Identification of Harmonic Sources by Underdetermined State Estimator

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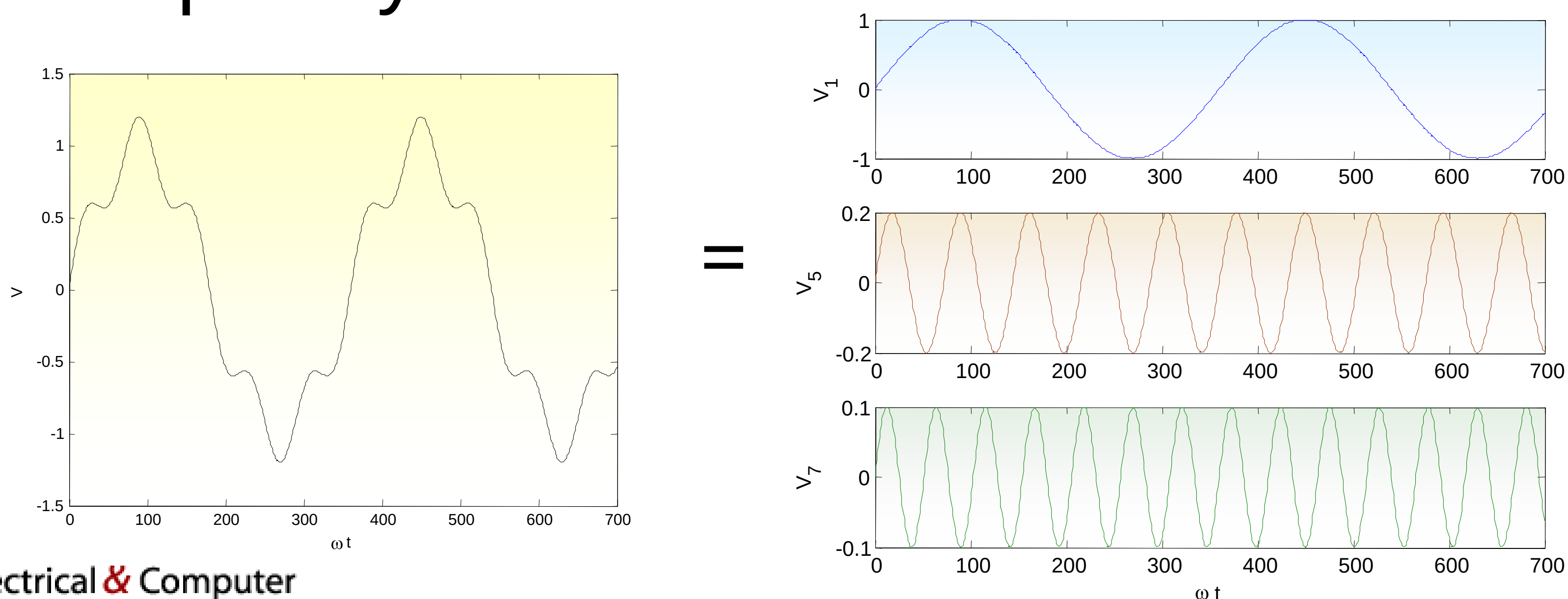
Overview

- Background
- Harmonic Source Identification
- Observability with Sparse Prior
- Source Identification via Sparsity Maximization
- Numerical Results
- Conclusions

Background

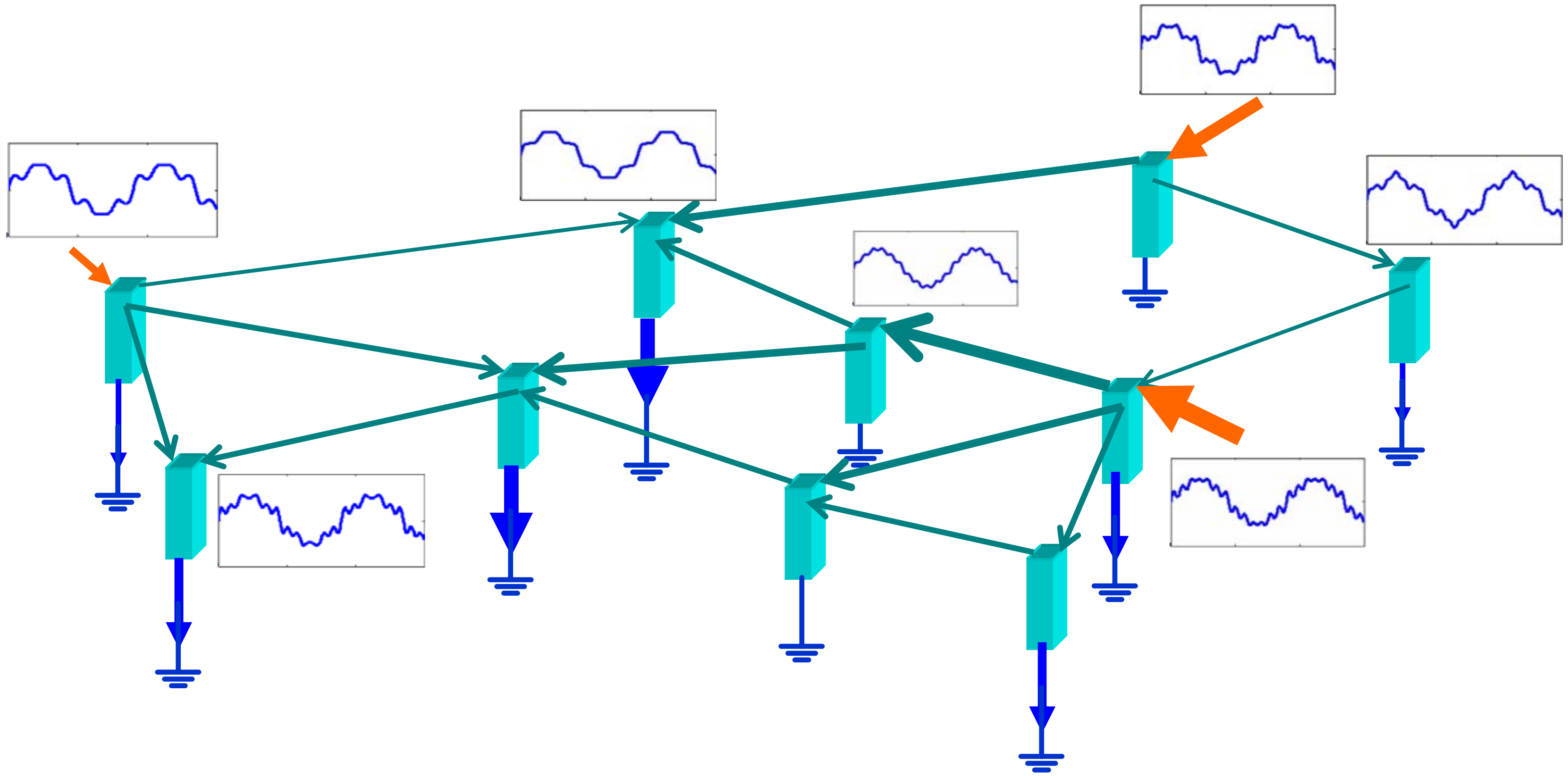
- Harmonics: Periodic distorted voltage/current waveform can be decomposed into components with whole multiples of the fundamental frequency.



Harmonic Pollutions

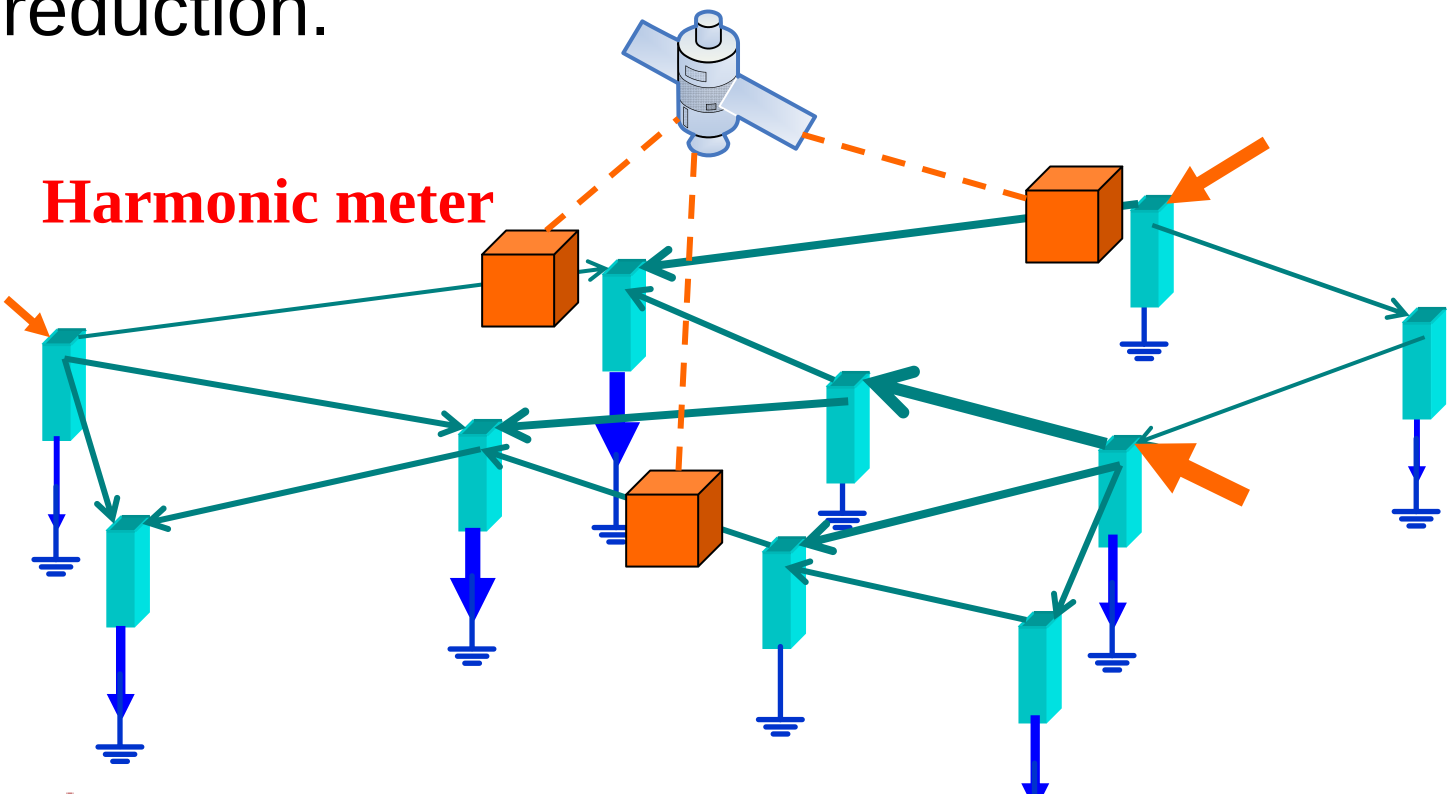
- Harmonic Sources: Converters, Inverters, static VAR compensators, switch-mode power supplies, pulse-wide-modulated drives
- Harmful Effects of Harmonics
  - shorten equipment life
  - Interfere communication
  - Induce malfunction of protective/control devices

Harmonics Propagation



Harmonic State Estimation

- Identify major harmonic sources by real-time harmonic measurements.
- Estimate harmonic distribution for harmonic reduction.



Harmonic State Estimation

Harmonic Order

Measurement Errors

$$\mathbf{z}(h) = \mathbf{H}(h)\mathbf{x}(h) + \mathbf{e}(h) \qquad (1)$$

Harmonic Measurements (Branch currents, nodal voltages)

Measurement Matrix

State Variables (Harmonic Injection Currents)

Observability

- Observability of (1) requires full rank of measurement matrix, i.e.

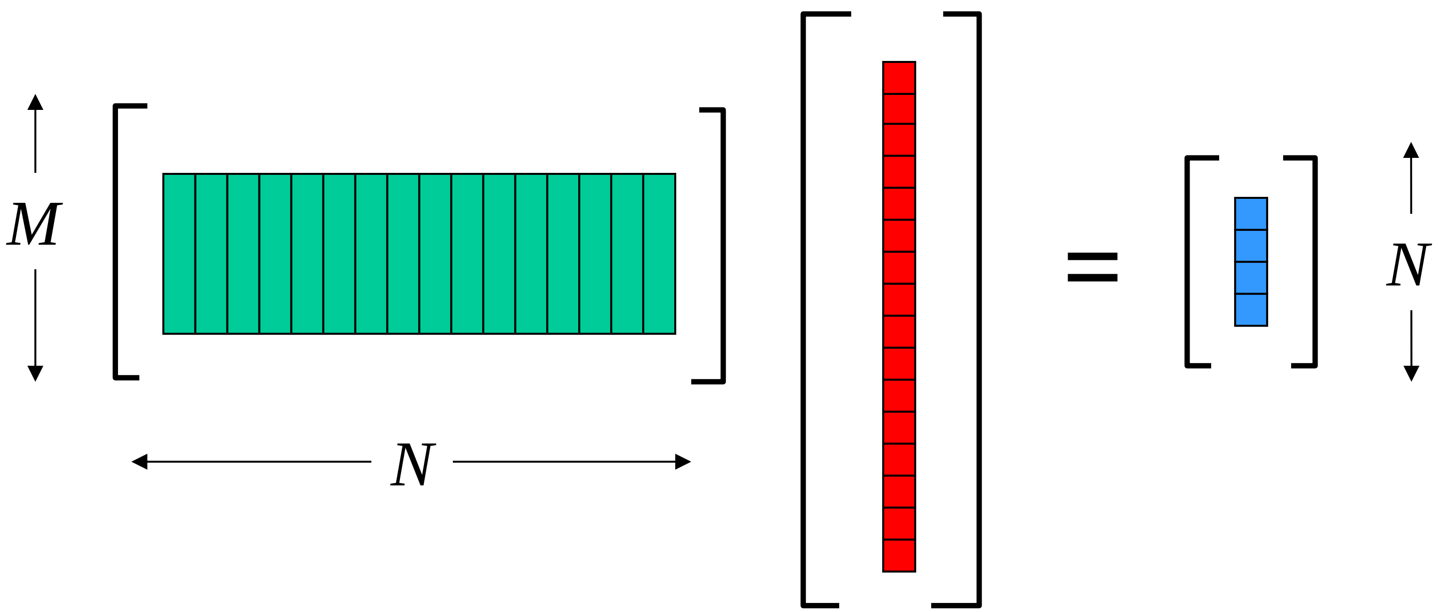
# of measurements ¤ # of state variables

- However, only limited # of harmonic meters available because
  - Harmonic meters are expensive
  - Extra cost of communication channels

The Difficulties

- Available meters # < Suspicious bus #

$$[\mathbf{H}]\,[\mathbf{x}]=[\mathbf{z}]$$



Harmonic State Estimator

- Underdetermined

M < N → **Infinite # Sol.**

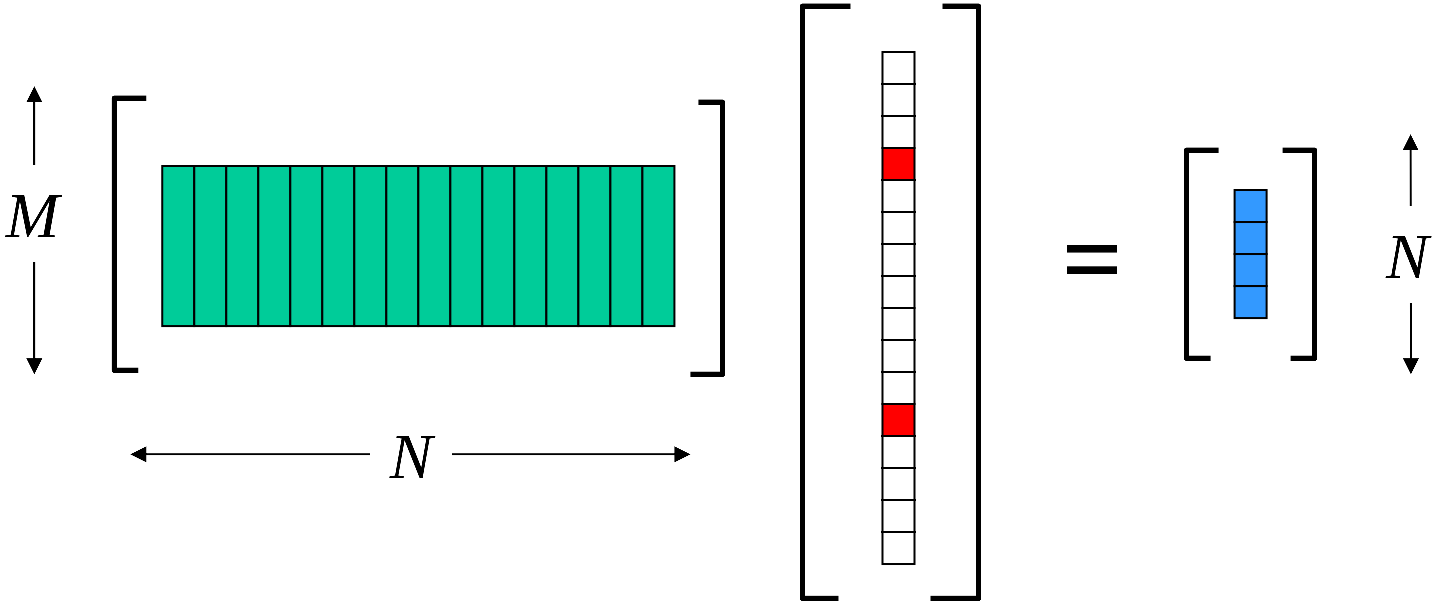
- M>=N, but H is ill-conditioned

ill cond. → **Unreliable estimate**

Existing Approach

- SVD (Singular value decomposition)-
  - Decompose the network into observable and unobservable parts
  - Estimate observable parts only
- Optimal Meter placement
  - Still need full rank of measurement matrix

An Observation : Spatial Sparsity of Sources



Could we solve it now?

Observability with Sparse Prior

- **Sparsity:** only a small portion of nodes have significant harmonic injections, while the rest have zero injections.
- **Spark**: smallest possible number of the matrix’s columns that are linearly dependent.

Example

- We know only one entry of x is non-zero
- We don't know which entry of x is non-zero.
- Spark(A)=3 : Any two columns are linearly independent
- The task: Solve x, given y and A

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{A}_{2\times 3}\mathbf{x}^*_{3\times 1} = [\alpha_1, \alpha_2, \alpha_3] \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$$

Uniqueness

Let

$$x_1 = \begin{bmatrix} k_1 \\ 0 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ k_2 \\ 0 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ k_3 \end{bmatrix}$$

The following must be true

$$Ax_1 - y \neq Ax_2 - y \neq Ax_3 - y$$

$$\alpha_1 x_1 - y \neq \alpha_2 x_2 - y \neq \alpha_3 x_3 - y$$

otherwise

$$\alpha_1 x_1 + \alpha_2 x_2 = 0 \quad \text{or}$$

$$\alpha_1 x_1 + \alpha_3 x_3 = 0 \quad \text{or}$$

$$\alpha_2 x_2 + \alpha_3 x_3 = 0$$

→ Existing two columns are linearly dependent =>Contradiction

Observability in Underdetermined Systems

Theorem: The underdetermined linear system

$$[\mathbf{H}]\,[\mathbf{x}]=[\mathbf{z}]$$

is observable if *x* has at most *s* non-zero entries and spark(H) > 2s.

Sparsity Maximization

- The sparest solution is the unique one.

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|_0 \qquad \text{Measure Sparsity}$$
$$\text{subject to} \quad \|\mathbf{z} - \mathbf{H}\mathbf{x}\|_\infty \leq \varepsilon$$

- However, to solve it, need combinatorial optimization methods

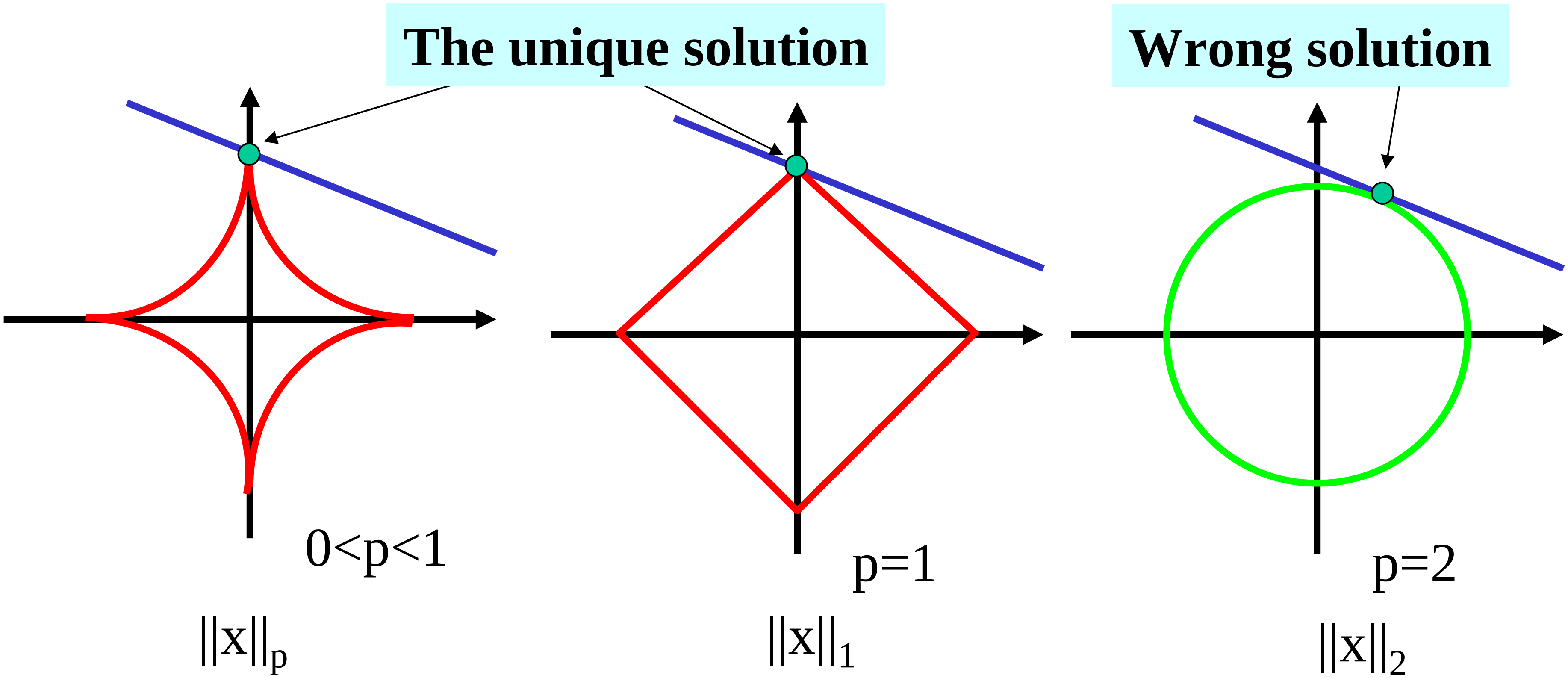
Sparsity Maximization by L1-norm

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|_0$$
$$\text{subject to} \quad \|\mathbf{z} - \mathbf{H}\mathbf{x}\|_\infty \le \varepsilon$$

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|_1$$
$$\text{subject to} \quad \|\mathbf{z} - \mathbf{H}\mathbf{x}\|_\infty \le \varepsilon \qquad (4)$$

Illustration

$$\underset{[x_1,x_2]}{Min} \quad |x_1|^p + |x_2|^p \quad s.t. \quad b = \phi_1 x_1 + \phi_2 x_2$$



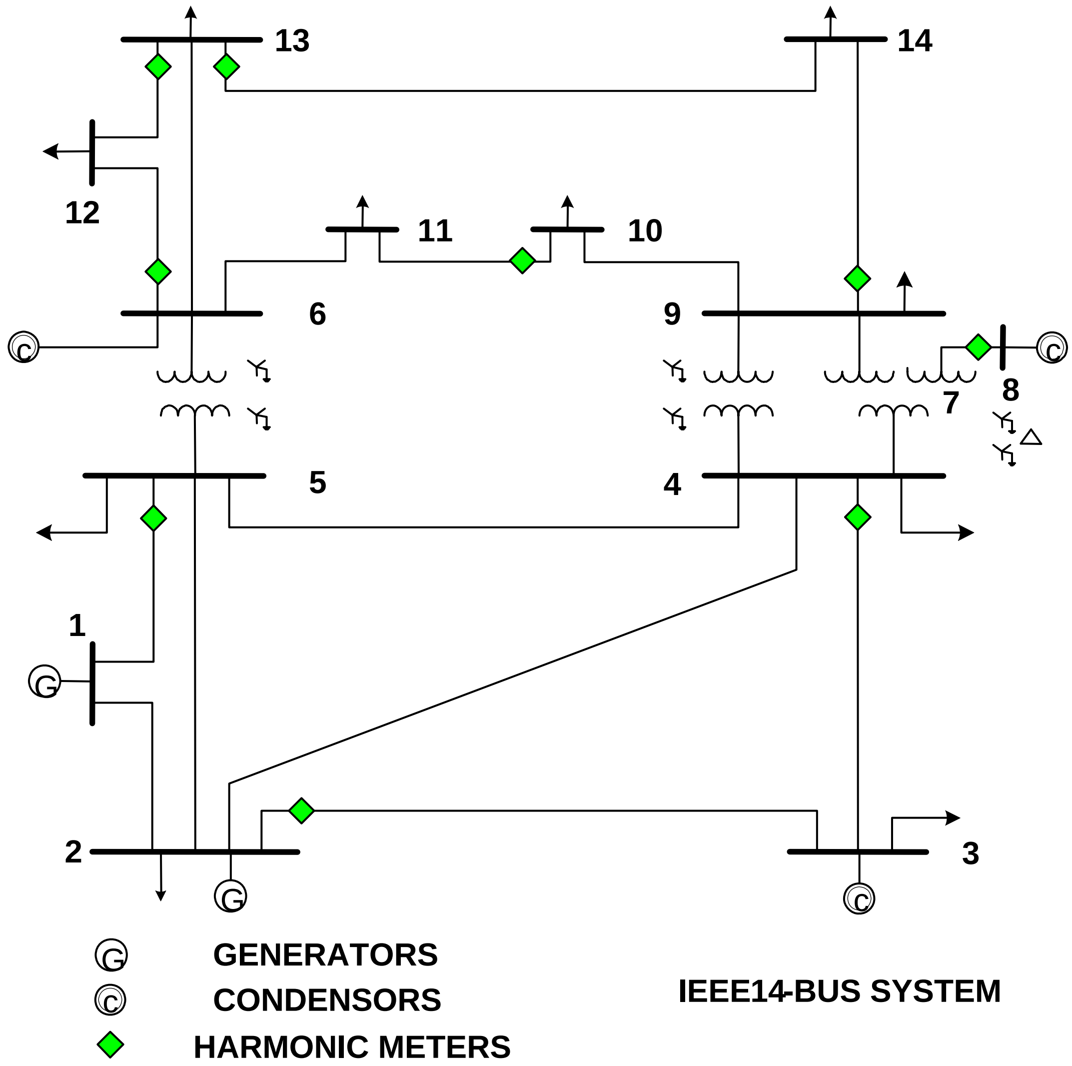
Solve Sparsity Maximization by Linear Programming

The optimization problem (4) can be cast into a standard convex program by applying $\mathbf{x} = \mathbf{x}_p - \mathbf{x}_n$, $\mathbf{x}_p \geq 0$, $\mathbf{x}_n \geq 0$. We have

$$\begin{aligned} \min_{\mathbf{x}_p, \mathbf{x}_n} \quad & f = \gamma \mathbf{1}^T (\mathbf{x}_p + \mathbf{x}_n) \\ \text{subject to} \quad & -\varepsilon \leq \mathbf{z} - \mathbf{H}(\mathbf{x}_p - \mathbf{x}_n) \leq \varepsilon \\ & \mathbf{x}_p \geq 0, \mathbf{x}_n \geq 0 \end{aligned} \tag{16}$$

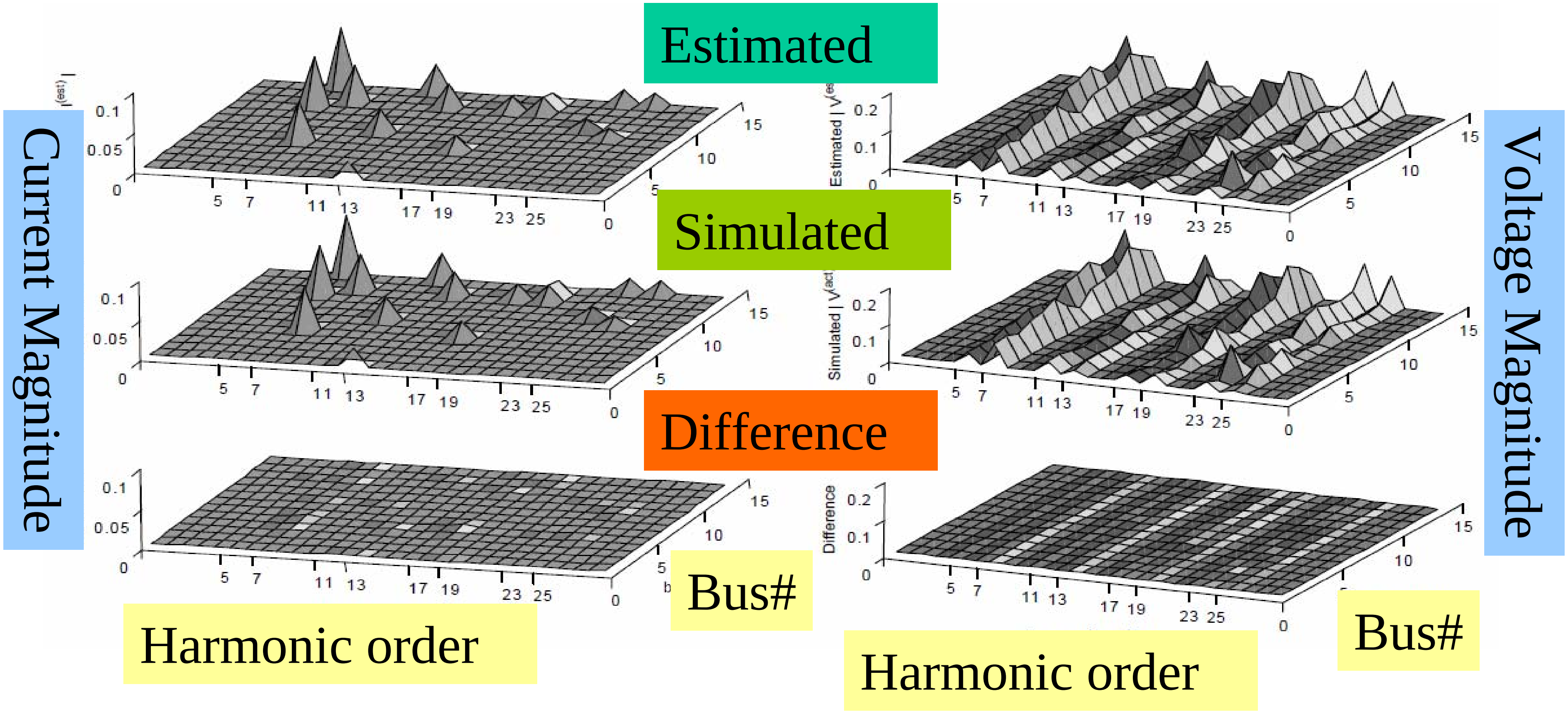
IEEE 14-bus Test System

- 13 suspicious nodes
- 9 meters
- Under-determined estimator

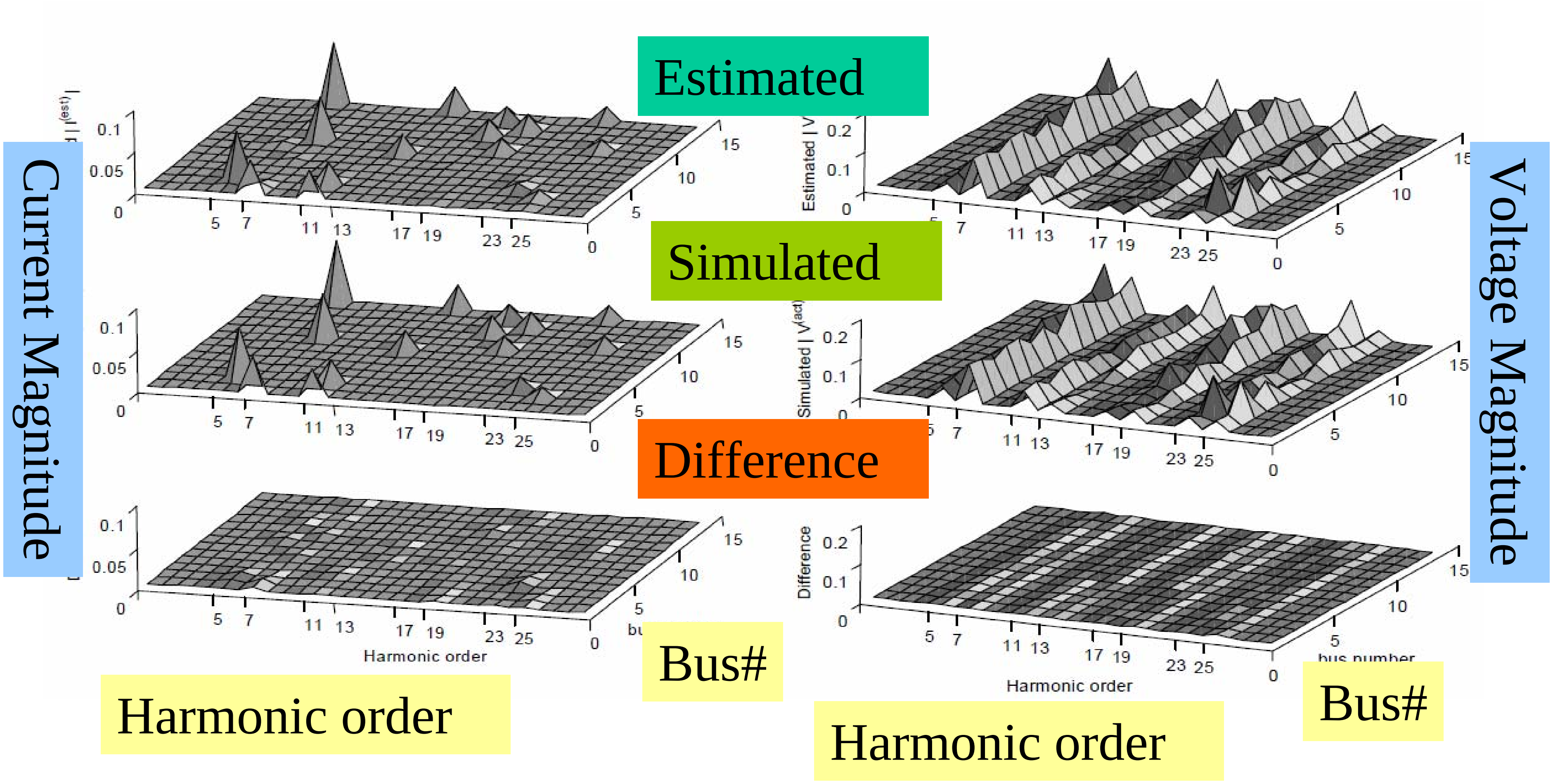


Electrical & Computer ENGINEERING

Noiseless Measurements



Noisy Measurements(5% noises)



Conclusions

- By utilizing sparsity, underdetermined systems can become observable
- The underdetermined state estimator can reliably identify harmonic sources

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Question?

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