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Cournot Gaming in Joint Energy and Reserve markets

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Outline

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  - Electricity markets overview
- Mathematical Framework
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  - ❑ Comparisons of different market clearing schemes
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Electricity Markets Overview

- Initiation of deregulation policies since 1990s [1][2]
- In a traditional oligopolistic market structure, Gencos maximize profits through strategic generation offers [3] [4]
- Analyze 'market power' by finding Nash Equilibrium [5]
- Cournot, Bertrand, Supply function Equilibrium [6]
- Importance of spinning reserve for secure operations under cases of contingencies and sudden change in demand combined with wind variations
- Ancillary services market: reserve priced separately
- Gencos' overall strategy of profit maximization may include strategic reserve offering [7] and even interaction with external markets

Electricity Market with Reserve Constraints

$$\sum_i r_i^{up} \geq g_j ; \forall j \qquad \left(\gamma_j^{up}\right) \qquad (1)$$

$$g_i + r_i^{up} \leq g_i^{\max} ; \forall i \qquad (2)$$

$$0 \leq r_i^{up} ; \forall i \qquad (3)$$

$$\gamma^{up} = \sum_j \gamma_j^{up}, \gamma^{down} \qquad (4)$$

$$\sum_i r_i^{down} \geq \Delta d \qquad \left(\gamma^{down}\right) \qquad (5)$$

$$0 \leq g_i - r_i^{down} ; \forall i \qquad (6)$$

$$0 \leq r_i^{down} ; \forall i \qquad (7)$$

- Up reserve compensates loss of largest generator
- Genco up reserve is limited by capacity
- Demand side can offer up reserve by voluntarily curtailing load
- Gencos may provide down reserve
- Δd is largest expected decrease in demand
- Associated Lagrange multipliers define the marginal cost (clearing price) of up and down reserves

Cournot Equilibrium without Reserve Constraints

$$B(d) = B_0 + \lambda_0 d - \frac{1}{2}\alpha d^2 \quad (8)$$

$$C_i^*(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2} b_i^* g_i^2 \quad (9)$$

$$MAXIMIZE \quad B(d) - \sum_i C_i(g_i)$$

$$0 \le g_i \le g_i^{\max} \quad (10)$$

$$\sum_i g_i = d \quad (\lambda) \quad (11)$$

$$pr_i = \lambda g_i - C_i^*(g_i) \quad (12)$$

$$C_i(g_i) \ge C_i^*(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2} b_i^* g_i^2$$

$$C_i(g_i) = C_i^c(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2}(b_i^* + \alpha) g_i^2 \quad (13)$$

- Consumption of d generates consumer benefit B(d)
- Parameters of Genco's true cost functions and B(d) are known
- System operator clears market by maximizing social welfare subject to power balance and capacity constraints
- λ is the price of electricity
- In Cournot gaming, Gencos maximize profits by gaming on quantity produced

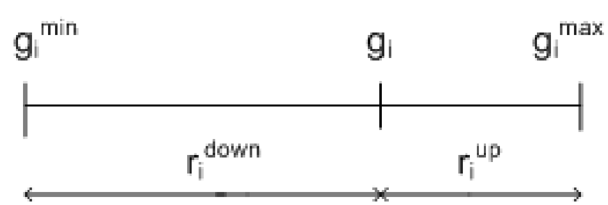
Effect of Reserve Constraints on Genco Profits

$$rev_i = \lambda g_i + \gamma^{up} r_i^{up} + \gamma^{down} r_i^{down} \qquad (14)$$

$$pr_i = \lambda g_i + \gamma^{up} r_i^{up} + \gamma^{down} r_i^{down} - C_i^*(g_i) \qquad (15)$$

$$g_i + r_i^{up} \le g_i^{\max}; \forall i \qquad (16)$$

$$0 \le g_i - r_i^{down}; \forall i \qquad (17)$$



- With no Genco reserve constraint active, the Cournot strategy without reserve constraints is still valid, however if one such constraint is active then the strategy does not correspond to the Nash equilibrium
- Thus, the Cournot strategy including power and reserve must be reformulated

Continent-wide Reserve Market

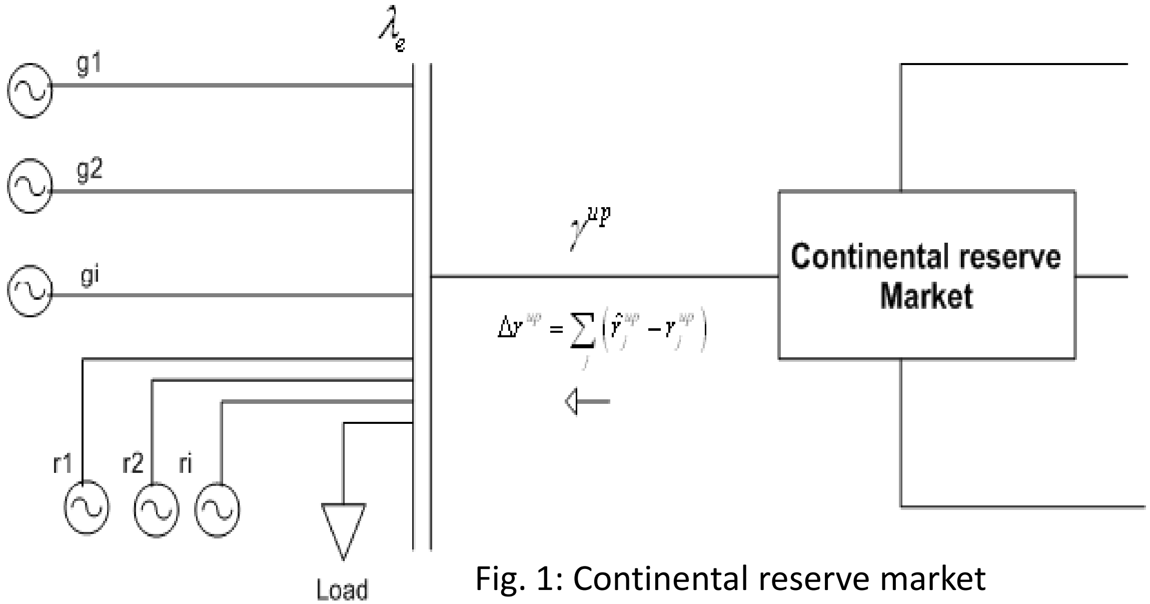


Fig. 1: Continental reserve market

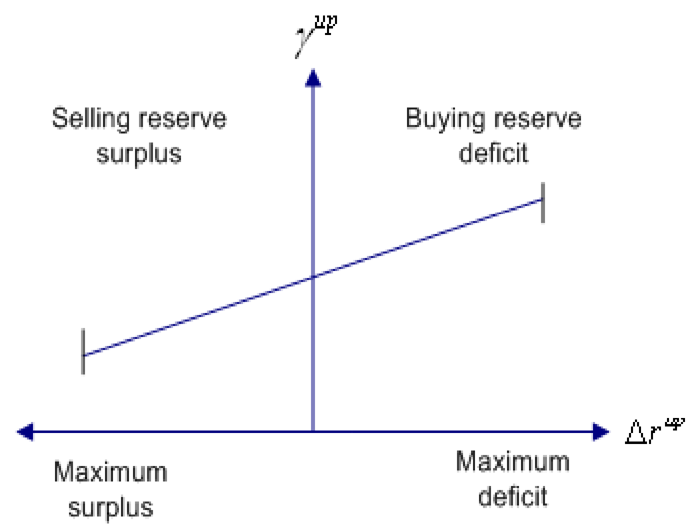


Fig. 2: Price of reserve in continental market

- System trades **reserve deficit** or **surplus** with continent-wide reserve market
- Continental reserve price increases when system is **net buyer** and vice versa
- Similarity to emissions cap and trade [8]

$$\Delta r^{up} = \sum_j \left(\hat{r}_j^{up} - r_j^{up} \right) \quad (18)$$

$$\gamma^{up} = \gamma_0 + \beta \Delta r^{up} \quad (19)$$

Reserve Requirements & Allocations

$$\hat{r}_i^{up} = \frac{g_i}{d} g_k \qquad (20)$$

Or,

$$\hat{r}_i^{up} = \frac{g_i^{max}}{\sum_i g_i^{\max}} g_k \qquad (21)$$

$$g_i + r_i^{up} = g_i^{\max} \qquad (22)$$

- Up reserve **"allocated"** to each Genco is the reserve for which Genco is 'responsible' by either providing it or buying it from the continental market
- Total reserve requirement, e.g. maximum single generation output, is distributed among all Gencos in a **pro-rata** manner, where g_k is the maximum generation produced by any Genco
- With a continental market, each Genco will have an incentive either to buy or sell as much reserve as possible within its generation capabilities

Necessary Conditions for NE of Joint Energy/Reserve Market

The profit of Genco i is, $pr_i = \lambda g_i - \gamma^{up}\left(\hat{r}_i^{up} - r_i^{up}\right) - C_i^*\left(g_i\right)$ (23)

$$dpr_i = d\left(\lambda g_i - \gamma^{up}\left(\hat{r}_i^{up} - r_i^{up}\right) - C_i^*\left(g_i\right)\right)$$

$$= \left(\lambda - IC_i^*\left(g_i\right) - \alpha g_i\right)dg_i - \gamma^{up}\left(\left(1 - \frac{g_i}{d}\right)\frac{g_k}{d}dg_i + dg_i\right) - \beta\left(\hat{r}_i^{up} - r_i^{up}\right)\left(\left(1 - \frac{g_i}{d}\right)\frac{g_k}{d}dg_i + dg_i\right)$$

$$Taking\ partial\ wrt\ g_i,\ for\ i \neq k, IC_i\left(g_i\right) = IC_i^*\left(g_i\right) + \alpha g_i + \left(\gamma^{up} + \beta\left(\hat{r}_i^{up} - r_i^{up}\right)\right)\left(\left(1 - \frac{g_i}{d}\right)\frac{g_k}{d} + 1\right) \quad (24)$$

$$Taking\ partial\ wrt\ g_i,\ for\ i = k, IC_i\left(g_i\right) = IC_i^*\left(g_i\right) + \alpha g_i + \left(\gamma^{up} + \beta\left(\hat{r}_i^{up} - r_i^{up}\right)\right)\left(\left(2 - \frac{g_i}{d}\right)\frac{g_k}{d} + 1\right) \quad (25)$$

- We assume gaming based on complete information (index k can be found iteratively)
- Gencos bid more aggressively than without considering reserve market
- Even if a Genco does not offer power, it has an incentive to sell reserve, which is why the offered marginal cost has a constant term γ^0

Market Clearing with Reserve Trading

$$MAXIMIZE \quad B(d) - \sum_i C_i(g_i) - \gamma^{up} \sum_i \left(\hat{r}_i^{up} - r_i^{up} \right)$$

$$s.t.$$

$$0 \le g_i \le g_i^{\max}$$

$$\sum_i g_i = d \qquad (\lambda)$$

$$g_i + r_i^{up} = g_i^{\max}$$

$$\hat{r}_i^{up} = \frac{g_i}{d} g_k$$

$$\Delta r^{up} = \sum_j \left(\hat{r}_j^{up} - r_j^{up} \right)$$

$$\gamma^{up} = \gamma_0 + \beta \Delta r^{up}$$

$$C_i(g_i) = C_i^*(g_i) + \frac{\alpha g_i^2}{2} + \left(\gamma_0 + \beta \sum_i (\hat{r}_i^{up} - r_i^{up}) \right) \left(\hat{r}_i^{up} - r_i^{up} \right) \qquad (26)$$

λ = Price of electricity

γ^{up} = Price of up reserve

Case Studies

- Economic dispatch schemes for comparison:
  - A No gaming, no reserve
  - B No gaming, internal reserve
  - C Gaming on power, only internal reserve market
  - D No gaming, continental market
  - E Gaming, continental market
- Effect of varying continental reserve market parameters

Simulation Results & Discussions

Table 1: Demand benefit function and continental reserve market parameters

Demand benefit parameter, B^0 \$	150
Demand benefit parameter, λ^0 \$/MWH	150
Demand benefit parameter, α \$/MW²H	0.05
Continental reserve market parameter, γ_0 \$/MWH	23.3
Continental reserve market parameter, β \$/MW²H	0.001

Table 2: Genco cost function parameters

	Genco 1	Genco 2	Genco 3
Cost function parameter, C_i^{0*} \$	100	200	200
Cost function parameter, a_i^* \$/MWH	20	30	40
Cost function parameter, b_i^* \$/MW²H	0.05	0.05	0.05
G_i^{max} MW	800	500	500

- For γ_0 we use the Lagrange multiplier value associated with security constraints in market clearing with internal reserve.
- We use commercial optimization solver MINOS (DNLP) in GAMS environment

Comparing Generation Levels, Load Served & Max SW

Table 3.1: Comparing results of different economic dispatch schemes

		A	B	C	D	E
		No gaming, no reserve	**No gaming, internal reserve**	**Gaming on g_i, only internal reserve**	**No gaming, continental market**	**Gaming, continental market**
Generation levels, MW	g_1	800	466.7	485.7	500 ↑	389.4
	g_2	500	466.7	464.3	500 ↑	389.4
	g_3	450	400	364.3	365.6	296.5
Load served, MW	d	1750	1333.3	1314.3	1365.6 ↑	1075.2 ↓
SW, $/H	z	109275	100983.3 ↑	86185.7 ↓	100870 ↑	88849.6 ↓

- In **case B,** due to inclusion of reserve constraints, total **load served** decreases.
- In **case C**, when Gencos game (without considering impact on reserve constraints) **maximum social welfare** degrades.
- In **case D**, by introducing continental reserve market, cheaper Gencos produce more g_i and total **load served** and **maximum SW** improve significantly.
- In **case E**, when Gencos bid strategically according to Cournot, **load served** and **max SW** drastically reduce

Comparing Profit Levels, Energy & Continental Reserve Prices

Table 3.2: Comparing results of different economic dispatch schemes

		A No gaming, No reserve	B No gaming, internal reserve	C Gaming on g_i, only internal reserve	D No gaming, continental market	E Gaming, continental market
Reserve deficit, MW	Δr	N/A	N/A	N/A	**65.6** +	**- 335.4** −
Energy price, $/MWH	λ	62.5	83.3	84.3	90.3	99.8
Continental reserve price, $/MWH	γ^{up}	N/A	23.3	7.9	23.3 ↑	22.9 ↓
Genco Profits, $/H	pr_1	17900	31788.9	27695.9 ↓	31527.7	33357.1 ↑
	pr_2	9800	20022.2	19895.7	19417.9	22474.2
	pr_3	4862.5	15466.7	13681.4	14857.4	17528.3

- In **case C**, when Gencos game without considering impact of reserve constraints, their **profits** do not improve, since the bidding strategies were not optimal.
- In **case D**, Gencos' **profits** do not change considerably compared to **case B**. Since the system is a net buyer of **reserve deficit** from external market, the price of reserve lowers.
- In **case E**, when Gencos bid strategically according to Cournot, **profits** improve. The system is a net seller of **reserve surplus** and price of reserve increases in continental market.

Overall Comparisons

Table 3.3: Comparing results of different economic dispatch schemes

		A	B	C	D	E
		No gaming, No reserve	**No gaming, internal reserve**	**Gaming on g_i, only internal reserve**	**No Gaming, continental market**	**Gaming, continental market**
Generation levels, MW	g_1	800	466.7	485.7	500	389.4
	g_2	500	466.7	464.3	500	389.4
	g_3	450	400	364.3	365.6	296.5
Load served, MW	d	1750	1333.3	1314.3	1365.6	1075.2
Allocated reserve, MW	$\hat{r}_1$	N/A	N/A	N/A	183.1	140.9
	$\hat{r}_2$	N/A	N/A	N/A	183.1	140.9
	$\hat{r}_3$	N/A	N/A	N/A	133.9	107.7
Dispatched reserve, MW	r_1	0	333.3	314.3	300	410.6
	r_2	0	33.3	35.7	0	110.6
	r_3	0	100	135.7	134.4	203.5
Reserve deficit, MW	Δr	N/A	N/A	N/A	65.6	- 335.4
Energy price, $/MWH	λ	62.5	83.3	84.3	90.3	99.759
Continental reserve Price, /MWH	γ^{up}	N/A	23.3	7.9	23.4	22.965
Genco Profits, $/H	pr_1	17900	31788.9	27695.9	31527.7	33357.1
	pr_2	9800	20022.2	19895.7	19417.9	22474.2
	pr_3	4862.5	15466.7	13681.4	14857.4	17528.3
Load profit $/H		76712	33705	39517	35067	25268
SW $/H	z	109275	100983.3	86185.7	100870	88849.6

Effect of Varying Continental Reserve Market Parameters

Table 4: Effect of Varying Continental Reserve Market Parameters

Cournot gaming at presence of continental reserve market		$\gamma_0 = 23.3$, $\beta = 0.001$	$\gamma_0 = 15$, $\beta = 0.001$	$\gamma_0 = 23.3$, $\beta = 0.01$
Generation levels, MW	g_1	389.4	427.3	414.0
	g_2	389.4	427.3	414.0
	g_3	296.4	350.7	313.3
Load served, MW	d	1075.2	1205.3 ↑	1141.4 ↑
Allocated reserve, MW	$\hat{r}_1$	140.9	151.5	150.1
	$\hat{r}_2$	140.9	151.5	150.1
	$\hat{r}_3$	107.3	124.3	113.6
Dispatched reserve, MW	r_1	410.6	372.6	385.9
	r_2	110.6	72.6	85.9
	r_3	203.5	149.3	186.6
Reserve Deficit, MW	Δr	- 335.4	-167.3	-244.5
Energy Price, $/MWH	λ	99.7	92.6	96.4
External Reserve Price, $/MWH	γ^{up}	22.9	14.8	20.8
Genco Profits, $/H	Pr_1	33357.1	29639.4 ↓	32200
	Pr_2	22474.2	20816.1 ↓	21703.4
	pr_3	17528.3	15541.5 ↓	16568.1
Load Profit $/H		25268	33016 ↑	28658
SW $/H	z	88849.6	86808.2	88104.3

- If γ_0 is high, Gencos improve profits by selling more reserve in continental market
-> system offers more **reserve surplus**.
- With low γ_0, cheaper Gencos produce more g_i
-> total **load served** is higher.
- Max SW does not vary significantly.
- With lower γ_0, **load profit** is higher, but Genco profits are lower.

Summary

- Gencos may game not only on generation, but also on reserve offers
- Initially, deterministic security constraints considered to simplify mathematical formulation
- Feasibility of continental reserve markets
- Effect on Genco strategic offers

Future

Under investigation:

- Hybrid Security model (LOLP, ENLS)
- Demand side participation in external reserve market through strategic load curtailing

Possible future extensions:

- Consider transmission charges and congestion
- SFE approach

Q&A

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