A Multi-Objective Formulation of ISO's Optimal Power Flow Problem[1]

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Outline

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A Nonlinear Optimization Problem

Decision Variables

Real/reactive power generation, voltage magnitude/angle, etc.

Objective Function

To minimize fuel costs/real power losses, or to maximize loadability, etc.

Constraints

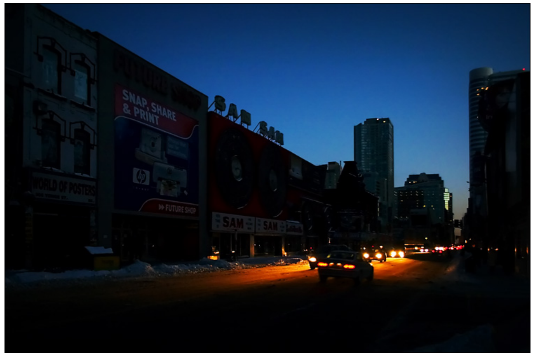
Thermal limits, Kirchhoff's laws, balancing constraint, voltage constraints, stability requirements, etc.

What Could Cause Component Failures?

- Lightning strike
- Fire
- Falling trees
- Harsh weather
- Overloading
- Terroristic vandalism

What Could Happen If They Fail?

- Cascading failure
- Loss of load
- System collapse
- Blackout



Can We Eliminate The Risk?

Yes, but No

- A tradeoff between **benefit** and **risk** is virtually inevitable
- Competition makes the deregulated power market more susceptible to blackouts

How to Reduce The Risk?

- On the hardware side
  - Sufficient production capacity
  - Transmission grid upgrade
- On the software side
  - **Ex ante planning (e.g., the conventional "N-k" criterion)**
  - Emergency response capability

Improving the "N-k" Criterion

Weaknesses of "N-k" Criterion

- Arbitrariness, rule of thumb
- Important information is ignored: probabilities of transmission line failures, topology of the network, etc.
- Sometimes "N-1" not reliable enough, but "N-2" often too expensive

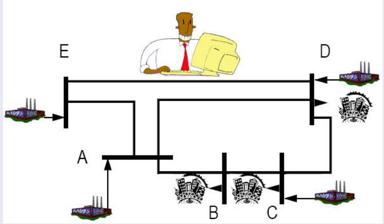
How To Improve "N-k" Criterion?

- Introduce utility function and probabilistic criteria for optimization
- Quantitatively take into account the probabilities of transmission line failures and network topology information
- Provide the class of optimal solutions

Assumptions On The ISO's OPF Problem

DC Lossless Transmission Network Setup

- A set of nodes $\mathcal{N}$ connected by a set of lines $\mathcal{L}$
- Demand function: $q_n^c \mapsto a_n^c - b_n^c q_n^c, \ \forall n \in \mathcal{C} \subseteq \mathcal{N}$
- Supply function: $q_n^p \mapsto a_n^p + b_n^p q_n^p, \ \forall n \in \mathcal{P} \subseteq \mathcal{N}$



Independent System Operator (ISO)

- Determines consumption q_n^c and production q_n^p at each node $n \in \mathcal{N}$
- To maximize a benefit (social welfare) function $B(q)$
- Subject to linearized thermal and balancing constraints

OPF Formulation

$$\max_q \quad B(q) = \sum_{n \in \mathcal{C}} \left[a_n^c q_n^c - \frac{1}{2} b_n^c \left(q_n^c\right)^2 \right] - \sum_{n \in \mathcal{P}} \left[a_n^p q_n^p + \frac{1}{2} b_n^p \left(q_n^p\right)^2 \right]$$

$$\text{s.t.} \quad \left| \sum_{n \in \mathcal{C}} PTDF_{l,n} q_n^c - \sum_{n \in \mathcal{P}} PTDF_{l,n} q_n^p \right| \le C_l, \ \forall l \in \mathcal{L}$$

$$\sum_{n \in \mathcal{C}} q_n^c - \sum_{n \in \mathcal{P}} q_n^p = 0$$

$$q_n^c \ge 0, \ \forall n \in \mathcal{C}; \ 0 \le q_n^p \le \overline{q_n^p}, \ \forall n \in \mathcal{P}.$$

Matrix Form

$$\max_{q \in \mathbb{R}_+^m} \left\{ B(q) = c^\top q + \frac{1}{2} q^\top Q q : Aq \le b \right\}$$

Contingency Scenario Considerations

Only Consider Transmission Line Failures

- Every transmission line has two states: working or failed
- Denote as $\mathcal{S}$ the complete set of scenarios in terms of transmission line availabilities, then $|\mathcal{S}| = 2^{|\mathcal{L}|}$
- For all $s \in \mathcal{S}$, denote as p_s the probability for scenario s to occur, then $\sum_{s \in \mathcal{S}} p_s = 1$
- Denote as $A^s q \le b^s$ the set of constraints under scenario s

Outline

To Maximize Social Welfare

OPF Under The Best Scenario s^0

$$\max_{q \in \mathbb{R}_+^m} \left\{ B(q) = c^\top q + \frac{1}{2} q^\top Q q : A^{s^0} q \leq b^{s^0} \right\}$$

Problem: Too Risky

- Objective value $B(q^*)$ is large, but optimal solution q^* is only guaranteed to be feasible under s^0
- Should some lines fail during power transmission, q^* could become infeasible, and the system would have to be shut down

To Minimize Risk

OPF Under All Possible Scenarios

$$\max_{q \in \mathbb{R}_+^m} \left\{ B(q) = c^\top q + \frac{1}{2} q^\top Q q : A^{s} q \leq b^{s}, \forall s \in \mathcal{S} \right\}$$

Problem: Too Conservative

- There is zero risk of infeasibility, but objective value is too small
- In fact, the entire transmission network is then idle

How Does "N-k" Criterion Balance These Objectives?

"N-k" Criterion

$$\max_{q \in \mathbb{R}_+^m} \left\{ B(q) = c^\top q + \frac{1}{2} q^\top Q q : A^{s} q \leq b^{s}, \forall s \in \mathcal{S}^{\text{N-k}} \right\}$$

where $\mathcal{S}^{\text{N-k}}$ is the subset of scenarios with at most k out of N lines failed

Problem: Selection of Scenarios May Be Inefficient

- **Considering** failures of **reliable** and/or **uncritical** lines, and
- **Ignoring** failures of **unreliable** and/or **critical** lines

Formulation and Explanation

$$\max_{x \in \mathbb{B}^{|\mathcal{S}|}, q \in \mathbb{R}^m} \left\{ f(x) B^\alpha(q) : A^s q \leq b^s \text{ if } x_s = 1, \forall s \in \mathcal{S} \right\}$$

- For all $s \in \mathcal{S}$, binary decision variable x_s indicates whether scenario s should be considered ($x_s = 1$) or not ($x_s = 0$)
- $f(x) = \sum_{s \in \mathcal{S}} p_s x_s$ is a feasibility function
- $f(x) B^\alpha(q)$ is a utility function
- α is a risk preference parameter
  - $\alpha = 1$: risk neutral
  - $0 \leq \alpha < 1$: risk averse
  - $1 < \alpha < +\infty$: risk loving

Multi-Objective Formulation vs. "N-k" Criterion

Multi-Objective Formulation

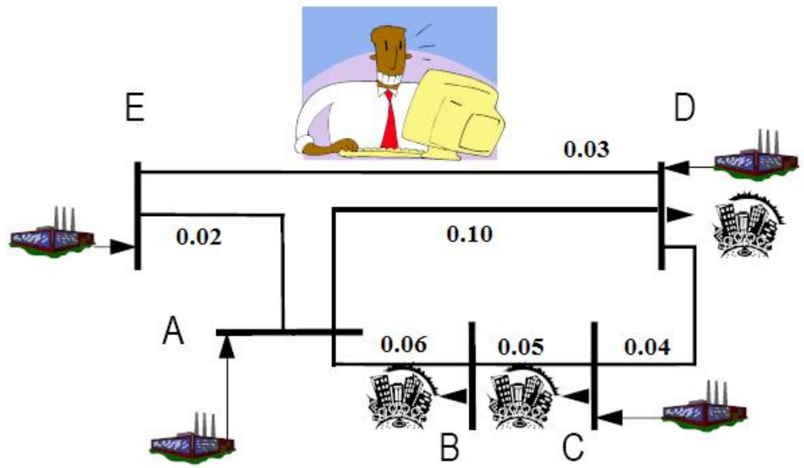
$$\max_{x \in \mathbb{B}^{|\mathcal{S}|}, q \in \mathbb{R}^m} \left\{ \left(\sum_{s \in \mathcal{S}} p_s x_s \right) B^{\alpha}(q) : A^s q \leq b^s \text{ if } x_s = 1, \forall s \in \mathcal{S} \right\}$$

"N-k" Criterion

$$\max_{q \in \mathbb{R}^m_+} \left\{ B(q) : A^s q \leq b^s, \forall s \in \mathcal{S}^{\text{N-k}} \right\}$$

where $\mathcal{S}^{\text{N-k}}$ is the subset of scenarios with at most k out of N lines failed

Multi-Objective Formulation vs. "N-k" Criterion: A Small Example



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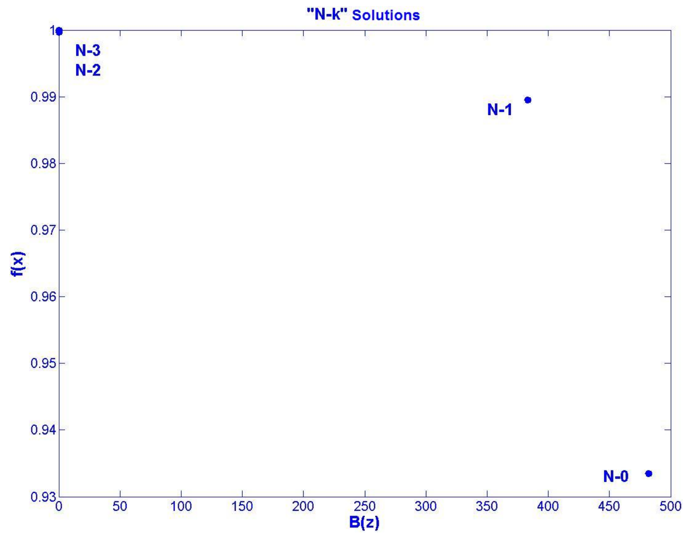

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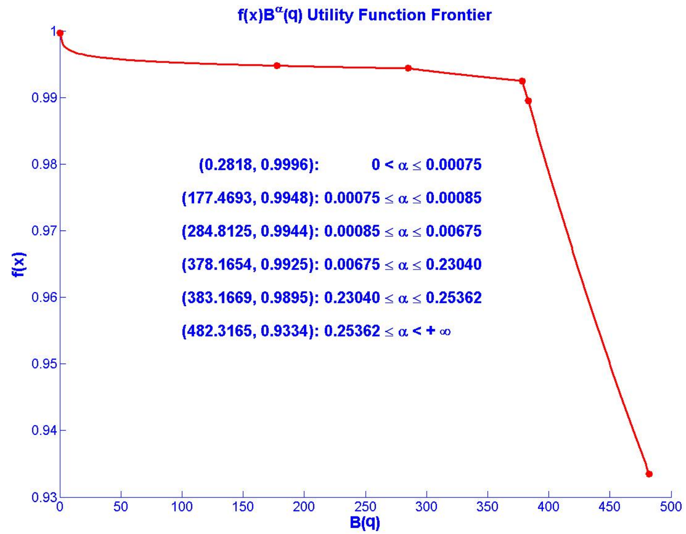

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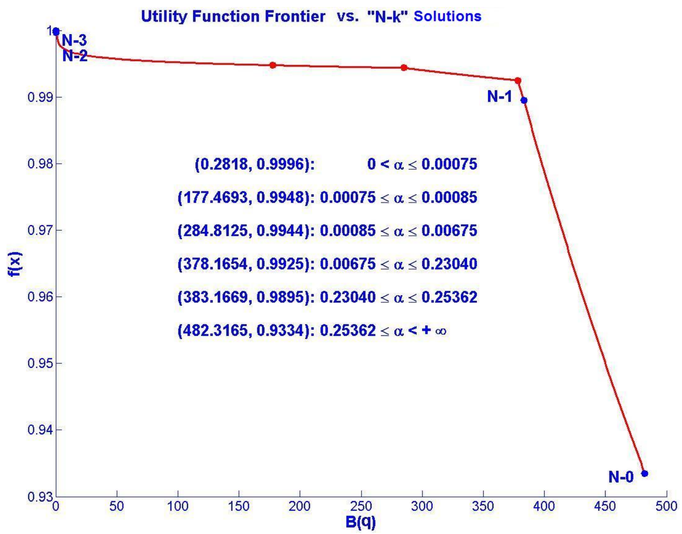

Outline

$$\max_{x\in\mathbb{B}^{|\mathcal{S}|},q\in\mathbb{R}^m}\left\{f(x)B^{\alpha}(q) : A^s q \le b^s \text{ if } x_s = 1, \forall s \in \mathcal{S}\right\}$$

1. Solve for (η^*, x^*) to the master problem:

$$\text{(PM)}: \max_{\eta\in\mathbb{R}_+,x\in\mathbb{B}^{|\mathcal{S}|}}\left\{f(x)\eta^{\alpha} : \text{ cuts, if any }\right\}.$$

2. Solve for q^* to the subproblem:

$$\text{(PS)}: \max_{q\in\mathbb{R}^m}\left\{B(q) : A^s q \le b^s + (1 - x_s^*)d, \forall s \in \mathcal{S}\right\}.$$

3. If $\eta^* \le B(q^*)$, then stop, and (η^*, x^*) is the utility function optimal solution for α. Otherwise add the following cut to (PM) and go back to step 1:

$$\eta + (d \otimes \lambda^*)^{\top} x \le B(q^*) + (d \otimes \lambda^*)^{\top} x^*.$$

1. Obtain $\left(\underline{B}, \overline{f}\right)$ and $\left(\overline{B}, \underline{f}\right)$.
2. Define $\mathcal{B} = \left\{\left(\underline{B}, \overline{B}\right)\right\}$, $\mathcal{F} = \left\{\left(\overline{f}, \underline{f}\right)\right\}$, and $\mathcal{P} = \left\{\left(\underline{B}, \overline{f}\right), \left(\overline{B}, \underline{f}\right)\right\}$.
3. **If** $\mathcal{B} \neq \emptyset$ and $\mathcal{F} \neq \emptyset$, then
   - Let (B_1, B_2) and (f_1, f_2) be the first elements in $\mathcal{B}$ and $\mathcal{F}$, respectively;
   - Calculate $\alpha = (\log f_2 - \log f_1)/(\log B_1 - \log B_2)$;
   - Solve for an optimal solution (B_α^*, f_α^*);
   - If $(B_\alpha^*, f_\alpha^*) \neq (B_1, f_1)$ and $(B_\alpha^*, f_\alpha^*) \neq (B_2, f_2)$, then set $\mathcal{B} = \mathcal{B} \cup \{(B_\alpha^*, B_1), (B_\alpha^*, B_2)\}$, $\mathcal{F} = \mathcal{F} \cup \{(f_\alpha^*, f_1), (f_\alpha^*, f_2)\}$, and $\mathcal{P} = \mathcal{P} \cup (B_\alpha^*, f_\alpha^*)$;
   - Set $\mathcal{B} = \mathcal{B}\backslash(B_1, B_2)$, $\mathcal{F} = \mathcal{F}\backslash(f_1, f_2)$; and
   - repeat step 3.

Otherwise stop, and $\mathcal{P}$ is the set of optimal solutions on the utility function frontier.

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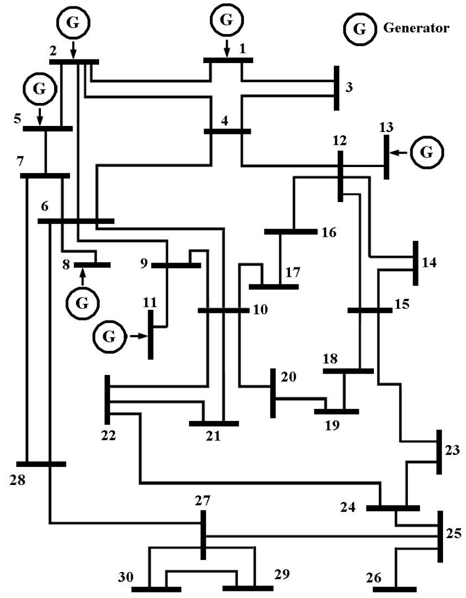
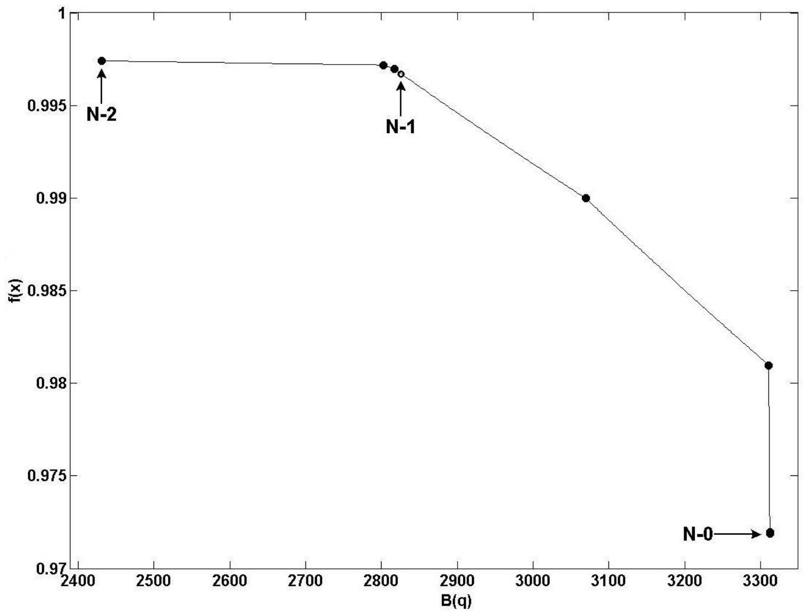
- 6 supply buses and 21 demand buses
- Probabilities of transmission line failures randomly generated from a uniform distribution U(0, 0.015)





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Conclusion

Contributions

- The multi-objective formulation proposes a new methodology to deal with uncertainties in optimization problems
- The utility function frontier provides **more** and **better** tradeoffs than the "N-k" criterion

Future Research

- Further improving algorithms
- Applying methodology to larger networks