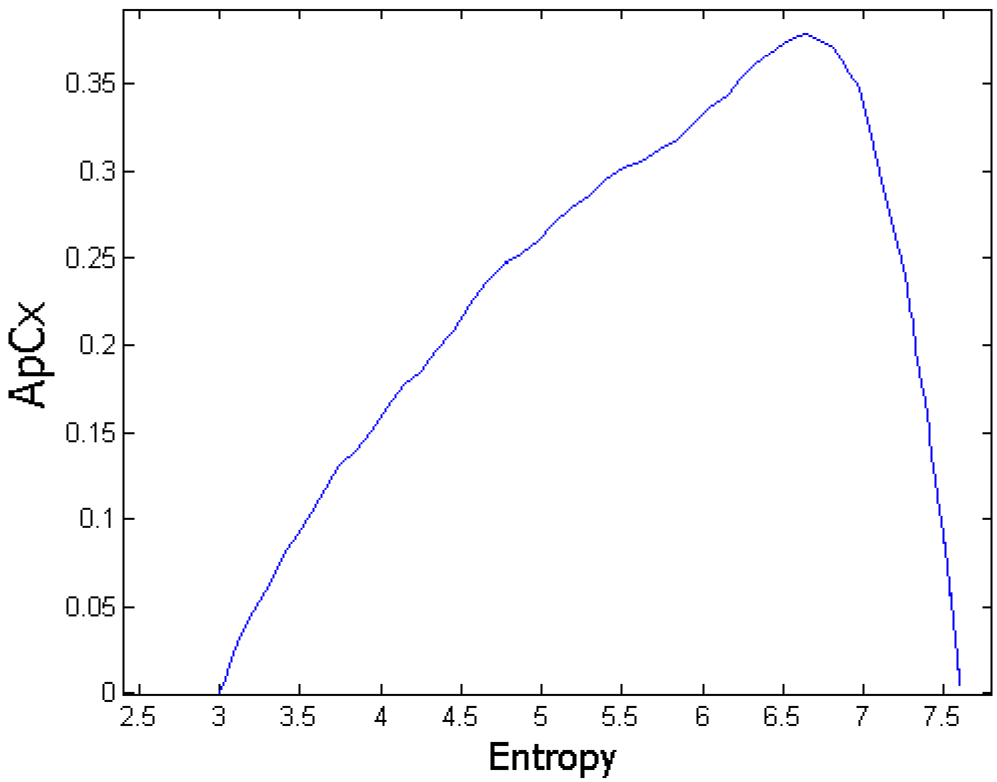
ApEn vs. ApCx on increasingly noisy Henon data



ApCx on periodic data becoming increasingly noisy

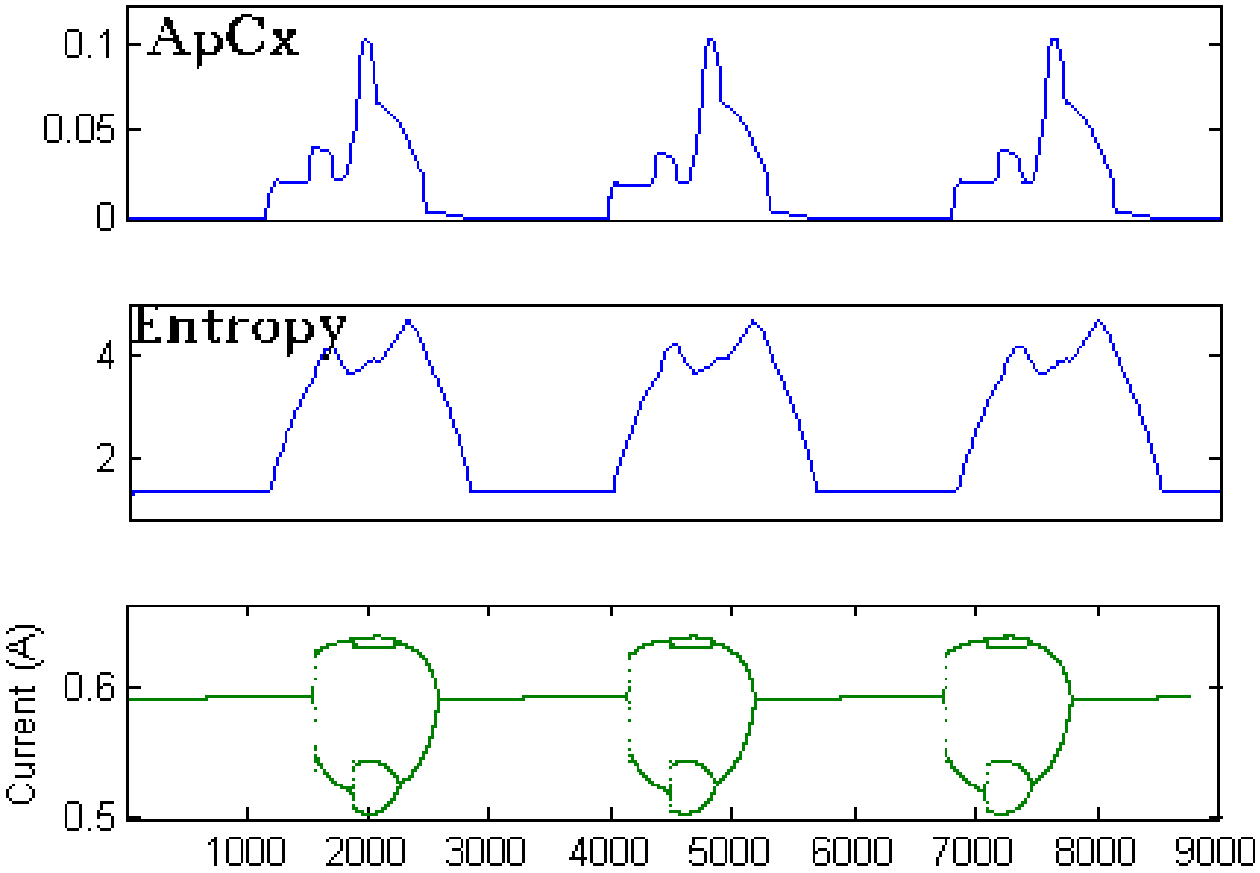


ApCx as a nonlinear function of entropy on increasingly noisy sine wave

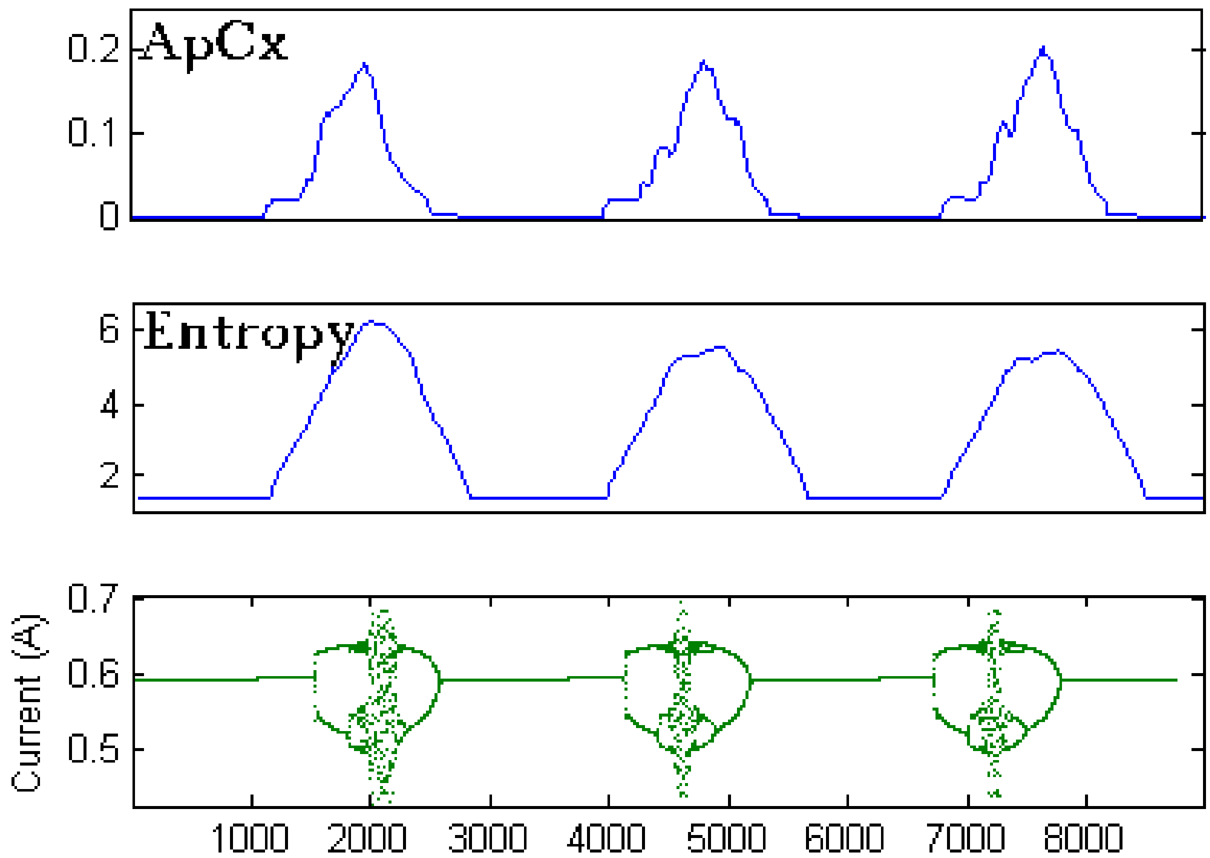


ApCx on systems

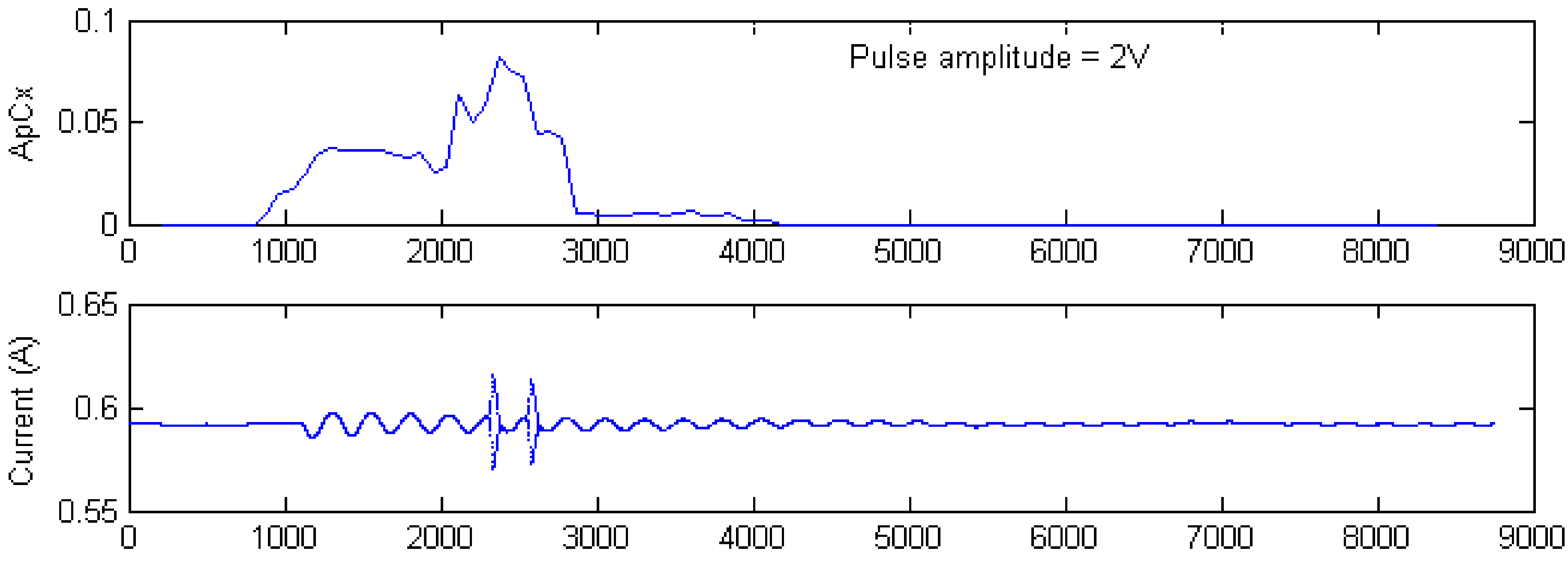
ApCx on buck converter (a)



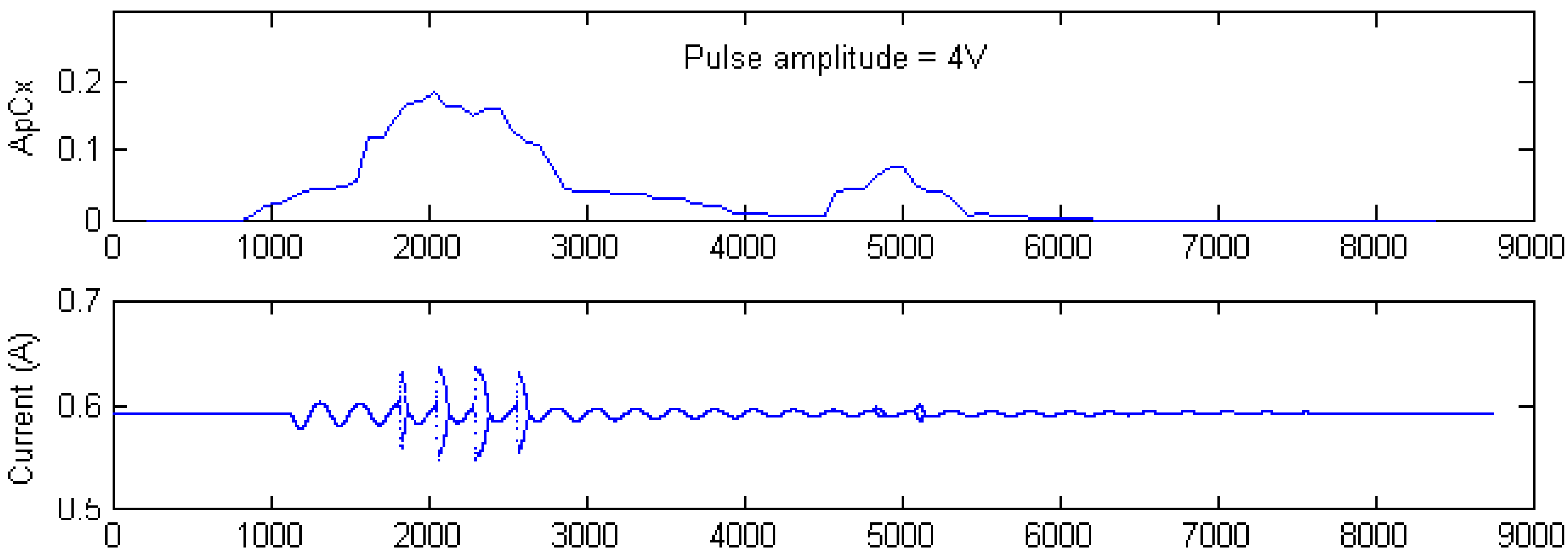
ApCx on buck converter (b)



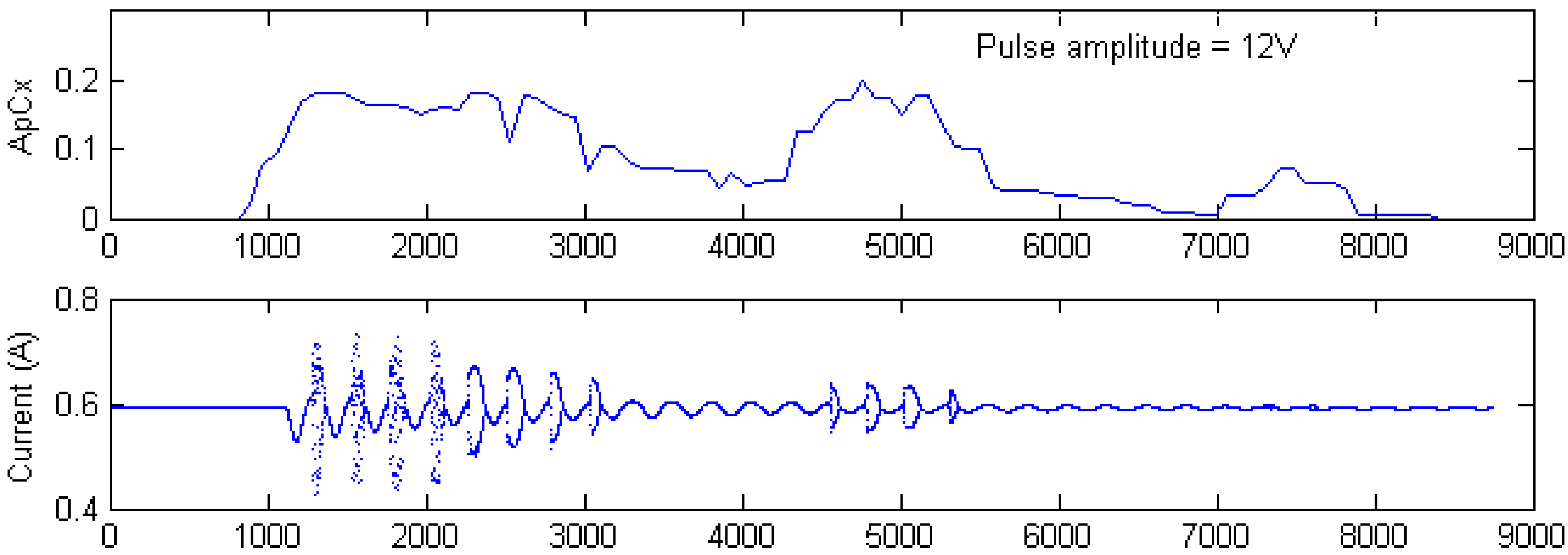
ApCx on buck converter (c)



ApCx on buck converter (d)



ApCx on buck converter (e)



Conclusion

- We can use measurements on state space structure as an indicators of complex behavior
- The time series analysis approach is fast, and requires little (well sampled) data

  -*useful in real time system analysis/control*

Future work

- Extend measures to signals from multiple components

  -*analogues of Conditional entropy and Mutual information*
- Account for complexity of a system as a function of scale (Bar-Yam)

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