Spectral Efficiency of Wireless Systems with Multiple Transmit and Receive Antennas

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Abstract—**Recent information-theory results have shown the enormous capacity potential of wireless techniques that use transmit and receive antenna arrays. As a result, a number of layered space-time (BLAST) architectures have been proposed wherein multiple data streams are transmitted in parallel and separated at the receiver on account of their distinct spatial signatures. While extremely promising, all analysis of BLAST to date were restricted to the context of a single-user link. In this paper, the system-level benefit of using BLAST in multicell scenarios is evaluated in comparison with other directive- and adaptive-array techniques.**

I. Introduction

RECENT information-theory results have shown the enormous capacity potential of wireless communication techniques employing antenna arrays at both transmitter and receiver, in particular when the channel and array structure are such that the transfer functions between different transmit and receive antenna pairs are largely independent [1][2][3]. To exploit this potential, a number of layered space-time (BLAST) architectures have been proposed [4][5]. In BLAST, multiple parallel data streams are transmitted—simultaneously and on the same frequency—in a multiple-input multiple-output fashion. With rich multipath propagation, these different streams can be separated at the receiver because of their distinct spatial signatures. In its original form, BLAST does not require the transmitter to possess any channel information; only the receiver is required to estimate the channel. Nonetheless, provided the scattering richness is sufficiently high, the spectral efficiency attainable—in this open-loop configuration—is very close to the spectral efficiency supported by the channel. Closed-loop versions of BLAST, where information on the channel and possibly also on the interference is available at the transmitter, have also been reported [3][6].

While extremely promising, all analysis of BLAST presented to date were restricted to the context of a single-user link. Thus, the impact of these techniques on the overall capacity of multicell systems had yet to be assessed. Furthermore, the system-level benefit of using BLAST over other adaptive-antenna techniques was still not quantified.

The spectral efficiency of single-antenna wireless systems has been extensively studied in the past, mostly with the assumption—determined by the interest in providing voice services—of constant and identical data rates for all users and minimal tolerance to delay [7]. In that case, there is a clear trade-off between the *link* spectral efficiency and the *system* spectral efficiency [8]. Since every user is exposed to interference from all other co-channel users, the highest system spectral efficiency is not attained when every individual user is independently attempting to maximize its own link spectral efficiency, but rather when every user selflessly reduces its transmit power to the lowest possible level that can sustain the target data rate [9]. Such a strategy requires the use of power control, which can be implemented in a distributed fashion with no loss of optimality [10]. If the traffic is dominated by delay-resilient data—as might be the case in emerging systems—and the data rates are variable and heterogeneous, rate adaptation becomes, not only an attractive complement but even an alternative to power control [11]. The system spectral efficiency with fixed-power and rate adaptation has been recently studied in [12]. Also, the impact of antenna diversity was investigated in [13]. Similar analysis with adaptive-antenna techniques were presented in [14].

In this paper, we evaluate the system spectral efficiency of BLAST in relative comparison with single- and adaptive-antenna solutions. These comparative evaluation is performed in a frequency-flat Ricean environment where FDMA/TDMA is employed for multiple access and the use of capacity-achieving codes is presumed. The emphasis of the paper is not on the choice of a multiple access scheme, which ultimately appears to have minor impact on the capacity of a well-designed system [15], but rather on how to use a given set of antenna resources most effectively. We restrict ourselves to the case where the total power per user is held constant while the data rate is being adapted.

II. Link Space-Time Processing Techniques

A. Link Model

Every user link consists of a transmitter and a receiver with M and N antennas, respectively. The channel responses from transmit antenna m to receive antenna n, denoted by h_{nm}, are assembled into a channel matrix $\mathbf{H}$. The N-dimensional received signal vector $\mathbf{x}$ depends on the M-dimensional transmit signal vector $\mathbf{s}$ via

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

where $\mathbf{n}$ is an N-dimensional interference-plus-noise vector with spatial covariance matrix $\mathbf{K}_n = E\{\mathbf{n}\mathbf{n}^\dagger\}$, which in-

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cludes a thermal noise term $\sigma^2\mathbf{I}$. The covariance matrix of the transmit signal is $\Phi=E\{\mathbf{s}\mathbf{s}^\dagger\}$ with the total transmit power limited to P_T irrespective of the number of transmit antennas, that is, $trace(\Phi)=P_T$. The covariance of the desired signal at the receiver is $\mathbf{K}_d=\mathbf{H}\Phi\mathbf{H}^\dagger$. We assume that $\mathbf{H}$ is known perfectly at the receiver, but not necessarily at the transmitter.

B. Channel Model

In order to simulate a wireless channel with different degrees of scattering richness, we use the well-known Ricean model [16]. Accordingly, the channel has two distinct components:

i) A specular component that illuminates the arrays uniformly and is thus spatially deterministic from antenna to antenna; and

ii) A scattered Rayleigh-distributed component that varies randomly from antenna to antenna.

In the limit of a purely scattering environment, the deterministic component vanishes. On the other hand, in the limit of a purely specular or line-of-sight environment, the deterministic component constitutes the entire channel response. Hence, the Ricean model comprises the rich-scattering and specular as particular (extreme) cases. Notice also that this model implicitly assumes that the only source of correlation among the array elements is the deterministic component. Since the base stations are usually located above the clutter while the user terminals are located within the clutter, we are therefore assuming a spacing of several wavelengths at the base stations for only a fraction of a wavelength at the terminals.

With the K-factor defined as the ratio of deterministic-to-scattered power, the channel response is given by

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}^{sc} + \sqrt{\frac{1}{K+1}}\mathbf{H}^{sp}$$

where

$$\mathbf{H}^{sc} = \sqrt{G}\,\tilde{\mathbf{H}}^{sc}$$

with G the large-scale (local average) path gain encompassing distance-dependent decay as well as shadow fading. The gain of the individual antennas is also absorbed into G. The elements of the normalized scattering component $\tilde{\mathbf{H}}^{sc}$ are statistically independent unit variance complex Gaussian random variables. The channel specular component, in turn, is given by

$$\mathbf{H}^{sp} = \sqrt{G}\,\tilde{\mathbf{H}}^{sp}$$

where

$$\tilde{\mathbf{H}}^{sp} = \mathbf{a}(\theta_t)\mathbf{a}(\theta_r)^T$$

with $\mathbf{a}(\theta_t)$ and $\mathbf{a}(\theta_r)$ the specular array responses at the transmitter and receiver, respectively. The array response corresponding to a N-element linear array, for instance, is given by $[1, e^{j2\pi d cos(\theta)}, \ldots, e^{j2\pi d(N-1)cos(\theta)}]$ where θ is the angle of arrival or departure of the specular component and d is the antenna spacing in wavelenths.

C. Space-Time Techniques

C.1 Closed-loop BLAST with Channel Information at the Transmitter

The maximum link spectral efficiency[1], achieved when the channel and interference spatial covariances are known at the transmitter and the covariance of the transmit signal is adjusted appropriately [6], is given by

$$C = \log_2(\Pi_{m=1}^{M}(1 + p_m\lambda_m)) \tag{1}$$

where p_m is the power assigned to each eigenmode, which is found by a water-fill process as

$$p_m = (\nu - \frac{1}{\lambda_m})^+, \quad m = 1, \ldots, M, \quad \sum_m p_m = P_T$$

with

$$\mathbf{H}^\dagger\mathbf{K}_n^{-1}\mathbf{H} = \mathbf{U}\Lambda\mathbf{U}^\dagger, \qquad \Lambda = diag(\lambda_1, \ldots, \lambda_M)$$

and with the constant ν chosen such that the total transmit power is equal to P_T. The function $(.)^+$ is zero when the argument is negative indicating that the corresponding mode is too weak and should be allocated no power.

The goal of this decomposition process is to find the channel eigenmodes in the presence of the interference $\mathbf{n}$ in order to send multiple independent data streams through those eigenmodes. The transmit covariance matrix that achieves (1) is given by

$$\Phi = \mathbf{U}\, diag(p_1, \ldots, p_M)\, \mathbf{U}^\dagger$$

The use of this algorithm within the context of a multicell environment poses some challenges. Since the spatial signature of every user has an impact on all other co-channel users, the arrangement of spatial signatures that would maximize the system capacity could only be computed and enforced by a centralized entity. A practical approach, based on only local information, would have to be iterative. Hence, every user would adjust the spatial characteristics of its output signal based on the structure of the interference, which would—in turn—trigger new adjustments by all other co-channel users, and so forth.

As an alternative, it is possible to formulate a form of closed-loop BLAST wherein the transmitter is supplied with information about the channel, but not about the interference. This form of BLAST is attractive because—without power control—it eliminates the need to iterate. In the remainder of the paper, we will concentrate on this simplified form of closed-loop BLAST.

With no information about the spatial characteristics of the interference, the default signaling is that for which the interference is spatially white and thus the link spectral efficiency reduces to (1) with the p_m values computed using $\mathbf{K}_n \propto \mathbf{I}_N$. The λ_n values, however, are still as before because—although unknown to the transmitter—the interference may still be colored at the receiver.

[1] Zero excess bandwidth is assumed throughout the paper.

C.2 Open-loop BLAST

When the transmitter is deprived of any channel or interference information, the optimal transmit covariance matrix is $\Phi = \frac{P_T}{M}\mathbf{I}_M$ [1]. Consequently, the link spectral efficiency is given by

$$C = \log_2 \det(\mathbf{I}_N + \frac{P_T}{M}\mathbf{H}\mathbf{H}^\dagger) \tag{2}$$

C.3 Adaptive Arrays with Channel Information at the Transmitter

In an adaptive array or beamformer, a single data stream is transmitted simultaneously from multiple antennas with proper weight coefficients [17]. The coefficients for the M transmit antennas are assembled into an M-dimensional vector $\mathbf{w}$. Therefore, the received signal can be expressed as

$$\mathbf{x} = \mathbf{H}\mathbf{w}s + \mathbf{n}$$

where s is here a scalar. The transmit covariance matrix is given by $\Phi = P_T\mathbf{w}\mathbf{w}^\dagger$ and the spectral efficiency is

$$\begin{aligned} C &= \log_2 \det(\mathbf{I}_N + P_T\mathbf{H}\mathbf{w}\mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{K}_n^{-1}) \\ &= \log_2(1 + P_T\mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{K}_n^{-1}\mathbf{H}\mathbf{w}) \end{aligned} \tag{3}$$

with optimal combining at the receiver [18]. The optimal $\mathbf{w}$ is the one that maximizes (3) with the constraint that the total transmit power (specified by the norm of $\sqrt{P_T}\mathbf{w}$) be limited to P_T, which is simply the principal eigenvector of $\mathbf{H}^\dagger\mathbf{K}_n^{-1}\mathbf{H}$ with norm set to unity.

As in closed-loop BLAST with interference information, iterative optimization approaches are necessary in order for the algorithm to operate in a distributed fashion within a multicell environment [14]. Therefore, we again choose to implement a sub-optimal version of the algorithm wherein the interference is unknown at the transmitter and hence—with no power control—there is no need to iterate. With that restriction, the optimal $\mathbf{w}$ is now the principal eigenvector of $\mathbf{H}^\dagger\mathbf{H}$ with norm set to P_T and the spectral efficiency can be obtained simply by plugging that value into (3).

C.4 Directive Arrays with Directional Location Information at the Transmitter

Implementing BLAST or adaptive-array techniques requires a Radio-Frequency (RF) chain per antenna at both transmitter and receiver. In order to simplify the RF requirements, the antennas can be arranged as a directive array.

In such configuration, the transmitter can steer a beam towards the receiver if only its directional location is known. That is,

$$\mathbf{w} = \frac{\mathbf{a}(\theta_r)}{\sqrt{M}} = \frac{1}{\sqrt{M}}[1, e^{j2\pi d cos(\theta_r)}, e^{j2\pi d(M-1)cos(\theta_r)}]^T$$

where θ_r defines the directional location of the receiver[2]. When the receiver uses a directive array as well, the combining vector at the receiver is given by $\mathbf{a}(\theta_t)/\sqrt{N}$, where θ_t is the angle to the desired transmitter. The received signal is given by

$$x = \sqrt{\frac{P_T}{MN}}\mathbf{a}^\dagger(\theta_t)\mathbf{H}\mathbf{a}(\theta_r)s + \frac{1}{\sqrt{N}}\mathbf{a}^\dagger(\theta_t)\mathbf{n}$$

and the capacity can now be evaluated as in architectures with a single antenna at both transmitter and receiver. The (scalar) desired signal variance is given by

$$K_d = \frac{P_T}{MN}|\mathbf{a}^\dagger(\theta_t)\mathbf{H}\mathbf{a}(\theta_r)|^2$$

and the (scalar) interference-plus-noise variance is given by

$$K_n = \frac{1}{N}E\{\mathbf{a}^\dagger(\theta_t)\mathbf{n}\mathbf{n}^\dagger\mathbf{a}(\theta_t)\} = \frac{1}{N}\mathbf{a}^\dagger(\theta_t)\mathbf{K}_n\mathbf{a}(\theta_t)$$

with the link spectral efficiency being

$$C = \log_2(1 + K_d K_n^{-1}). \tag{4}$$

[2]The formulation we present herein corresponds specifically to linear arrays. The analysis can be generalized to any arbitrary array geometry by replacing $\mathbf{a}$ with the corresponding array response.

III. System Model and Simulation Environment

We consider a multicell system layout with three sectors per cell, where some combination of FDMA and TDMA is employed. Users are uniformly distributed throughout the system and connected to the sector from which they receive the strongest signal. In the remainder, we concentrate on the downlink only, which has the most stringent capacity demands for data applications. Focusing on a set of co-channel sectors, the signal at the receiver in sector i is given by

$$\mathbf{x}_i = \sum_j \mathbf{H}_{ij}\mathbf{s}_j + \mathbf{n}_i$$

where $\mathbf{H}_{ji}$ is the matrix channel response from the transmitter in sector j to the receiver in sector i, $\mathbf{s}_j$ is the transmit vector intended for user j with covariance $\Phi_j = E\{\mathbf{s}_j\mathbf{s}_j^\dagger\}$ and $\mathbf{n}_i$ is the noise vector at receiver i. The total covariance matrix at receiver i is given by

$$E\{\mathbf{x}_i\mathbf{x}_i^\dagger\} = \sum_j \mathbf{H}_{ij}\Phi_j\mathbf{H}_{ij}^\dagger + \sigma_i^2\mathbf{I}_N.$$

The desired-signal covariance matrix is

$$\mathbf{K}_{d,i} = \mathbf{H}_{ii}\Phi_i\mathbf{H}_{ii}^\dagger \tag{5}$$

and the interference-plus-noise covariance matrix is

$$\mathbf{K}_{n,i} = \sum_{j\neq i} \mathbf{H}_{ij}\Phi_j\mathbf{H}_{ij}^\dagger + \sigma_i^2\mathbf{I}_N.$$

It is known that, in a multicell interference channel, joint Gaussian signaling falls short of maximizing the total system capacity [19]. In fact, the optimal signaling in that general case is an unsolved information-theory problem. However, if the structure of the interference from other cells is not exploited but only regarded as noise, Gaussian signaling *is* optimal. Thus, we assume Gaussian signaling throughout our system, which makes the interference also Gaussian. Furthermore, in our search for general results and relative performance levels, we also postulate the use of codes tending towards achieving capacity in the Shannon sense. With that, we avoid invoking specific modulation formats or code structures.

We conduct Monte-Carlo simulations on a wrapped-around universe with 100 perfectly sectorized hexagonal cells arranged in a 10×10 grid. The terminal antennas are omnidirectional. The propagation exponent is set to 3.5 and the shadow fading is log-normally distributed with an 8-dB standard deviation. The cell size, transmit power and noise floor are scaled to ensure that the system is mostly interference-limited[3]. We consider three different reuse factors which cover a wide range of system arrangements. In order of increasing tightness, we consider reuse 3/9 (every unit of bandwidth is used in one sector of every third cell), 1/3 (every unit of bandwidth is used in one sector of every cell) and 1/1 (universal reuse—every unit of bandwidth is used in every sector of every cell).

IV. Performance Evaluation

First, we evaluate the impact of scattering on the various techniques. Shown in Fig. 1 is the 10%-outage user spectral efficiency as a function of the K-factor with reuse 3/9. To emphasize the differences, the analysis is performed using a large number of antennas (M=12, N=16). BLAST clearly outperforms all other techniques in highly scattering scenarios, although its advantage diminishes with decreasing scattering. Nonetheless, its robustness is remarkable for its spectral efficiency does not drop significantly with factors as large as K=10. Therefore, its performance is superior in most cases of practical interest. Furthermore, the closed-loop version is never inferior to any of the other schemes. The adaptive- and directive-array techniques, on the other hand, improve monotonically with K and, in the limit of $K \rightarrow \infty$, the adaptive-array efficiency becomes identical to that of closed-loop BLAST, clearly indicating that forming beams is the most adequate solution in such conditions. With respect to a baseline system with M=N=1, the spectral efficiency advantage is enormous in all cases, particularly when using BLAST with sufficient scattering.

Next, we focus on the rich-scattering case by presenting, in Fig. 2, the system spectral efficiency cumulative distributions with K=0. Again, in order to emphasize the behaviors, the number of antennas is large (M=N=16). With adaptive arrays, the spectral efficiencies—for every reuse factor—have a relatively small spread. Clearly, this type of processing effectively mitigates and controls co-channel interference. In fact, as long as the number of dominant interferers is smaller than the number of receive antennas, the receiver can push the interference level down to the noise floor. Therefore, the lower tail of the cumulative distribution, corresponding to the worst locations within each cell, is very well behaved. However, adaptive arrays are unable to provide further efficiency increases in those locations within every cell—corresponding to the upper tail—where conditions are favorable. In those locations, additional antennas only contribute array gain, which results in a slow—asymptotically logarithmic—efficiency improvement. With BLAST, on the other hand, the spectral efficiency growth is much faster—asymptotically linear—with the number of antennas [1]. Thus, users in favorable locations can utilize their antennas to attain much larger efficiencies and, as a result, the spectral efficiency spread is much larger and its peak is almost an order of magnitude higher. Nonetheless, since the interference caused by a BLAST transmitter has components in multiple spatial dimensions, it also requires multiple spatial degrees of freedom for proper mitigation. As a result, interference control is much more difficult except possibly if $N \gg M$. Thus, in our example with M=N=16, the lower tail of the open-loop BLAST cumulative does not improve with tightening reuse. Furthermore, the lower tail of the open-loop BLAST cumulative is always behind the corresponding adaptive-array cumulative, with the cross point increasing as reuse tightens and interference levels increase. With closed-loop BLAST, that effect is almost completely eliminated as users in detrimental locations signal in fewer spatial dimensions and thus better operating points are found.

Finally, we look at how the spectral efficiency scales with the number of antennas. To reduce the number of parameters, both M and N are scaled simultaneously. It appears clear from the previous section that the reuse that yields the highest spectral efficiency is different at 50%, 90% or 99% support. In addition, the optimal reuse also varies with the number of antennas. Hence, since an advanced TDMA system would most likely adapt its reuse dynamically [20] based on whichever operating point is chosen, a true measure of the spectral efficiency growth with the number of antennas should be calculated with the most adequate reuse at every point. The resulting set of curves, with K=0, are presented in Fig. 3. Notice how the slopes oscillate because of the reuse factor granularity. With true dynamic channel assignment, the granularity would disappear. At sufficiently small outage levels, adaptive array processing outperforms BLAST. With respect to average or peak efficiencies, however, BLAST is vastly superior

In all cases, it must be taken into account that absolute system spectral efficiencies are very sensitive to propagation parameters such as the exponent, shadow fading standard deviation, etc. Therefore, it is the relative scaling rather than the absolute numbers themselves that is relevant.

V. Conclusions

In summary, BLAST is vastly superior—with sufficient scattering—to any other space-time technique at most levels in its open-loop form and at virtually all with closed loop. With decreasing levels of scattering, the spectral efficiency diminishes for all algorithms. In highly specular environments, the performance of closed-loop BLAST is always equal or better than those of adaptive- and directive-arrays and still largely superior to the baseline reference posed by a single-antenna system.

[3]The signal-to-noise ratio is higher than 25 dB in 90% of every cell.

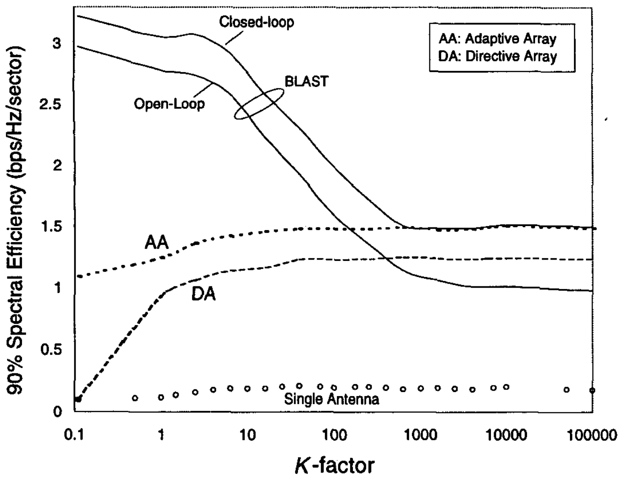

Fig. 1. 90% system spectral efficiency (bps/Hz) as a function of the Ricean K-factor with reuse 3/9 and M=12, N=16.

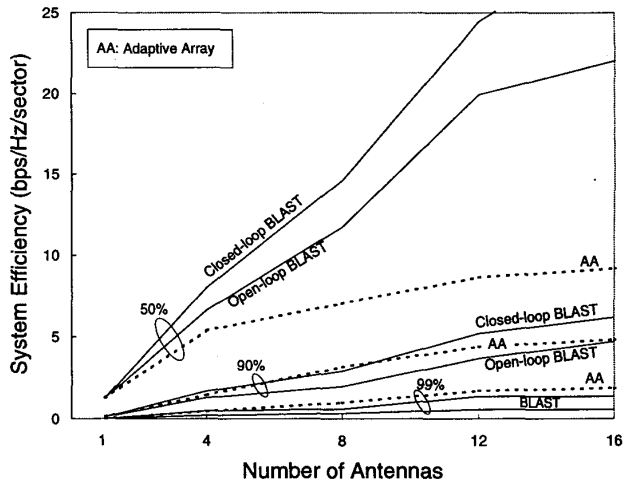

Fig. 3. 50%, 90% and 99% system spectral efficiency as a function of the number of antennas, M=N, with optimized reuse and K=0.

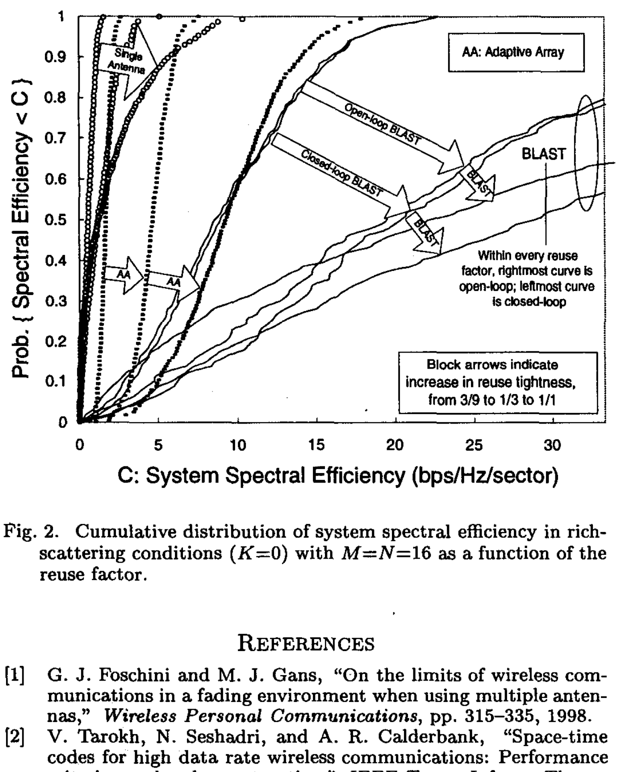

Fig. 2. Cumulative distribution of system spectral efficiency in rich-scattering conditions (K=0) with M=N=16 as a function of the reuse factor.

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