Simplified Models for Use in the Analysis of Future Power Systems

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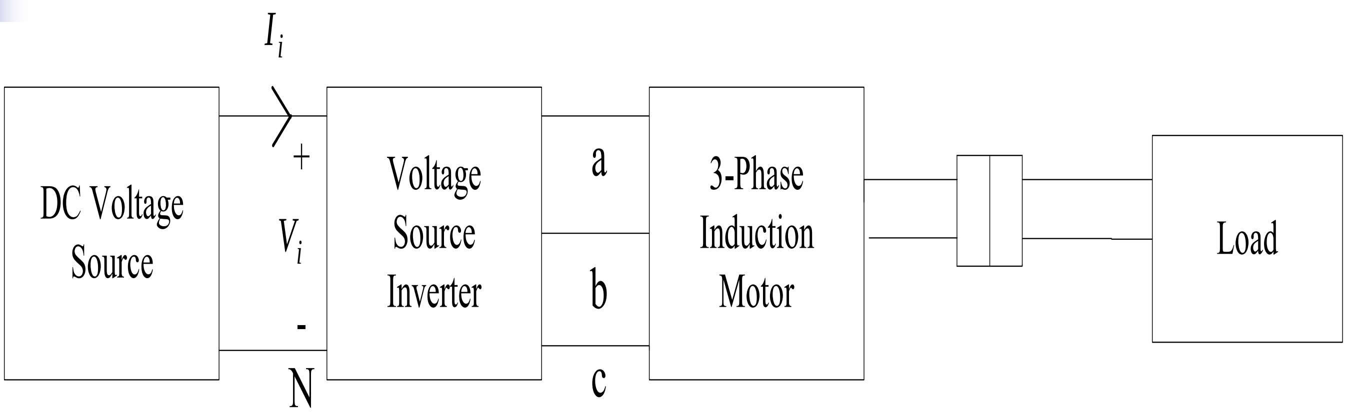
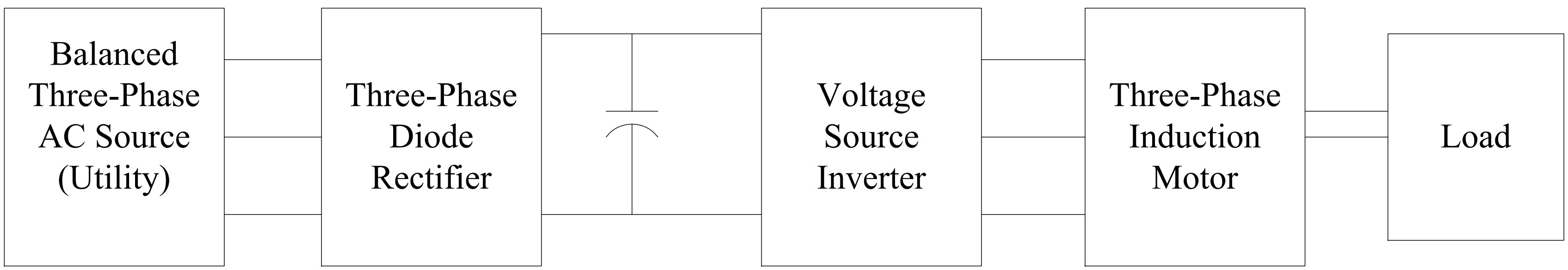
Development of a Simplified Motor-Drive System Model

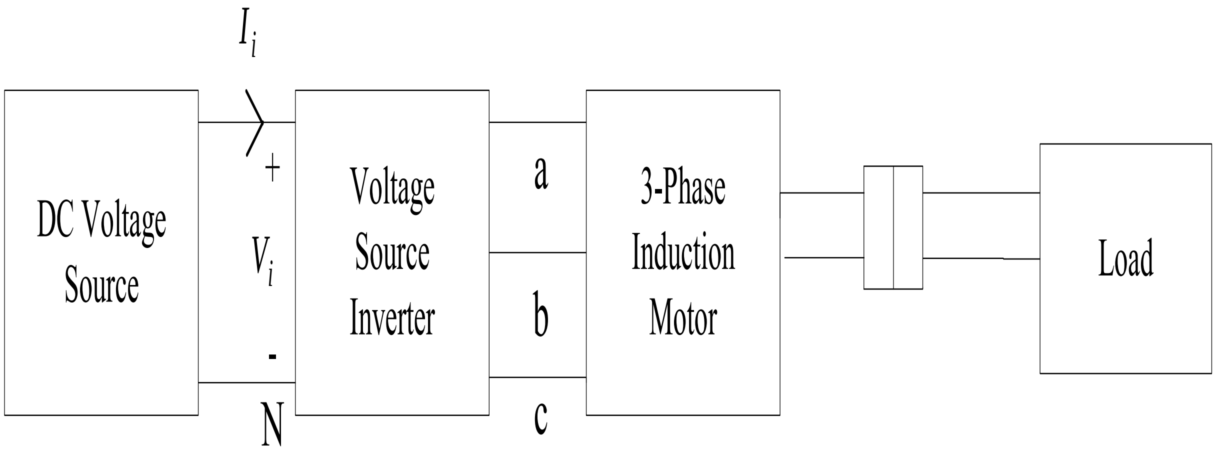


- Model is based on [1]:
  - Induction motor equivalent circuits
  - Input-output relationships of the voltage source inverter
  - Power balance at the input and output terminals of the voltage source inverter

The Simplified Model Can be Used in other Topologies

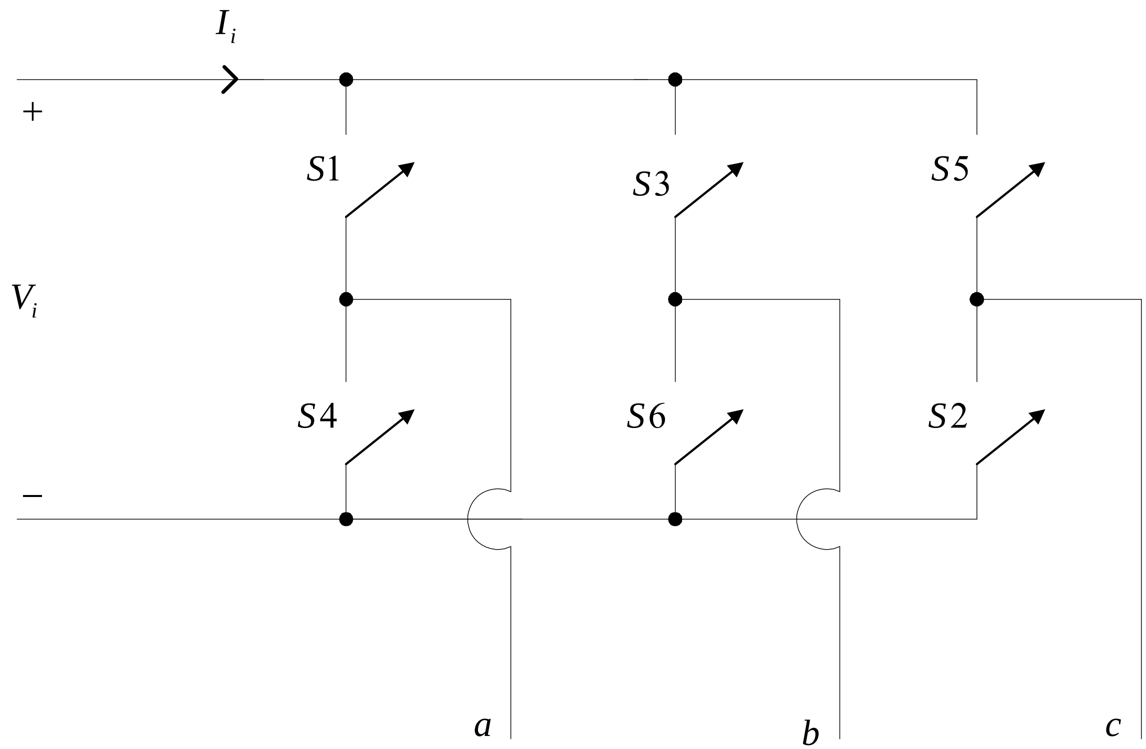


Motor-Drive System Model



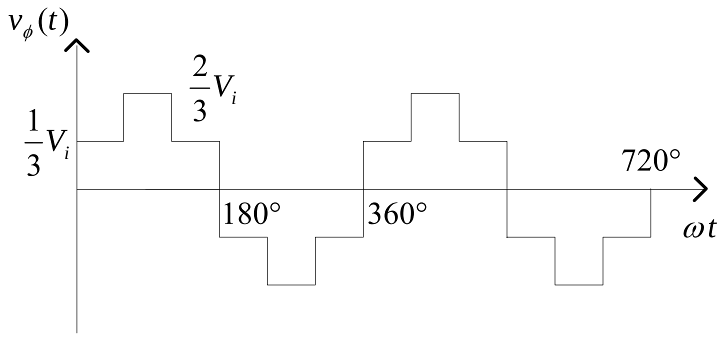
- Develop an analytical method to analyze this system.
- Avoid using simulation packages such as PSPICE and Simulink.
- Develop a simplified model of the system.
- Extend the simplified model to a DC power system containing multiple motor-drive loads.

Three-Phase Voltage Source Inverter

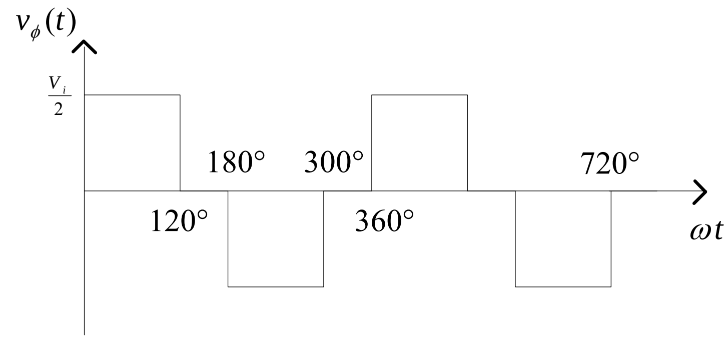


(S1, S4), (S3, S6), (S5, S2) are switched as pairs.

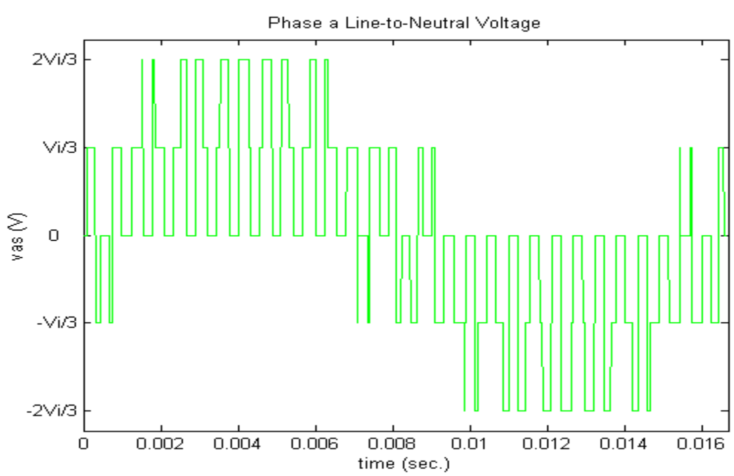
Inverter Output Voltage Waveform



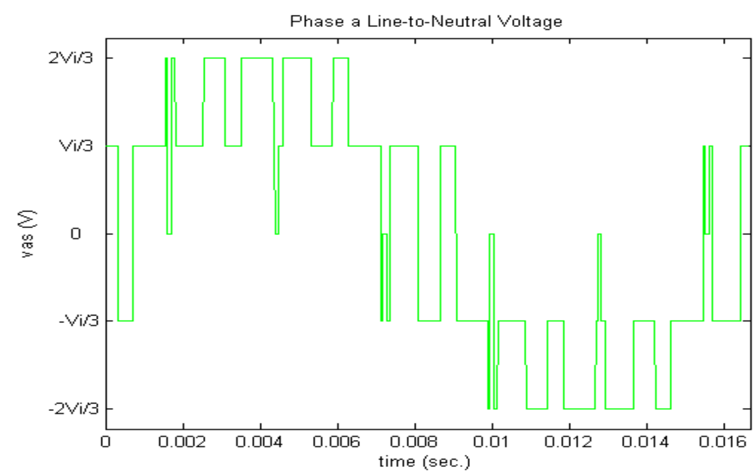
Six-Step with 180° Conduction



Six-Step with 120° Conduction

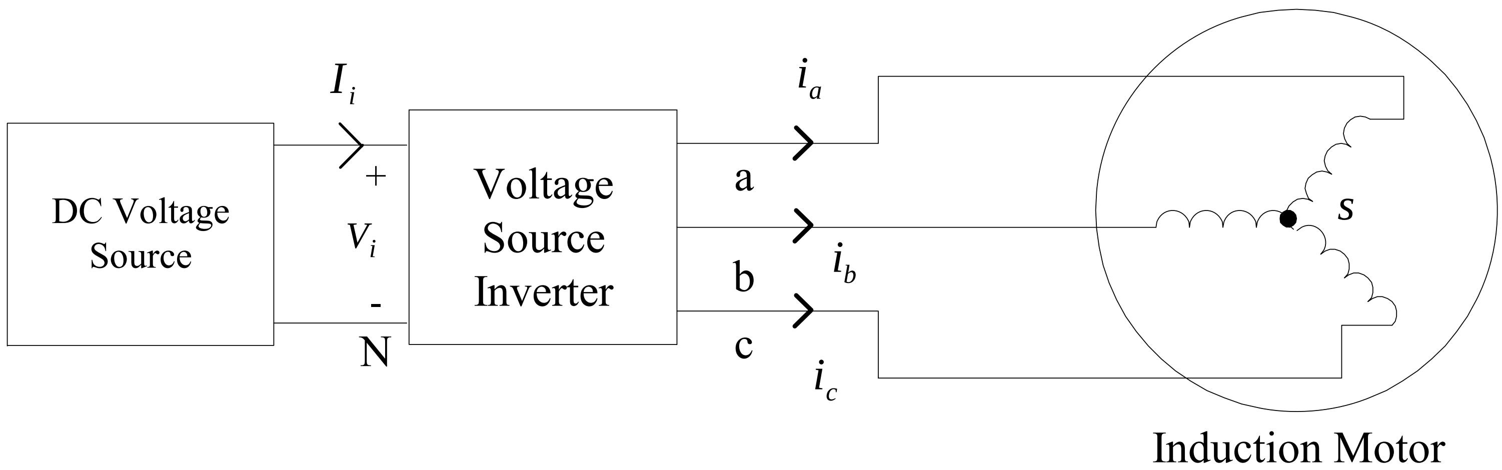


Space Vector PWM



Two-Level Sinusoidal PWM

Three-Phase Inverter Block Model



- Develop a Fourier series for:
  - Line-to-neutral voltage

Phase *a* Line-to-Neutral Voltage Fourier Series

- The Fourier series can be expressed in the following form for two-level sine PWM and space vector PWM [2]:

$$v_{as}(t) = \sum_{\substack{n=1 \\ n\neq 3k \\ k=1,2,3,\ldots}}^{\infty} C_n \cos\left(\frac{2\pi n}{T}t + \delta_n\right)$$

$$C_n = \sqrt{a_n^2 + b_n^2} \qquad \delta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$$

Phase *a* Line-to-Neutral Voltage Fourier Series

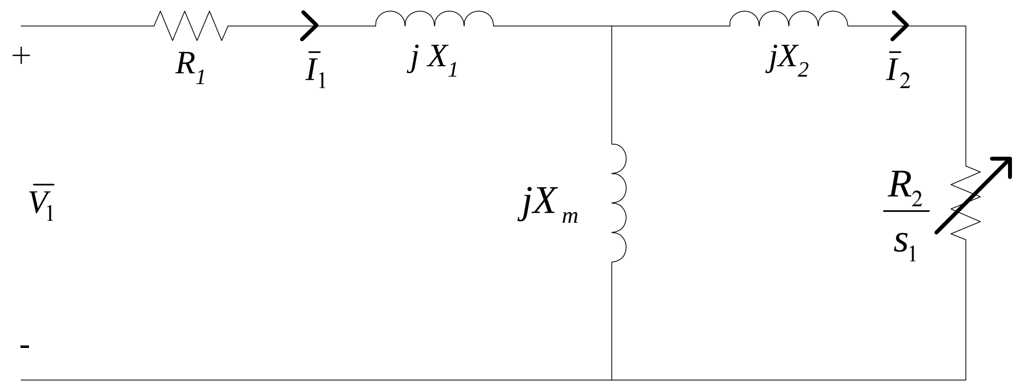
- The Fourier series of the six-step inverter phase *a* line-to-neutral voltage waveform with 120° conduction can be expressed as [3, 4]:

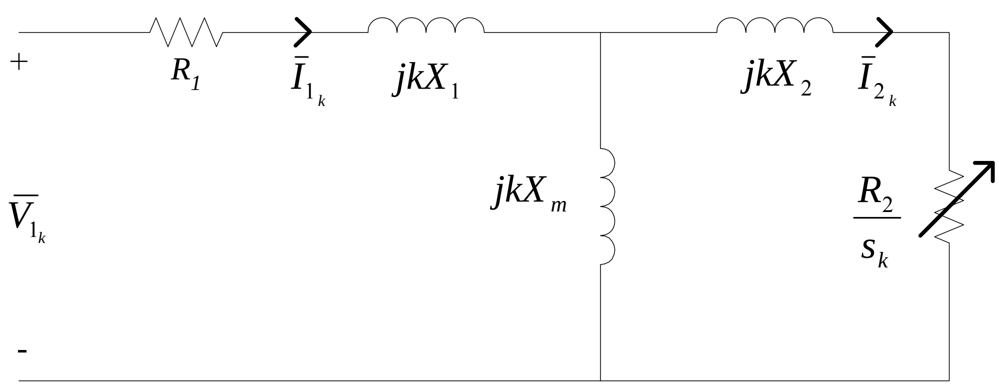
$$v_\phi(t) = \frac{\sqrt{3}}{\pi} V_i \left[\cos(\omega t + 30°) - \frac{1}{5}\cos(5\omega t + 30°) - \frac{1}{7}\cos(7\omega t + 30°) + \frac{1}{11}\cos(11\omega t + 30°) + \ldots \right]$$

- The Fourier series of the six-step inverter phase *a* line-to-neutral voltage waveform with 180° conduction can be expressed as [3, 4]:

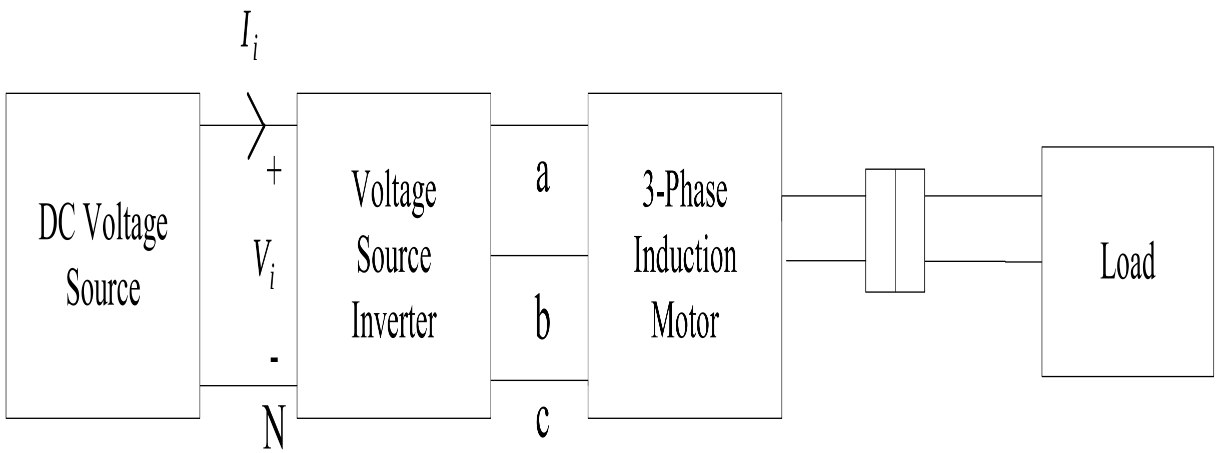
$$v_\phi(t) = \frac{2}{\pi} V_i \left[\cos\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t - \frac{1}{11}\cos 11\omega t + \ldots \right]$$

Induction Motor Circuit Models





Inverter Power Balance



- If a value of V_i is assumed at the input terminals of the inverter, a corresponding I_i can be found using a power balance.
- Inverter power balance:

$$V_i I_i = \sum_{k=1}^{\infty} \frac{3}{2} V_k I_k \cos\theta_k$$

V-I Characteristic Curves

- Varying V_i over a range of values produces a V-I load characteristic curve having the following form:

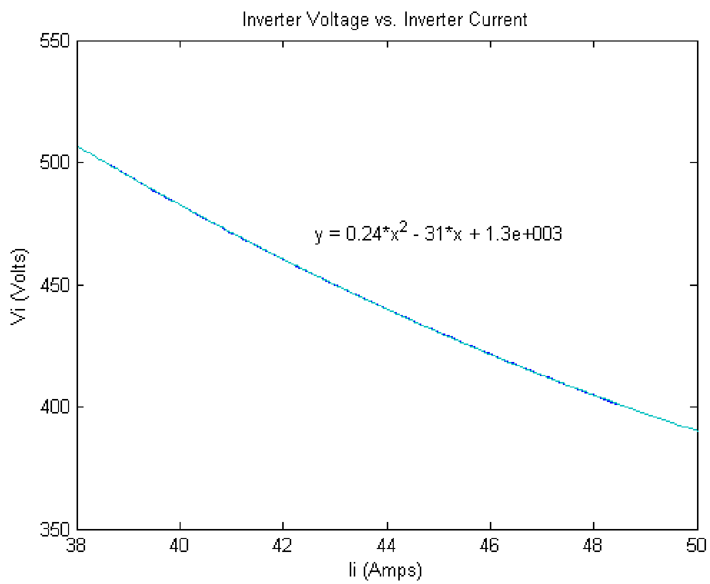
$$V(I_i) = aI_i^2 + bI_i + c$$

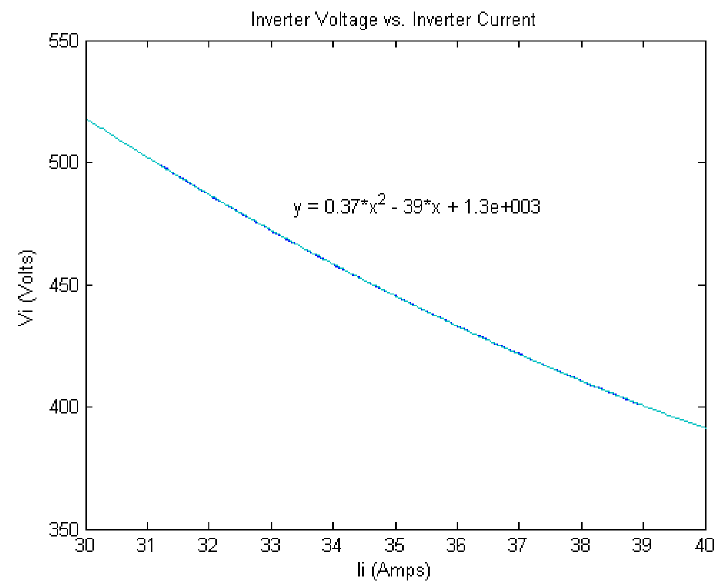
- The *polyfit* command in MATLAB can be used to curve fit the generated V-I data.

V-I Characteristic Curve Developed for a 50 Hp Motor and Inverter

- 50 Hp Motor-Drive System was analyzed using MATLAB
- The source voltage was varied over a range of 401V-500V with all other parameters remaining unchanged.
- Induction motor parameters: $f = 60$ Hz, $P = 4$, $R_1 = 0.087\Omega$, $R_2 = 0.228\Omega$, $X_1 = 0.302\ \Omega$, $X_2 = 0.302\ \Omega$, and $X_m = 13.08\ \Omega$

V-I Characteristic Curves Developed for a 50 Hp Motor and Inverter





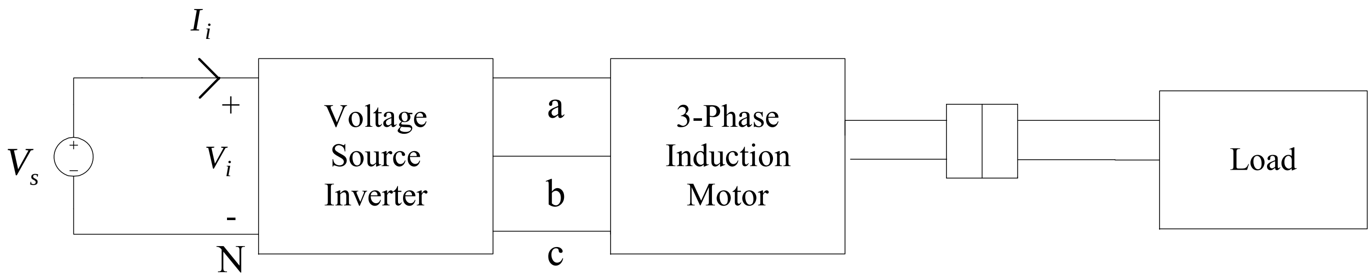
Two-Level Sinusoidal PWM Inverter

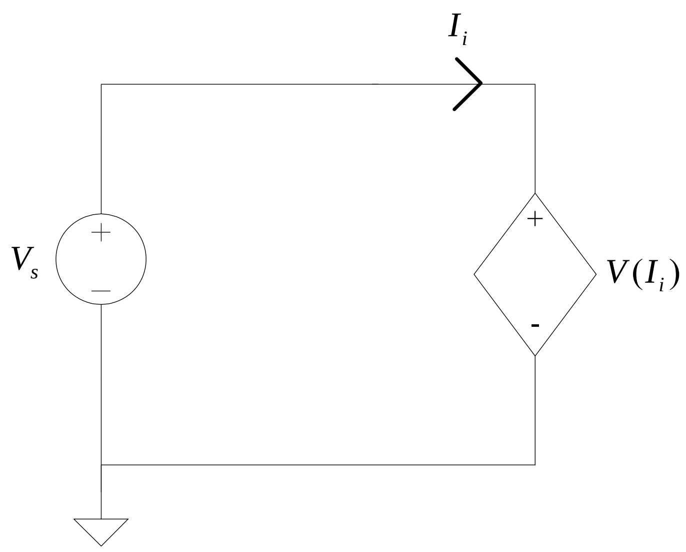
$m_a = 1.4$ $m_f = 15$ $T_L = 100$ N-m

Space Vector PWM Inverter

$M = 0.7$ $m_f = 15$ $T_L = 80$ N-m

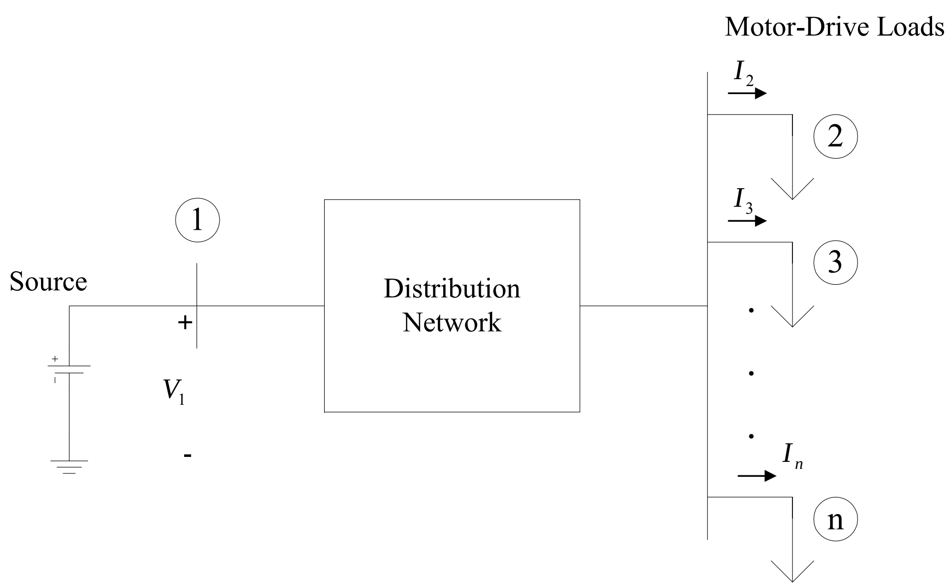
Simplified Motor-Drive System Model





- The inverter drive system can now be replaced by a current-controlled voltage source [1].

Multiple Motor-Drive Systems



- The simplified model can be extended to a system containing more than one motor-drive.
- Use the V-I characteristic curve of each motor-drive load.
- Incorporate the simplified model into an iterative procedure based on the Newton-Raphson method.

The network can be represented as [5]:

$$\tilde{I} = G\tilde{V}$$

The Bus Voltages

- Since the system studied contains motor-drive loads, each bus element of $\tilde{V}$ will have the following form:

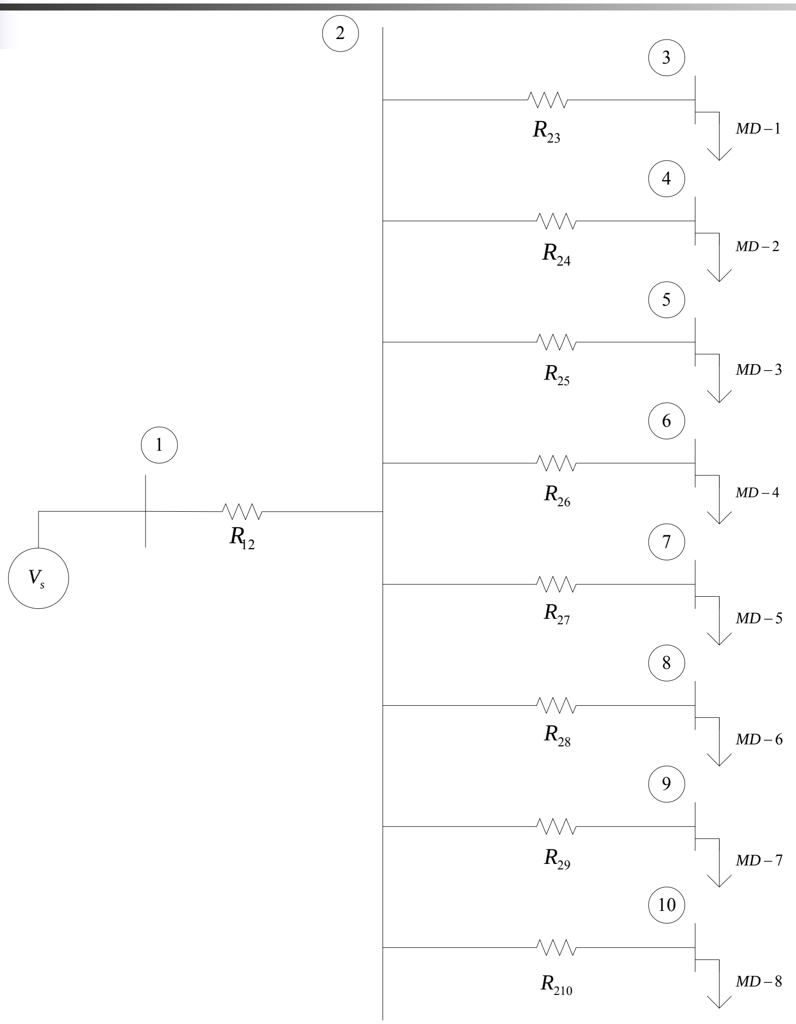
$$\tilde{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} V_1 \\ a_2 I_2^2 + b_2 I_2 + c_2 \\ a_3 I_3^2 + b_3 I_3 + c_3 \\ \vdots \\ a_n I_n^2 + b_n I_n + c_n \end{bmatrix}$$

(where, V_1 is the swing bus, and n is the number of buses)

A Newton-Raphson Based Method

- The network is a system of simultaneous nonlinear algebraic equations.
- An iterative procedure based on the Newton-Raphson method can be used to solve for the load currents [1].
- After the currents have converged, the bus voltages can be found by substitution into the voltage vector.

10-Bus System with Two-Level Sinusoidal PWM Inverter Drives



Bus Number	Load Torque (N-m)		Line Section	Line Resistance (mΩ)
1				
2			1 - 2	10
3	70		2 - 3	20
4	65		2 - 4	30
5	100		2 - 5	40
6	60		2 - 6	50
7	50		2 - 7	60
8	40		2 - 8	70
9	30		2 - 9	80
10	80		2 - 10	90

10-Bus System with Two-Level Sinusoidal PWM Inverter Drives

- The swing bus voltage was chosen to be 550V.
- The inverter parameters used for all of the inverters in the system were: f_1=60 Hz, m_a=1.4, and m_f=15.
- The induction motor equations, the two-level PWM inverter relationships, and the power flow equations were all coded in MATLAB.
- The voltage at each load bus of the system was varied over the range of 496V-595V, with all other parameters in the system remaining unchanged.
- Induction motor parameters: 50Hp, f = 60 Hz, P = 4, R_1 = 0.087Ω, R_2 = 0.228Ω, X_1 = 0.302 Ω, X_2 = 0.302 Ω, and X_m = 13.08 Ω

Power Flow Results for the Two-Level Sine PWM Inverter

Bus Number	Current from PSPICE (A)	Converged Current (MATLAB) (A)	ΔI (A)	Current Percent Error (% of PSPICE)	Voltage from PSPICE (V)	Converged Voltage (MATLAB) (V)	ΔV (V)	Voltage Percent Error (% of PSPICE)
3	24.6824	24.6512	0.0312	0.1262	547.7546	547.7583	0.00370	0.000675
4	22.9430	22.9104	0.0326	0.1422	547.5599	547.564	0.00410	0.000749
5	35.2968	35.253	0.0438	0.1241	546.8364	546.8412	0.00480	0.000878
6	21.2136	21.1773	0.0363	0.1710	547.1876	547.1924	0.00480	0.000877
7	17.7230	17.6864	0.0366	0.2063	547.1849	547.1901	0.00520	0.000950
8	14.2392	14.2022	0.0370	0.2595	547.2515	547.2571	0.00560	0.001023
9	10.7636	10.726	0.0376	0.3490	547.3871	547.3932	0.00610	0.001114
10	28.3157	28.2638	0.0519	0.1834	545.6998	545.7076	0.00780	0.001429

Advantages of the Analytical Method Presented

- Faster than simulation packages such as PSPICE
- Produces results that are comparable to other simulation packages
- The analytical method presented can be used regardless of the switching scheme employed in the inverter (two-level sinusoidal PWM, space vector PWM, etc.)

Advantages of the Analytical Method Presented

- The simplified model developed can be utilized in the analysis of a multiple-bus power system containing multiple motor-drive loads

- The simplified model can be used in DC or AC system analysis

References

- [1] A.W. Leedy and R.M. Nelms, “Analysis of DC Power Systems Containing Multiple Induction Motor-Drive Loads”, Proceedings of the 39th IEEE Southeastern Symposium on System Theory, Mercer University, Macon, GA, March 4-6, 2007, pp. 52-57.
- [2] A.W. Leedy and R.M. Nelms, “Harmonic Analysis of a Space Vector PWM Inverter Using the Method of Multiple Pulses”, Proceedings of the International Symposium on Industrial Electronics (ISIE2006), ETS Downtown, Montreal, Canada, July 9-13, 2006.
- [3] J.M.D. Murphy and F.G. Turnbull, *Power Electronic Control of AC Motors*, NY: Pergamon Press, 1988.
- [4] R.A. Pearman, *Power Electronics: Solid State Motor Control*, Reston Publishing Company, Inc., VA: 1980, pp. 170-182.
- [5] C.A. Gross, *Power System Analysis*, 2nd ed., John Wiley & Sons, Inc., NY: 1986, pp. 255-273.

Questions???