Propagation modelling of complex HVAC networks using transfer matrix method

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Abstract

Use of heating, ventilation, and air conditioning (HVAC) ducts for indoor communications is a topic of recent interest. Real HVAC networks are complex systems, which contain multiple bends, tapers, etc. In this paper, we present a propagation model for such cascaded networks, based on the transfer matrix method. As an example, we theoretically analyze and experimentally characterize a system composed of straight sections, bend and taper. Measured data are in agreement with our theoretical predictions.

1 Introduction

The HVAC duct system in a building is a multimode waveguide structure. Efficient modelling of propagation in such a complicated network is a challenging task. From the point of view of radio propagation, this system consists of multiple cascaded elements, where each element can be considered as a microwave device with one physical input and one physical output and can be characterized with its transfer matrix. A transfer matrix method provides a good frequency-domain description of wave propagation in a cascaded system. This method has been widely used in optics [1]. It works well if reflections from the element junctions due to mismatch are small. In this paper, we apply this method to model propagation in an HVAC duct system.

2 Propagation model

Consider an arbitrary HVAC duct system shown in Figure 1 with two antennas coupled into it. The k-th element contained between transmitter and receiver is characterized by its transfer matrix $\hat{Q}_k$. The compound transfer matrix of the system portion between transmitter and receiver can be written as $\hat{Q} = \hat{Q}_K \hat{Q}_{S-1} ... \hat{Q}_k ... \hat{Q}_1$. Note that the order of multiplying transfer matrices is important, and non-diagonal elements of each matrix represent coupling between different modes. System portions to the left of the transmitter and to the right of the receiver are described by matrices $\hat{P}$ and $\hat{R}$. Reflections from the ends are characterized by reflection matrices $\hat{F}$ and $\hat{G}$.

A transmitting antenna excites waveguide modes, described by a vector of mode amplitudes $\vec{C}_T$. The modes observed at the receiver are related to the modes at the transmitter

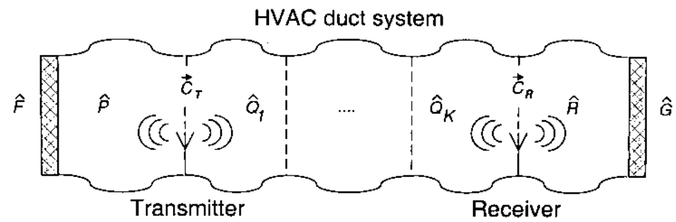

Figure 1: Transmitting and receiving antennas in an arbitrary HVAC duct system.

as $\vec{C}_R = \hat{T}\,\vec{C}_T$, where subscripts $_T$ and $_R$ denote cross-sections of transmitting and receiving antenna respectively and $\hat{T}$ is the compound transfer matrix of HVAC duct system. This matrix includes the effects of reflections from system ends (reflections from element junctions are neglected to keep problem tractable). Matrix $\hat{T}$ can be found from an infinite series arising from multiple reflections from the ends:

$$\hat{T} = \left(\hat{Q} + \hat{R}\,\hat{G}\,\hat{R}\,\hat{Q} + \hat{Q}\,\hat{P}\,\hat{F}\,\hat{P} + \hat{R}\,\hat{G}\,\hat{R}\,\hat{Q}\,\hat{P}\,\hat{F}\,\hat{P}\right)\left(\hat{I} + \hat{\Upsilon} + \hat{\Upsilon}^2 + \ldots\right), \tag{1}$$

where $\hat{I}$ is the identity matrix and $\hat{\Upsilon} = \hat{R}\,\hat{G}\,\hat{R}\,\hat{Q}\,\hat{P}\,\hat{F}\,\hat{P}\,\hat{Q}$. If the loads are matched, there are no reflections from the terminated ends: $\hat{F} = \hat{G} = 0$ and $\hat{T} = \hat{Q}$. The frequency response between the ports of two antennas coupled into this system can be written as:

$$H(\omega) = \frac{2Z_o}{(Z_o + Z_{a,T})(Z_{a,R} + Z_o)} \sum_{n=1}^{N} Z_{n,R} \frac{p_{n,R}}{\int_R \vec{e}_{n,R} \cdot \vec{J}_R \, dS} \sum_{m=1}^{M} T_{nm} \frac{\int_T \vec{e}_{m,T} \cdot \vec{J}_T \, dS}{p_{m,T}}, \tag{2}$$

where Z_o is the impedance of the transmitter and the receiver, Z_a is the impedance of the antenna in waveguide, N and M are the number of modes at the transmitter and the receiver respectively, p_n is the normalized power density of mode n, T_{nm} are the elements of the transfer matrix $\hat{T}$, $\vec{e}_n$ is the normalized electric field of mode n, $\vec{J}$ is the current density on the antenna, and the integration is performed over the antenna surface (or length, in case of wire antennas). Details of the above derivation can be found in [2].

3 System example

Consider the configuration shown in Figure 2, which consists of cascaded straight sections, a bend, and a taper (all cylindrical). This configuration is typical to HVAC duct systems. To find the transfer matrix of this system, we need transfer matrices for straight sections ($\hat{P}$, $\hat{Q}_1$, $\hat{Q}_3$, $\hat{Q}_5$, $\hat{R}$), bend ($\hat{Q}_2$), and taper ($\hat{Q}_4$), as well as reflection matrices ($\hat{F}$ and $\hat{G}$). We give those below, using a notation Q_{nm} to denote the elements of each transfer matrix.

Straight sections do not cause coupling between propagating modes. The transfer matrix that describes a straight section is

$$Q_{nm} = e^{-\gamma_n L_s}\delta_{nm}, \tag{3}$$

where γ_n is the complex propagation constant of mode n, L_s is the length of the straight section, and δ_{nm} is the Kronecker delta.

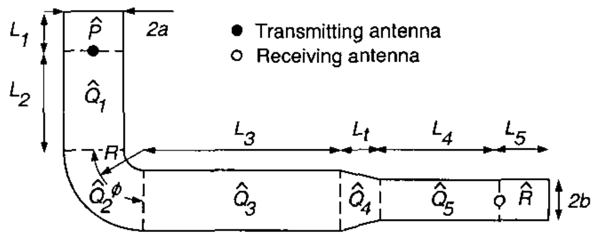

Figure 2: Example HVAC system.

Bends are well known in microwave and optical engineering and can be modelled rigorously [3], but the solution is usually rather complicated. A simplified transfer matrix for a bend can be found by treating the bend as a section of toroid. The complex propagation constant in a bend γ_n^B can be expressed via cutoff wavenumbers of toroid eigenmodes, which are well known [4]. Neglecting mode conversion effects in a gentle bend $(a/R < 1)$ allows the bend transfer matrix to be written as:

$$Q_{nm} = e^{-\gamma_n^B R\phi}\delta_{nm} , \quad (4)$$

where ϕ is the angle of the bend and R is its center radius.

Tapers can be treated as cylindrical waveguides with a changing radius [5], if the change in radius is gentle: $|a-b|/L < 1$. Enforcing a power conservation (to relate mode amplitudes on input and output) and averaging the waveguide propagation constant over the taper radius (to obtain an equivalent propagation constant in taper γ_n^T) allows a taper transfer matrix to be written as

$$Q_{nm} = a^2 b^{-2}\sqrt{\beta_n^a/\beta_n^b}\, e^{-\gamma_n^T L_t}\delta_{nm} , \quad (5)$$

where β_n^a, β_n^b are propagation constants in waveguides with diameter a and b, and L_t is the taper length.

Reflections from open ends are small in multimode waveguides. Reflection matrices can be approximated as $F_{nm} = F\delta_{nm}$ and $G_{nm} = G\delta_{nm}$, where reflection coefficients are assumed to be the same for all modes.

4 Comparison with experiment

To verify the model for the system shown in Figure 2, we used a network analyzer to experimentally measure frequency responses in the 2.4-2.5 GHz band between 3.1 cm monopole probes coupled into this system. Other system parameters were: $a = 15.25$ cm, $b = 7.63$ cm, $L_1 = 0.45$ m, $L_2 = 2.6$ m, $R = 45.75$ cm, $\phi = 90^o$, $L_3 = 3.05$ m, $L_t = 0.16$ m, $L_4 = 2.6$ m, $L_5 = 0.45$ m, $F = G = 0$. Figure 3 shows theoretical and experimental frequency responses. It can be seen that the curves are in reasonable agreement. The theoretical curve reproduces major minima observed in the experiment. Variations are caused by reflections from open ends, back-scattering from bend and taper junctions, and imperfections of ducts, which are not precision waveguides.

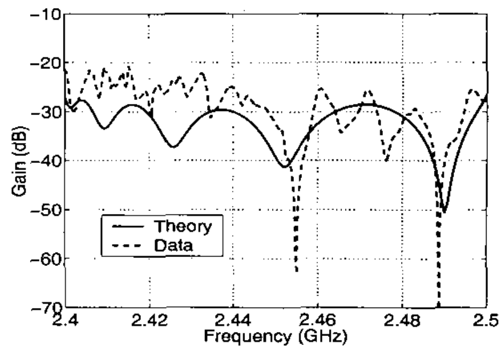

Figure 3: Theoretical and experimental frequency responses between antennas coupled into the system shown in Figure 2 with parameters given in Section 4.

5 Conclusions

In this paper, we applied the transfer matrix method to modelling propagation in complex cascaded multimode waveguide networks (HVAC duct systems) and demonstrated that experimental results confirm theoretical predictions. This method is an attractive approach to efficient modelling of propagation in duct systems, where back-scattering from element junctions can be neglected. The accuracy of this method depends on the accuracy of transfer matrices used to model individual network elements.

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