Determining the Nash Equilibrium of 'Black-box' Electricity Markets

Nguyen, D. H. M., Wong, K. P., and Ilic, M.

***Abstract*— This paper presents a stochastic method for determining the Nash equiquilibrium of arbitrarily defined electricty markets. The new method make direct use of the definition of Nash equilibrium to search for the required market outcome. The guided stochastic search mechanism of an evolutionary algorithm is employed by the new method. The evolutionary algorithm is guided by an appropriately defined fitness mapping that captures the Nash equilibrium condition. Direct use of the Nash equilibrium property free the new method from dependence on the underlying market model. The new method is validated with a comparative study involving linear, analytic, and general (i.e. non-analytic) electricity markets. In sum the method allows the Nash equilibrium of arbitrary electricity markets to be examined under a single common framework, which reveals that common approximation of quadratic costs are not alway appropriate.**

***Index Terms*— Electricity market, Nash equilibrium, and evolutionary algorithm.**

I. INTRODUCTION

THE power industry worldwide have under gone and continues to under go restructuring and deregulation. This process generally results in a market oriented framework of many interdependent enties working together to provide generation transmission and distribution of electricity for society. These market entities includes generation companies, large consumers, retail distributors, wholesale traders, market and system operators, and transmission and ancinary service providers.

Each of the entities have their own independent objectives, which can only be achieved in an interdepedent manner, as the action of each entity is restrained by the reaction of other entites in the market. Unfortunately since the new market system has such multiple independent and uncomaprable objectives, traditional single objective optimisation techniques common in engineering can not be employed to determine the system outcome. Instead equilibrium concepts such as those of Nash and Pareto must be employed.

Yet it is useful to know the likely outcome and resulting resource allocation patterns as this would allow us to determine whelther such pattern is socially desireable and what avaliable remedial actions are effective. For example to engineer appropriate regulatory and technical measures for the emerging markets, it is necessary to predict the kind of equilibrium a market can naturally arrive at, given some inherent characteristics of the participants under a given market structure.

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With reliable quantitative models market designers can test out different market scenarios to determine how to best restructure the industry or plan for growth. For example knowledge of the expected equilibrium utilisation of generator capacity and the level economic profit at equilibrium would reveal which subdivision of generation infrastructure is stable in the longer term. This information can be used to guide the disaggregation of regulated utilities into competing gencos, and to assess meger and takeover propositions.

In seeking to model the outcome(s) for the multiple objective market system there are two fundamental difficulties: first is the dificulty in quantifying the behaviour of some of the market entities; and the second is determining what would constitute an outcome for the decentralised market system. Although the second issue is more fundamental in nature, both issues has to be addressed adequately before any predicted outcome can have any validity.

Much research has been conducted on this subject in the fields of micro-economics and games-thoery, and there is currently renewed interest in the subject from the power industry. The effort has resulted in various representations for market entities ranging from characteristic functions, to dynamical agents, and even evolving population(s). Likewise market outcomes have been characterised by various concepts including societal optimality, dynamic equilibrium, Pareto optimality and Nash equilibrium (NE).

A. Nash Equilibrium

Among the concepts of market outcome, the NE predominates due to its solid mathematical foundation and general applicability [1]. The NE $\{\boldsymbol{s}_1, \cdots, \boldsymbol{s}_M\}$, of a market with M participants, denumerated by participant set $\mathcal{M} = \{1, \cdots, M\}$, with individual strategy $\boldsymbol{s}$, strategy space $\mathcal{S}$, and corresponding outcome π, is given by (1). In essence (1) states that the NE of the system is any set of all participants strategy where a unilateral deviation by any participant would result in an outcome for the participant which is inferior to that of the original NE.

$$\pi_i(\boldsymbol{s}_1, \cdots, \boldsymbol{s}_i, \cdots, \boldsymbol{s}_M) \geq \pi_i(\boldsymbol{s}_1, \cdots, \boldsymbol{s}_i^\star, \cdots, \boldsymbol{s}_M) \quad \forall \boldsymbol{s}_i^\star \in \mathcal{S}_i, \quad \forall i \in \mathcal{M}, \quad \boldsymbol{s}_i^\star \neq \boldsymbol{s}_i \tag{1}$$

When participants charactersitics are analytic the solutions for NE invariably results in the NE being given as the solution of an algebraic or differential equation system. These systems are then to be solved for using numerical techniques: of linear, quaradtic programming and linear/nonlinear complementarity for algebraic systems; and of dynamic programming and numerical integration for diferenntial systems [2]–[4].

This analytical approach while most efficient, required the underlying market model to be analytical and the resulting equation sytem to be ameneable to the available numerical techniques. To over come this limitation to allow for more complex and realistic market models to be examined various non–analytical methods have been applied to find the NE. Prominent amongst these approaches is the use of agents in their corresponding ecosystems, to represent participants in the market environment.

This agent–ecosystems approach simulates a participant's behaviour in a market by embedding a learning mechanism, a market inferencing engine, and the participant's characteristic and objective, in software algorithms. These algorithms are then executed interactively and the collective convergent behaviour is taken as the outcome of the market. Applications of electricity market simulations using agents includes coevolution algorithms, market dynamics and iterative systems [5]–[7].

Although the non–analytic agent approach is highly flexible, it has two fundamental weaknesses: firstly the agent model may not adequately simulate participant behaviour, consequently the simulated time dyanmics may not be representative; secondly, the converged steady state may not corresponds to the system's NE. To date the only agent approach which can demonstrate that its steady state outcome represents a NE is the evolutionary stable strategies of evolutionary games thoery [8].

The evolutionary games thoery approach uses a coevolutionary framework to evolve stable populations of strategies, these stable strategies, if they exists, are also NE strategies, however not all NE are evolutionary stable. Although this approach can find NE which are also evolutionary stable, it may fail to find any NE for an electricity market where there are valid NE which are evolutionary unstable.

II. Direct Nash Equilibrium with Evolutionary Algorithm

This paper presents a new approach to determining the NE which also uses evolutionary computation, however conceptually it is not an agent model as there is no attempt at simulating individual participants behaviour. The new approach use an evolutionary algorithm to perform a guided stochastic search for a set of participants strategy that is likely to be a NE. The search is guided directly by the condition of (1) which ensures convergence to the NE.

The direct use of the Nash condition *only* requires information on participants payoffs $\boldsymbol{\pi}$ for a given participants' strategy $\boldsymbol{s}$, and not the explicit form of the mapping of $\boldsymbol{s} \rightarrow \boldsymbol{\pi}$. This implies that the underlying market system and participants characterisitics can be arbitrary 'black–boxes' as only a set of input output pairs, $(\boldsymbol{s}, \boldsymbol{\pi})$, are required. However, being a stohastic method this approach is naturally more computationally intensive then the analytical approaches.

The basic difference between guided and un–guided stochastic search mechanism is that the first uses information of previous samples in order to bias future sampling to improve the search, where as the second have no such sampling bias. The un–guided meachisms are predominately Monte–Carlo simulations, where as there are many different guided search mechanism such as simulated annealling, particle swarm, and evolutionary algorithms (EAs). For this paper the guided search mechanism of EA is choosen.

EA's allows very arbitrary systems to be represented via its genetic encoding of problem variables and its good performance on abitrary systems is extensively documented [9]. Essentially EAs operates by incrementally improving a collection of candidate solutions with each iteration, these improvements arises as the genetic operator(s) randomly introduces new candidates, and the selection operator(s) eliminates less fit candidates. The particular EA used is the basic Evolutionary Programming (EP) algorithm, its structure is illustrated below in Figure 1.

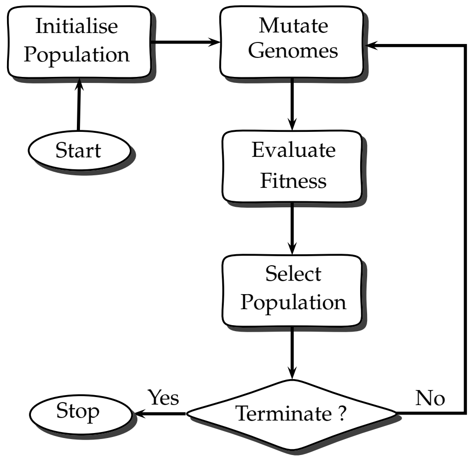

Fig. 1. Evolutionary Programming Algorithm

The EP of Figure 1 achieves guided stochastic search by first initialising a population of L genomes $\mathcal{S} = \{{}^{i}\mathsf{s} \mid i \in \mathcal{L}\}$, where set $\mathcal{L} = \{1, \cdots, L\}$ denumerates the individual genome, with each genome s representing a candidate solution for the search, the fitness of these initial L genomes in solving the search problem is then evaluated. Next for each iteration k, a new offspring genome is created by random perturbation from each of the current L genomes via a genetic operator, and the fitness of the offsprings are calculated. Using the fitness values of the $2 \cdot L$ original and offspring genome, a selection operator is then applied to choose L genomes for the next iteration.

This process of random sampling about a population that is biased by fitness selection is repeated until a candidate solution genome with a sufficient fitness emerges from the population. Thus it can be seen the stochastic component of the search is implemented by the genetic operator acting on the current genome distribution, whereas as the guided dimension is provided by the selection operator acting on the current genome fitness distribution.

Application of the EP to find the NE of a market system requires: the specification of an effective genome representation for the participants strategies; the tayloring genetic operators to enable effective exploration of the solution space; and the design of an appropriate fitness mapping that captures NE condition. It is the synthesis of these three requirements together with the underlying idea of using guided stochastic search to directly find the NE, that constitute the novel contributions of this paper.

In the published literature there exists other scalar optimisation approaches to the NE problem, the most popular being the application of a Liapunov function over the payoff space to induce a scalar measure for NE suitability [10], [11]. However, to the best of the author's knowledge no attempts has been made to determine the NE by direct use of the raw Nash condition of (1). With this direct and exclusive use of the Nash condition, the new approach can determine solutions for electricity markets, or other thoeretic game constructs, by treating all systems as pure input–output 'blackboxes'.

A. Genome Representation and Mutation

A matrix genome structure s given in (2) is used to encode the M *active* participants market strategies as a candidate solution. A matrix row $\boldsymbol{s}$ represents a participant's strategy vector, and it is assumed that all participants have the same N degrees of freedom in choosing their strategies. If the number of strategies avaliable to each participants is different, then N can be set to the number of strategies of the participants with the most strategies. The 'extra' strategies of any participants can simply be ignored.

$$\mathsf{s} = \begin{bmatrix} \boldsymbol{s}_1 \\ \vdots \\ \boldsymbol{s}_M \end{bmatrix} = \begin{bmatrix} s_{1,1}, & \cdots, & s_{1,N} \\ \vdots & \ddots & \vdots \\ s_{M,1}, & \cdots, & s_{M,N} \end{bmatrix} \tag{2}$$

The genetic operator in EP is known as mutation, this operator randomly generate a new genome $\dot{\mathsf{s}}$ from an existing s one at a time. The mapping $\mathsf{s} \mapsto \dot{\mathsf{s}}$ used is given by (3), where i^{th} row vector is chosen randomly each time, and $\mathcal{N}$ is a vector of random variables normally distributed with zero means and a deviation vector $\boldsymbol{\sigma}$. The component σ_j of $\boldsymbol{\sigma}$ is defined so that the deviation is proportional to the extent, $\overline{s}_j - \underline{s}_j$, with the proportionality constant ρ controlling the localisation of $\dot{\mathsf{s}}$ to s.

$$\dot{\mathsf{s}} = \mathsf{s} + \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathcal{N}(\mathbf{0}, \boldsymbol{\sigma}_i) \\ \vdots \\ \mathbf{0} \end{bmatrix}, \quad \text{where} \quad \boldsymbol{\sigma}_i = \begin{bmatrix} \sigma_{i,1}, & \cdots, & \sigma_{i,N} \end{bmatrix}$$
$$\text{and} \quad \sigma_{i,j} = \rho \cdot \left(\overline{s}_j - \underline{s}_j \right) \tag{3}$$

B. Nash Equilibrium Fitness and Selection

In order to guide the stochastic search a figure of merit f, to indicate how close a genome is to satifying the NE condition of (1), has to be assigned to each s to represent its fitness for the selection operator. Equation (1) can not be directly used to determine f since it is impractical to exhaustive tests over $\mathcal{S}_1 \times \cdots \times \mathcal{S}_M$. Even if exhaustive testing is achieved the outcome for s is either that it is or it is not a NE. When s is not a NE there is no of indication how close it is to a NE, yet EAs are implicitly dependent on such proximity information to guide its search.

To form an appropriate f to measure how close s is to a satisfying the NE condition, the consistency test of (4) is employed. This test simply compares s to an arbitrary $\mathsf{s}^\star$ via (4) and if the inequality holds then s is found to be *consistent* with being a NE, although it may not necessary be one. The test of (4) can be repeated T times, with a randomly chosen $\mathsf{s}^\star$ each time. The number of times $t \in [0; T]$ that s is found to be consistent can be counted and the frequency of consistency can be computed. The significance of using a frequency measure is that it is applicable to both continuous and discrete systems.

$$\pi_i \left(\boldsymbol{s}_1, \cdots, \boldsymbol{s}_i, \cdots, \boldsymbol{s}_M \right) \geq \pi_i \left(\boldsymbol{s}_1, \cdots, \boldsymbol{s}_i^\star, \cdots, \boldsymbol{s}_M \right)$$
$$\boldsymbol{s}_i^\star \in \mathcal{S}_i, \quad \forall i \in \mathcal{M}, \quad \boldsymbol{s}_i^\star \neq \boldsymbol{s}_i \tag{4}$$

The consistency frequency can be used to defined f per (5) since it is expected that a genome close to a NE would have have higher consistency frequency then one further away. An illustration of a fitness landscape induced by this definition of fitness is illustrated in Figure 2, for a two producer Cournot market with volume strategies $\boldsymbol{s}_1 = q_1$ and $\boldsymbol{s}_2 = q_2$, quadratic costs, affine demand and $T = 100$ Nash consistency tests per fitness evaluation. As can be seen fitness definition of (5) induces a fitness ridge along the participants best reaction curves, shown as the vertical walls cutting through the landscape, and a peak at their intersection which is the NE.

$$f = \frac{t}{T} \tag{5}$$

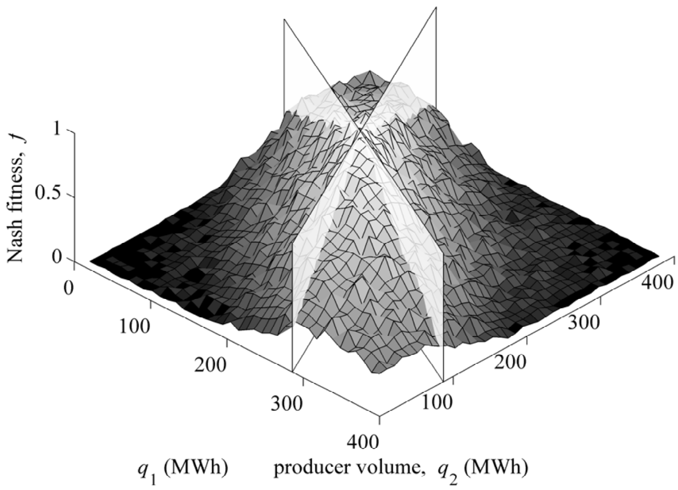

Fig. 2. Induced Fitness Landscape

The selection operator select a genome to survive into the next generation based on its fitness relative to the fitness of other genomes. The particular selection operator used by EP is tournament selection. This method of selection chooses the genome that is the winner of a contest which compares constestants' fitness. Each tournament has a fixed number of l contestants which are randomly chosen from the population without replacement. By varying the number of constestants the level of deteministic and randomness of the selection process can be set. At one extreme when $l = 1$ selection is purely random, at the other extreme when $l = 2 \cdot L$ selection is totally deterministic with genome selected strictly according to their fitness ranking.

III. Electricity Market Applications

The method of directly finding the NE with a stochastic search as described previously is a very general technique as it imposes no condition on the participants underlying characteristics or their payoff function. That is there is no requirements for the system to be low order or analytic, in fact the internals of the system is not even required to be explicitly defined. All that need to be known is the payoff of each particpants for any valid strategy set, consequently the method can be appllied to 'black box' market model where only input-output pairs, of participant strategies-payoffs, are defined.

Although the new method can handle complex markets models, the paper will focus on simple models with well define solutions to allow comparison with existing approaches. Specifically three cases of the Cournot electricity markets are examined: a linear market; an analytic market; and a general or non-analytic market. The old classical approaches can solve the linear and analytic markets cases but can not handle the general market case. In constrast the new direct method uniformly handle all three cases. The remainder of the paper will give a comparative study of the equilibirum outcomes of the three cases found by the new and old methods.

A. Market Model and Participants Characterisations

In a Cournot electricity market the active participants are the producers, with each producer having a single strategy variable $\boldsymbol{s} = q$ where q is their production volume. Consequently, with $N = 1$, the market strategy simplifies to (6). Consumers in the market are *not* considered to be active participants as they are assumed to only react to producers strategic behaviour according to a predefined characteristic $p(\mathsf{s})$, where p is a market price that is accepted by all producers.

$$\mathsf{s} = \boldsymbol{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_M \end{bmatrix} \tag{6}$$

With (6) the consumer characteristic $p(\mathsf{s})$ becomes $p(\boldsymbol{q})$, which for Cournot markets is further simplifies to $p(Q)$ where $Q = \boldsymbol{q} \cdot \mathbf{1} = \sum_{i \in \mathcal{M}} q_i$ is the aggregate supply quantity which will be taken by consumer at price p. The consumer demand characteristic considered in the paper is an affine characteristic define in (7). Affine demand is a first order function that is often used to approximate other demand characteristics as it is the simplest approximation possible. The parameters used are $A = -1/3$ \$/T(Wh)^2 and $B = 200$ \$/MWh.

$$p(Q) = A \cdot Q + B \tag{7}$$

Producers participanting in the market are modelled with a combination of a characteristic function $k(q)$ and a payoff or objective $\pi(\mathsf{s}, k)$ that its seeks to optimise. The characteristic function relates the monetary costs k to producers of generating a volume q of electricity. The demand price function $p(\mathsf{s})$ is common to all producers and represent the aggregated reaction, in the form of a uniform market price p, of all consumers to the action of producers as expressed in s. However, with $\mathsf{s} = \boldsymbol{q}$ and $p(\mathsf{s}) = p(Q)$ per (7), the payoff $\pi(\mathsf{s}, k)$ simplifies to (8).

$$\pi = p \cdot q - k(q) \tag{8}$$

Three producer cost characteristics k are considered in the paper, the first is quadratic costs, the second is higher order analytic, and the third is non-analytic per (9). Quadratic costs is the lowest order charactersitics that is can characterise nonlinear costs. The general quadratic–sinusiodal costs is a more realistic model of generation costs that factored in the cost of valve point loading losses [12]. The analytic quadratic–sinusiodal costs is an analytic approximation of the generator valve point loading costs. The values of parameters a, b, c, d, e, $\underline{q}$ and $\overline{q}$, of the $M = 3$ producers considered is given in Table I.

$$k(q) = \begin{cases} a \cdot q^2 + b \cdot q + c & \text{: quadratic} \\ a \cdot q^2 + b \cdot q + c + d \cdot \sin^2\left(e \cdot (q - \underline{q})\right) & \text{: analytic} \\ a \cdot q^2 + b \cdot q + c + d \cdot \left|\sin\left(e \cdot (q - \underline{q})\right)\right| & \text{: general} \end{cases} \tag{9}$$

TABLE I
Producers Costs Parameters.

i	a \$/(MWh)^2	b \$/MWh	c \$	d \$	e 1/MWh	$\underline{q}$ MWh	$\overline{q}$ MWh
1	$.280/10^3$	8.10	550.	300.	.035	0.00	680.
2	$.560/10^3$	8.10	309.	200.	.042	0.00	360.
3	$3.24/10^3$	7.74	240.	150.	.063	60.0	180.

B. Market Cases and Classical Solutions

When costs are quadratic and demand is affine the Cournot market is linear in the sense that the equilibrium is described by a system of linear equations. Explicitly applying the NE condition $\frac{\partial \pi_i}{\partial q_i} = 0$, $\forall i \in \mathcal{M}$ result in the equation set $p + q \cdot \frac{\mathrm{d}p}{\mathrm{d}Q} - \frac{\mathrm{d}k}{\mathrm{d}q} = 0$, $\forall i \in \mathcal{M}$ with the constraint $Q = \sum_{i \in \mathcal{M}} q_i$. These equations evaluates to linear system (10) when demand $p(Q)$ is affine and costs $k(q)$ is quadratic. The equilibrium volume vector $\boldsymbol{q}^*$ can be solved for numerically by simply inversing system (10) to get $\boldsymbol{q}^* = -\mathsf{A}^{-1} \cdot \boldsymbol{b}$.

$$\mathsf{A}\cdot\boldsymbol{q}+\boldsymbol{b}=\boldsymbol{0},\quad \text{where}\quad \boldsymbol{b}=\begin{bmatrix}B-b_1\\B-b_2\\\vdots\\B-b_M\end{bmatrix}\quad\text{and}$$

$$\mathsf{A}=\begin{bmatrix}2\cdot(A-a_1) & A & \cdots & A\\ A & 2\cdot(A-a_2) & \cdots & A\\ \vdots & \vdots & \ddots & \vdots\\ A & A & \cdots & 2\cdot(A-a_M)\end{bmatrix} \quad (10)$$

When costs and demand characteristics are analytic, but not quadratic and affine, then the Cournot market is analytic in the sense that the equilibrium is described by a set of analytic but non-linear equations. For a market with affine demand and 'analytic' costs in (9), applying the NE condition produces equation system (11). This system can be solved for using iterative nonlinear equation system solvers such as Newton–Raphson iterations (NRI). NRI method is not deteministic, especially if there are multiple solutions, since it is dependent on the initial conditions which are generally random.

$$\mathsf{A}\cdot\boldsymbol{q}+\boldsymbol{b}-\begin{bmatrix}d_i\cdot e_i\cdot\sin\Big(2\cdot e_i\cdot(q_i-\underline{q}_i)\Big)\\ \vdots\\ d_M\cdot e_M\cdot\sin\Big(2\cdot e_M\cdot(q_M-\underline{q}_M)\Big)\end{bmatrix}=\boldsymbol{0} \quad (11)$$

IV. Results and Discussion

The Nash equilibrium solutions determined by 'old' classical methods and the 'new' direct method is compared in Table II, the solutions for the general market is not included since a classical solution does not exists. The 'old' solutions were obtained for the linear market by matrix inversion, and for the analytic market solution by NRI with 100 trials of random initial conditions uniformly distributed in the set $\mathcal{Q}=[\underline{q}_1;\overline{q}_1]\times\cdots\times[\underline{q}_M;\overline{q}_M]$. It was found that there were four distinct Nash equilibrium for the nonlinear analytic market, with each of the trial solutions precisely converging to one of these four multiple equilibria.

The new solutions were obtained by evolving the EP for $K=300$ iterations, with $T=100$ Nash tests per fitness evaluation, a localisation constant of $\rho=1/6$, tournament size of $l=5$, and a population of $L=300$. For each market case 100 trials were conducted and the fittest genome is selected as the trial's solution. The trials solutions are then clustered using K–means clustering to identify the clusters' centriod which is taken as a Nash equilibrium. In total three clusters can be uniquely identified for the analytic market, but only one sought for the linear market since it has only one equilibrium solution. With the general market no clustering were performed since it is not known if the Nash equilibria is distinct.

A. Equilibrium Results

From Table II it can be seen that there are four solutions to the analytic markets, and that the old method and new methods are more efficient at finding different Nash equilibrium. For solution 1 the old method only have a 1% chance of finding the solution, where as the new method can find this solution 37% of the time. Conversely for solution 2 the new method could not find the solution in 100 trials yet the old method could find this solution 14% of the time. Both methods however are able to find analytic solution 3 in more then 50% of the times. It can also be observed that the new method consistently produces solutions with a percentage error less 3.223% irrespective of whelther market is linear or analytic.

TABLE II
Solutions of 'New' & 'Old' Methods.

Market equilibrium	Nash volumes $\boldsymbol{q}^*$ (MWh)	Max. % error	No. of solution[1] 'Old' %	'New' %
Linear	$(144.3, 144.0, 142.8)$	2.702	100.	100.
Analytic				
solution 1	$(123.3, 148.7, 156.5)$	3.223	1.00	37.0
solution 2	$(167.0, 147.4, 117.9)$	0.460	31.0	12.0
solution 3	$(157.5, 141.5, 150.5)$	2.559	54.0	51.0
solution 4	$(162.4, 144.0, 136.0)$	–	14.0	0.00

[1]As a percentage of the number of trials.

Although the new method performed well giving a percentage error bound of 3.223% for $T=100$ Nash tests per fitness evaluation under all scenarios studied. There is scope for improving the computation efficiency by considering how the Nash tests are performed as well as how the fitness is defined in terms of the Nash tests statisitics. For example computation savings may be made by biasing Nash tests perturbation to regions occupied by the current population, or by argumenting fitness assignment with neighbourhood interpolation. Investigations in these directions are not pursued due limitation of space and because the primary goal of the paper is to establish the valididty of the new method and not to optimise its perfomance.

B. Market Cases Discussion

Figure 3 shows the distribution of the solutions of the general market found using the new method together with the solution of the other market cases. It can be observed that the general market tends to cluster about solution 1 in constrast to the analytic market prevelance for solution 3. It is also apparent that, unlike the new solutions for the analytic market, there is some clustering about solution 4. Infact, the general market solution distribution tends to interpolate between all four multiple analytic solutions, but not the linear solution. This is expected since the linear market has no sinusoidal costs, and the sinusiodal costs of the analytic market only differs from the general market in that it is squared instead of rectified.

The aggregated profitability π_{margin}, is plotted in Figure 4 for the various equilibria, here profitability is defined as ratio of agrregated profit to aggreagate revenue per (12). It is clear there is a trend for π_{margin} to decrease with increasing volumes Q, this indicates that there is diminishing gains for producers to increase output. It can also be seen that the linear market have the highest profitability. In constrast the general market tends to have the lowest profitability at all market volumes.

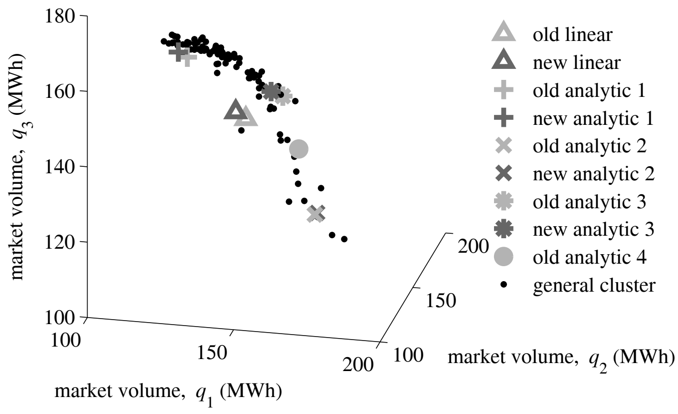

Fig. 3. Producers' Equilibrium Volumes

This indicates that when the costs of valve-point loading are realistically accounted for, the market may be less profitable then is expected from models with simplified quadratic costs.

$$\pi_{\text{margin}} = \frac{\sum_{i \in \mathcal{M}} \pi_i}{p \cdot Q} = 1 - \frac{1}{p \cdot Q} \cdot \sum_{i \in \mathcal{M}} k_i \tag{12}$$

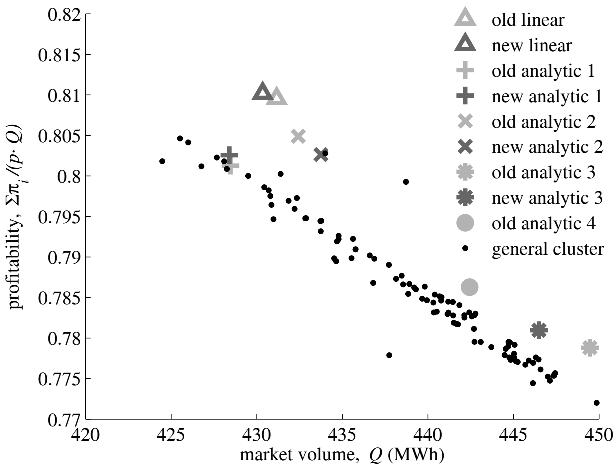

Fig. 4. Equilibrium Profitability

The mean of the average profits π_{average}, defined in (13), is shown in Figure 5. It reveals that π_{average} is positve for all market cases, and decreases with increasing volumes for the analytic and general markets. Positive π_{average} indicates producers have super–normal or excess profits and means that there is incentive for new firms to enter the market. Decreasing π_{average} means diminishing returns and a lack of incentive to operate at equilibrium with high output volumes. Furthermore given producer 1, 2, and 3 are operating at less then 25%, 42%, and 87% of capacity, it is apparent that producer 1 and 2 are oversized for the given market scenerio. All these observation takened together indicates that it is likely that in the long term producer 1, and 2 will exit the market and be replaced with a producers of similar size to producer 2.

$$\pi_{\text{average}} = \frac{\pi}{q} = p - \frac{1}{M} \cdot \sum_{i \in \mathcal{M}} \frac{k_i}{q_i} \tag{13}$$

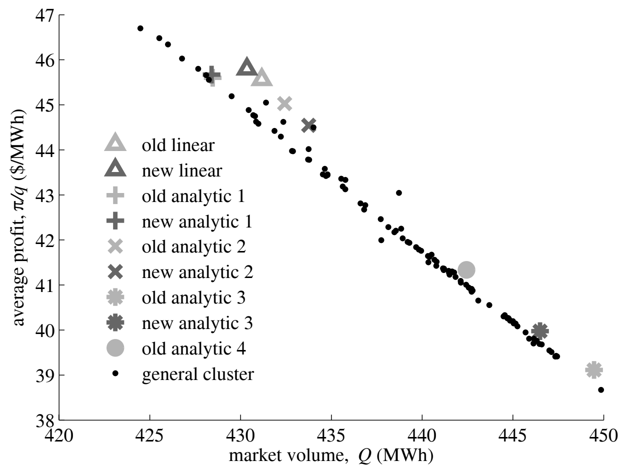

Fig. 5. Equilibrium Average Profits

The mean of the marginal profits π_{marginal}, defined by (14), is illustrated in Figure 6. It can be noted that π_{marginal} is negative and decrease with volume, which means that producers are disinclined to increment increase output as it results in reduced profits and this effect is more pronounced at larger volumes. Furthermore unlike the common trends for π_{average}, there is a divergence in the trends of π_{marginal}, with the general market trend sloping steeper then the analytic market trend, which means that the general market even less likely to increment output. Thus it can be inferred that approximate cost characterisation can underestimate the economic value of a marginal increase in production output from equilibrium, to cater for demand surges.

$$\pi_{\text{marginal}} = \frac{\mathrm{d}\pi}{\mathrm{d}q} = \frac{\mathrm{d}(p \cdot Q)}{\mathrm{d}Q} - \frac{1}{M} \cdot \sum_{i \in \mathcal{M}} \frac{\mathrm{d}k_i}{\mathrm{d}q_i} \tag{14}$$

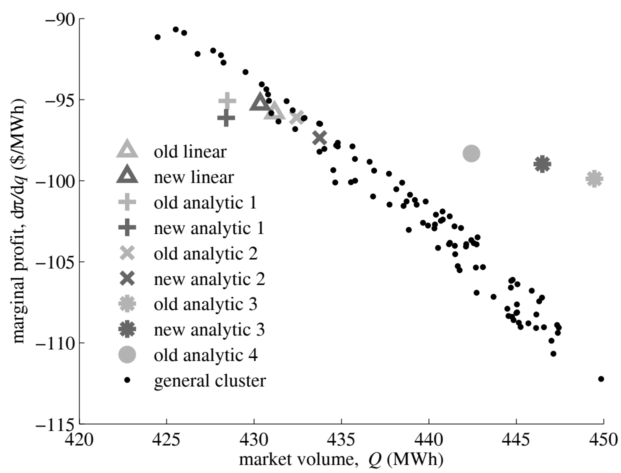

Fig. 6. Equilibrium Marginal Profits

V. Summary and Conclusions

The paper presents a new stochastic method for directly determining the NE of arbitrary electricity markets modelled as 'black boxes'. The method uses a series of Nash consistency tests to measure how well a given production vector is at being a Nash equilibrium. The stochastic search is conducted with a basic evolutionary programming algorithm, that employs the Nash consistency frequency as a fitness measure. The new method is deployed to determined the Nash equilibrium a small analytic market with known Nash solutions so that its performance can be compared. A linear simplification as well as a nonanalytic extension of the small analytic market was also considered.

It was found that the new method and the classical methods are more efficicent at finding different equilibrium, and that analytic approximations of valve point loading costs do not always give consistent results with more realistic nonanalytic costs models. Simplified cost models can exaggerate profit margins of the market, and under estimates the disincentive for producers to marginally increase production to meet unexpected additional demand. It is also demonstrated that the new methods can be used to accurately study, through simulations, the level of economic profits and the corresponding potential for the rationalisation in the market.

In conclusions this paper presents a new general numerical method for the analysis of electricity markets which is independent of how the market is characterised. The method is robust and give accurate results although there is room to improve the computational efficiency of the method. Work also remains to be done on scaling the method to large systems as well as applying the method to nonconventional market models. In sum this work represents a promising new avenue in the development of robust tools that is can be used to analyse and understand the new decentralised power infrastructure.

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