Time-Frequency Distribution Analysis of Waveguide Modes

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1 Introduction

In the analysis of scattering targets, wide-band frequency data are used to generate an approximation of the impulse response via an inverse Fourier transform (IFT). The peaks in this impulse response then correspond to scattering centers of the target [1]. For frequency dispersive scatterers, such as an open waveguide duct or inlet, due to frequency dependent features of the propagating modes, the impulse response alone cannot provide information about the individual modes.

In this paper, we shall show that an analysis of scattering from an open ended waveguide cavity can be accomplished by time-frequency distribution (TFD) techniques. Three TFDs are used for scattering analysis of a circular waveguide cavity. Propagating modes and cutoff frequencies can readily be determined from the TFD, whereas neither time nor frequency representations will provide such information.

2 Time-Frequency Distributions

Time-frequency representations (TFD) describe a signal in terms of its joint time and frequency content [2]. Such a distribution can be used to give the energy of a signal contained in certain frequency and time intervals. If the TFD is represented by $P(t,\omega)$, the energy in any time-frequency interval (t_1,t_2) and (ω_1,ω_2) is given by the integral of the distribution over that time and frequency interval.

$$E = \int_{t_1}^{t_2} \int_{\omega_1}^{\omega_2} P(t,\omega) dt d\omega \qquad (1)$$

Time-frequency distributions were originally proposed for time-varying signals in the context of time-to-frequency transformations. In this paper, however, the TFDs are presented in the frequency-to-time transformation context. Three TFD are considered; the running-window Fourier transform, the Wigner distribution, and the running-window autoregressive spectral estimation.

2.1 Running-Window Fourier Transform

The most commonly used technique for analysis of time varying signals is the short-time Fourier transform (STFT). The STFT of a time signal $s(t)$ is defined as

$$STFT(t,\omega) = \int_{-\infty}^{\infty} s(\tau) w(\tau - t) e^{-j\omega\tau} d\tau. \qquad (2)$$

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Similar to the STFT, for frequency dispersive scenarios, we use a partitioning of the entire frequency band into smaller bands, and generate the impulse response for the sub-bands. In this method, we take a slice of the frequency data about a center frequency, and compute the inverse Fourier transform of the windowed data. For the frequency domain data $S(\omega)$, we define the running-window Fourier transform (RWFT) as

$$RWFT(t,\omega) = \int_{-\infty}^{\infty} S(\Omega)W(\Omega-\omega)e^{j\Omega t}d\Omega, \tag{3}$$

where $W(\omega)$ is the windowing function.

2.2 The Wigner Distribution

Analogous to the time signal TFD, we define the Wigner distribution of the frequency signal $S(\omega)$ as:

$$WD_s(t,\omega) = \int_{-\infty}^{\infty} S(\omega+\frac{\Omega}{2})s^*(\omega-\frac{\Omega}{2})e^{j\Omega t}d\Omega. \tag{4}$$

It has been shown [3] that the Wigner Distribution has the highest signal concentration in the time-frequency plane. A more concentrated distribution would violate the time-frequency uncertainty principle [3]. Unfortunately, the Wigner distribution introduces cross-terms between multi-component signals which makes the interpretation of the Wigner distribution difficult in some applications.

2.3 Running-Window Autoregressive TF Distribution

In the last decade a variety of high resolution spectral estimation techniques have been introduced. The major advantage of these techniques is their ability to resolve closely spaced spectral peaks for short data records. The high resolution method used in this study is an autoregressive (AR) spectral estimation of the sub-band frequency data.

The AR formulation of scattering data is based on the point-scatterer (non-dispersive) formulation of the radar target [4]. Although the point-scatter assumption is violated for dispersive features of a target, as we shall show, if a narrow window is used, reasonable distributions can be obtained.

3 TFD of a Circular Waveguide Cavity

The electromagnetic (EM) scattering from a 2-ft long, 1.75-inch diameter circular cavity with an open end (Figure 1) is analyzed using three time-frequency distribution techniques.

Referring to Figure 1, scattering from the interior of the cavity $\vec{E}^s_{cav}$, is the frequency dispersive part of the total scattered signal. For frequencies below the cutoff frequency of the cavity, the $\vec{E}^s_{cav}$ does not contribute to the total scattering. For frequencies greater than cutoff, as the operating frequency increases, the group velocity of the propagating mode increases. The number of propagating modes also increases with frequency.

For the 0^o incidence case, the magnitude of the measured frequency data, and the band limited impulse response are shown in Figure 2. From this figure, the scattering from the front and back ends of the cavity can be seen at -2.0 and 2.0 nano-seconds respectively. From the impulse response, we can also see a diffuse response after 2.0 nano-seconds. The diffuse feature can be an indication of frequency dispersion. However, neither the frequency nor the time-domain data of Figure 2 provides specific information about the frequency dispersive nature of the cavity.

The magnitude of the RWFT distribution versus time and frequency is shown in gray-scale levels in Figure 3. In the same figure, along the time and frequency axes, the impulse response, and the magnitude of backscattered signal are also included. From the TFD, we can identify the leading and trailing ends of the cavity, and the two cutoff frequencies at 3.9 GHz and 11.5 GHz. These values agree with the theoretical results. The shape of the modes in the time-frequency plane also indicate the rapid variation of the group velocity for frequencies greater than cutoff.

The second TFD technique applied to the cavity data is the Wigner distribution. The Wigner distribution for the measured data is shown in Figure 4. As expected, the Wigner distribution provides a higher resolution for the leading and trailing ends of the cavity as well as the propagating modes. However, for some sections of the distribution, the cross-terms obscure the distribution to a point where it is difficult to distinguish between the co- and cross-components.

The running-window autoregressive (RWAR) spectral estimation technique is the third TFD technique applied to the cavity data. The RWAR time-frequency distribution for the measured cavity data is shown in Figure 5. As seen in the figure, the frequency resolution of the propagating modes is not as good as the RWFT distribution, whereas the resolution for the leading and trailing ends of the cavity is superior to that of RWFT or the Wigner-distribution.

References

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[4] E. K. Walton, "Comparison of Fourier and Maximum Entropy Techniques for High-Resolution Scattering Studies," *Radio Science*, vol. 22, pp. 350–356, May-June 1987.

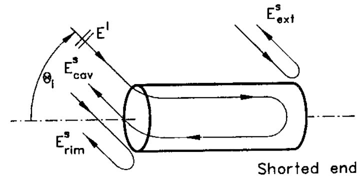

Figure 1: Geometry of the open-ended circular cavity and major scattering mechanisms

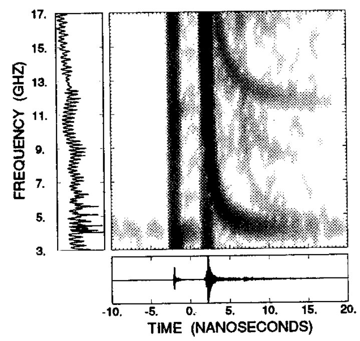

Figure 3: Running-window FT Time-frequency distribution using a 2-GHz wide Kaiser-Bessel window.

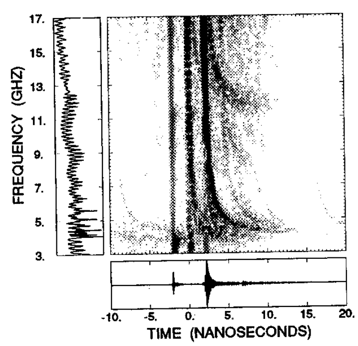

Figure 4: Wigner Time-frequency distribution

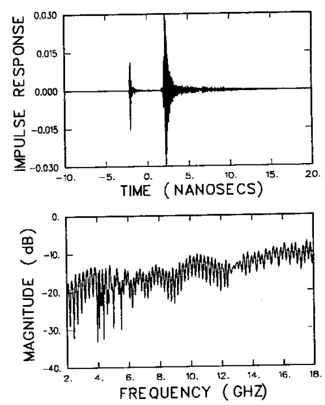

Figure 2: Magnitude of the measured frequency scan data, and the band limited impulse response for 0° incidence

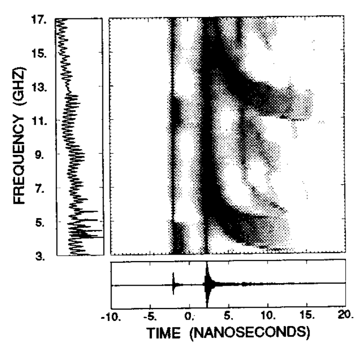

Figure 5: Time-frequency distribution obtained from the running-window autoregressive spectral density.