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Novel mode content analysis technique for multimode waveguides

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Abstract—**This paper presents a novel technique for analyzing the mode content in multimode waveguides. The technique is based on measuring the frequency response between the two antennas coupled into a multimode waveguide and using that information to extract the frequency-averaged mode content at the location of the transmitting antenna. The technique is applicable to cases in which the complex mode amplitudes are approximately constant over the frequency range of interest. This method is valuable for determining the mode mix generated by arbitrary transmitting antennas in a multimode waveguide propagation environment, such as HVAC duct system used for indoor wireless communications.**

I. INTRODUCTION

Using the heating, ventilation, and air conditioning (HVAC) duct system in buildings for communications is a promising way to provide a high-speed network access to offices [1]. A typical HVAC duct system is a complex network of hollow metal pipes of rectangular or circular cross-section which may extend to hundreds of meters. These pipes behave as multimode waveguides when driven at RF and microwave frequencies. The signal is coupled into and out of the ducts using coaxially-fed probe antennas mounted on duct walls.

Multimode dispersion in ducts and re-distribution of energy between different modes in such HVAC elements as T-junctions can significantly affect the capacity and the signal-to-noise ratio in the duct communication channel. Knowing the mode content excited by the transmitting antenna in an HVAC duct system is important for characterizing channel properties and understanding the behavior of complicated HVAC elements.

When experimentally characterizing the performance of various antennas in an HVAC duct system, the mode content must be measured multiple times at various locations. That means that the mode content measurement technique must be simple, efficient, and non-destructive to an existing duct system. Moreover, only the modes to which a receiving antenna is the most sensitive to are important for analysis.

This paper describes a novel technique for the mode content analysis that satisfies all the aforementioned criteria. The remainder of this paper is organized as follows. Section II describes the previous work in the area of mode content measurement. The technique description is presented in Section III. Section IV contains experimental results and technique validation. Conclusions are given in Section V.

II. PREVIOUS WORK

All existing mode content determination approaches can be divided into four groups: scanning the field pattern, using mode-selective couplers, measuring open-end radiation pattern, and array processing. The first and the last approaches may overlap.

The scanning field pattern technique has been used by many researchers. Forrer and Tomiyuasu [2] used a moving probe to measure the electric field magnitude and phase at the walls of a waveguide and a Fourier analysis to compute the power flow in each mode. Fixed multiple-probe arrays were used by Price [3], Taub [4], and Levinson and Rubinstein [5]. Klinger [6] used a fixed probe and a moving short termination to measure the multimode content. Glock [7] used a fixed probe, a fixed termination, but moving (adjustable length) waveguide to perform necessary measurements.

The mode-selective coupling has been used by Lewis [8] and Beck [9], who employed a series of specially designed mode couplers to couple a mode to its own output port. Seguinot [10] used mode coupling technique for characterizing multimode microstrip lines.

Measuring the radiation pattern of an open-ended waveguide has typically been used in high-power microwave engineering for extracting the mode content of a high-power source (magnetron, gyrotron, etc.) [11].

Array processing involves measuring the signal at the elements of an antenna array mounted on the waveguide and using those measurements for mode content extraction. To achieve good results, antenna locations must be carefully chosen and sometimes even optimized in the process of measurement. An excellent example of using an antenna array for mode measurement is described by Roper [12]. Other related work is presented in [13]-[14].

All of the techniques described above require a complicated experimental setup and a lengthy process of mode content measurement. Below we describe a novel technique for mode content analysis that requires only one sensing antenna for determining the average mode content in the frequency range of interest.

III. TECHNIQUE DESCRIPTION

Consider a conceptual setup shown in Fig. 1: a multimode waveguide with two coaxially-fed antennas coupled into it, where one antenna is transmitting, and another antenna is receiving (sensing the mode content). Assume that the waveguide is straight and is terminated on both ends with matched loads which prevent end reflections. The mode mix is to be determined at the location of the transmitting antenna, which is located at a distance L from the receiving antenna. The key information needed for mode content determination is the frequency response between the two antennas, measured by a network analyzer. Fixing a sensing antenna position and sweeping the frequency allows one to obtain independent measurements, somewhat similar to fixing the frequency and moving the antenna along a slit in a waveguide wall.

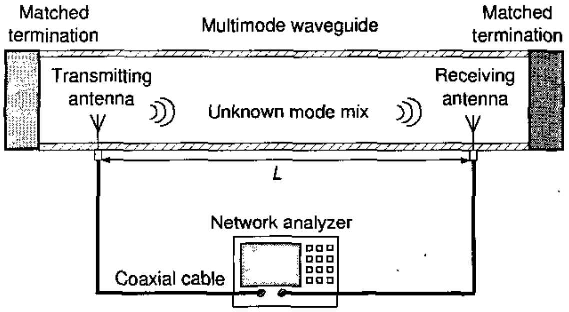

Fig. 1. Mode content measurement setup.

Assume that complex mode amplitudes excited by the transmitting antenna are weak functions of frequency in the range of interest. This important assumption is critical to our method. Then system frequency response values measured at different frequencies can be used as independent sources of information to resolve the mode content. This allows one to find a set of approximate mode amplitudes, constant over the frequency band, which best approximate the measured frequency response, such as the one shown in Fig. 2.

Assume for simplicity that both probe antennas are identical and oriented in the same way. Assume further that N modes can propagate in our waveguide (if antennas are sufficiently far apart, evanescent modes can be neglected). The unknown N-dimensional vector of frequency-dependent complex mode amplitudes $\vec{X}(\omega)$ is to be measured at the transmitting antenna location. The frequency response measured between the transmitting and the receiving antenna can be written in terms of $\vec{X}(\omega)$ as a scalar product:

$$H(\omega) = \vec{X}(\omega)\,\vec{C}(\omega)\,, \qquad (1)$$

where the frequency-dependent vector $\vec{C}(\omega)$ describes a coupling between waveguide modes and a sensing antenna.

Since we assumed that complex mode amplitudes are weak functions of frequency, they can be approximated as frequency-independent constants over the frequency range of interest:

$$\vec{X}(\omega) \approx [X_1\, X_2 \ldots X_N]\,. \qquad (2)$$

Let us perform the measurements of $H(\omega)$ at M discrete frequency points. Then (1) can be rewritten as

$$\vec{H} = \vec{X}\,\hat{A}\,, \qquad (3)$$

where the matrix $\hat{A}$ consists of elements $A_{nm} = C_n(\omega_m)$ and the M-dimensional vector $\vec{H}$ is

$$\vec{H} = [H(\omega_1)\; H(\omega_2)\; \ldots\; H(\omega_M)]\,. \qquad (4)$$

Linear system given by (3) contains N unknowns and can be solved for $\vec{X}$ if the number of independent frequency measurement points is greater or equal to the number of modes to be determined.

Matrix element A_{nm} represents the coupling of mode n excited by the transmitting antenna into the receiving antenna at the frequency ω_m and can be calculated for a generic antenna in a straight waveguide as

$$A_{nm} = \frac{2Z_o}{(Z_o + Z_a)^2}\,\frac{Z_n p_n}{\mathcal{I}_n}\,e^{-\gamma_n L}\,, \qquad (5)$$

where Z_o is the coaxial cable impedance, Z_a is the antenna impedance in a waveguide, p_n is the normalized power flow density in mode n, Z_n is the antenna impedance due to mode n, $\mathcal{I}_n$ is the integral that describes the interaction of antenna current with electric field of mode n, and γ_n is the waveguide propagation constant of mode n. Quantities Z_a, Z_n, $\mathcal{I}_n$, γ_n depend on the frequency ω_m and can be calculated for given waveguide cross-section and antenna geometry. The analytical formulas for those quantities in a special case of monopole and dipole probe antennas in cylindrical and rectangular waveguides and derivation details for (5) can be found in [15]. The distance L between the transmitting and receiving antennas must be large enough to ensure a reasonably-conditioned matrix $\hat{A}$ over the frequency range of interest.

The precision of our method depends on the degree of variation of complex mode amplitudes with frequency at the transmitting antenna location. This variation depends on the waveguide size, operating frequency range, transmitting antenna characteristics and should be minimal for good extraction results. A necessary condition is that no cutoff frequencies of the analyzed modes must be near or within the frequency range of interest.

The presented technique is not limited to straight waveguides but requires negligible mode conversion, reflection, and scattering between the transmitting and receiving antennas to minimize the variation of mode amplitudes with frequency.

IV. RESULTS AND VALIDATION

To validate our mode extraction technique, we performed experimental measurements on a straight cylindrical multimode waveguide where amplitudes of modes excited by monopole probe antennas can be theoretically calculated. We used straight metal cylindrical HVAC duct of 30.5 cm (12 inches) in diameter as a waveguide. This is a typical duct used in the US. We performed measurements in the 2.4-2.5 GHz frequency range which contains the popular unlicensed ISM band. For 30.5 cm cylindrical ducts, this frequency range allows propagation of 17 modes and does not contain any mode cutoff frequencies. The antennas used were 3.1 cm long (approximately quarter-wavelength at 2.45 GHz) coaxially-fed monopole probes located 14.6 m apart. In the experiment, duct ends were left open, which approximated matched load terminations. The distances from the antennas to open ends were 0.25 m and 0.38 m.

Fig. 2 shows the frequency response measured between the antennas. The observed frequency response shape depends on the excited mode distribution and the distance between the antennas. Interference between the modes results in maxima and minima with specific widths, depths, and positions. In the validation case considered here, it can be shown analytically that only the complex amplitude of mode TE_{61} notably changes over the 2.4-2.5 GHz frequency range(the change in magnitude is about 40%). Note that TE_{61} is a higher order mode, whose cutoff frequency is 2.35 GHz, which is close to 2.4 GHz.

The spacing between the frequency points must be such that the responses measured at different frequencies are sufficiently independent and solution to system given by (3) can be found. The minimum spacing distance can be estimated from the autocorrelation function $S(\omega)$ of the frequency response defined as:

$$S(\omega) = \int_{\omega_1}^{\omega_2} H(\omega + y) H^*(y)\, dy\,, \tag{6}$$

where ω_1 and ω_2 are the lower and the upper frequencies in the band. Fig. 3 shows the normalized magnitude of the autocorrelation function computed for a frequency response shown in Fig. 2. The width of the central peak at the 50% signal correlation level (dashed line) can serve as an estimate for the coherence bandwidth, which gives a minimum frequency spacing for measurements. One can estimate from Fig. 3 that the coherence bandwidth is about 3.3 MHz, which means that 30 independent frequency measurements can fit into a 100 MHz band.

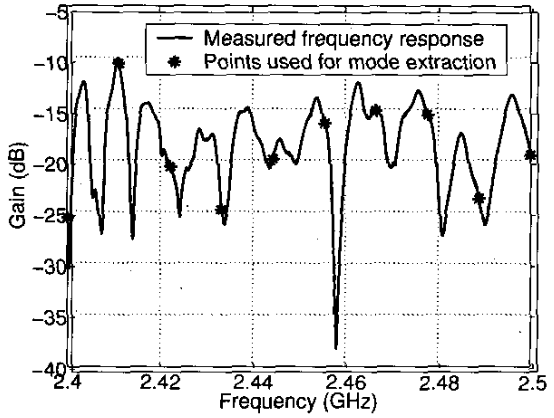

Fig. 2. Frequency response measured between two 3.1 cm monopole antennas located 14.6 m apart in a 30.5 cm straight cylindrical duct with open ends and frequency measurement points used for mode extraction.

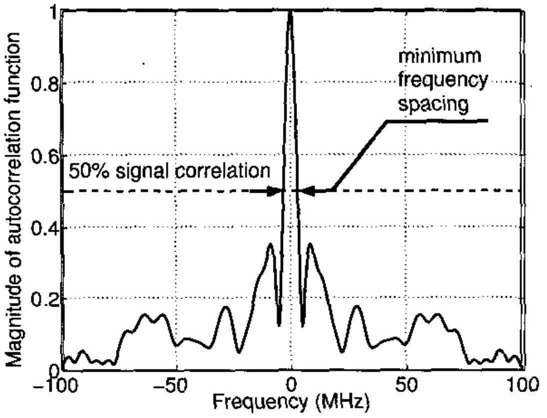

Fig. 3. Normalized magnitude of the frequency autocorrelation function for the frequency response shown in Fig. 2.

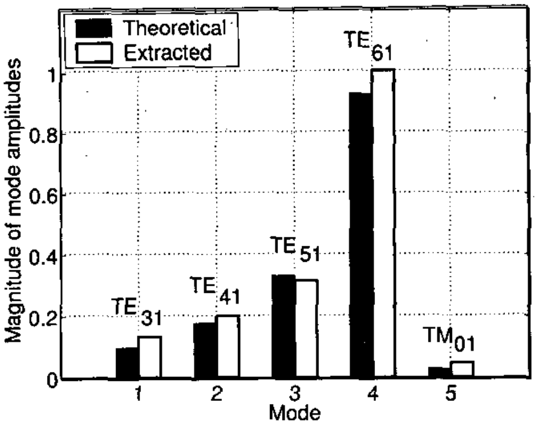

Fig. 4. Normalized magnitude of theoretically calculated mode amplitudes and their values extracted from the frequency response shown in Fig. 2 using our technique.

The accuracy of the mode extraction is determined by the condition of matrix $\hat{A}$, which depends on the distance L and the characteristics of the sensing antenna. Although the number of potential frequency points that can be used for mode content analysis is larger than the maximum number of propagating modes, some modes are interacting with the sensing probe antenna very weakly, which leads to an ill-conditioned matrix $\hat{A}$.

We extracted mode amplitudes for five modes sensed and excited best by the 3.1 cm monopole probe antenna: TE_{61}, TE_{51}, TE_{41}, TE_{31}, and TM_{01}. Linear system given by (3) was solved in *Matlab*[1] using the pseudo-inverse function (pinv) of matrix $\hat{A}$ for 10 equally spaced frequency points in the 2.4-2.5 GHz band.

Fig. 4 shows the normalized magnitude of theoretically calculated amplitudes of the five aforementioned modes and their values extracted from the measured frequency response using our technique. Theoretical values were averaged over the frequency band. One can see that theoretical and extracted mode amplitudes are in good agreement. The largest relative error is observed for the modes TE_{31} and TM_{01} whose interaction with the receiving antenna is weak.

V. CONCLUSIONS

A novel mode content measurement technique for multimode waveguides is presented in this paper. The technique is based on using a single sensing antenna and measurements at different frequency points to determine the mode content at the location of the transmitting antenna. The method works whenever the complex amplitudes of modes excited by the transmitting antenna are weak functions of frequency in the frequency range of interest and there is negligible mode conversion, reflection, and scattering between the antennas.

Comparison of the mode content calculated analytically and extracted from experimental measurements confirm the validity of our technique. The main advantage of the presented method is its simplicity, which makes it very attractive for quick estimation of mode content generated by arbitrary transmitting antennas in such multimode waveguides as HVAC ducts.

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[1] MATLAB is a registered trademark of The Mathworks, Inc.