Propagation and Capacities of Multi-element Transmit and Receive Antennas

Dmitry Chizhik, Gerard J. Foschini, Michael J. Gans and Reinaldo A. Valenzuela

Abstract

Multi-element system capacities are usually thought of as limited only by correlations between elements. It is shown here that degenerate channel phenomena, called "keyholes" may arise under realistic assumptions which have zero correlation between the entries of the channel matrix **H** and yet only a single degree of freedom. Canonical physical examples of keyholes are presented. For outdoor environments, it is shown that roof edge diffraction is perceived as a "keyhole" by a vertical base array that may be avoided by employing instead a horizontal base array.

I. Introduction

Single user communication with M transmit and N receive antennas can achieve very high spectral efficiencies in highly scattering environments [1]. For example, BLAST (Bell-labs LAyered Space-Time) communication technique has been proposed by Foschini [2], and demonstrated experimentally by Golden, et. al. [3] and Wolniansky, et. al. [4]. These high spectral efficiencies are enabled by the fact that a scattering environment makes the signal from every individual transmitter appear highly uncorrelated at each of the receive antennas. As a result, the signal corresponding to every transmitter has a distinct spatial signature at the receiver. In a sense, the scattering environment acts like a very large aperture that makes it possible for the receiver to resolve the individual transmitters.

The high spectral efficiency is reduced if the signals arriving at the receivers are correlated. A narrowband channel may be described in terms of a complex channel transfer matrix **H**, whose entry h_{nm} corresponds to the response of the n^{th} receiver to the signal sent by the m^{th} transmitter. When the entries of **H** are distributed as complex Gaussians, maximum capacity is achieved when $\langle h_{nm}h_{kl}^*\rangle=0$ for $n\neq k$ and $m\neq l$. Correlation between antennas may be reduced in actual deployments by, say, separating the antennas spatially [5,6,7]. However, it has been shown by Chizhik, et. al. [8] and by Gesbert, et. al. [9] that low correlation is not a guarantee of high capacity. In [8] the existence of degenerate channels, called "keyholes" has been proposed, and demonstrated through physical examples, that have uncorrelated transmit and receive signals, and yet only a single degree of freedom. In this paper outdoor propagation channels are analyzed to allow predictions of capacity for multiple transmitter, multiple receiver systems.

II. Outdoor propagation

A. Analysis

The outdoor environment is modeled as a dielectric slab, shown in Figure 1, which represents large-scale clutter, e.g. houses, trees, etc. The signal radiated by the remote antennas is scattered in the vicinity of the remote and produces a field U in the horizontal plane lying above the street where the remote is located and at the height of the top of the large-scale clutter.

The scalar field U represents the horizontal H field in the case of vertical polarization and the horizontal E field in the case of horizontal polarization. This scalar field representation is strictly valid only where there is no cross-polarization coupling, but is used here for simplicity to demonstrate the effect of roof edge diffraction. As the fading between the signals on the two polarizations is uncorrelated (Jakes [10]), the capacity is expected to be doubled through the use of both polarizations at both the transmitter and the receiver, regardless of the strength of cross-polarization coupling. It is assumed that the medium has no preferential treatment of either polarization [12]. The field U therefore satisfies the scalar Helmholtz equation $\nabla^2 U + k^2 U = 0$. The field radiated by the remote and measured at the base station may be expressed in terms of the values of the field at the boundary which is a horizontal plane just above the dielectric slab, by using Helmholtz-Kirchhoff theorem:

$$U(\underline{r}) = \iint \underline{dA'} \cdot (U\nabla G - G\nabla U), \quad G(r') = \frac{e^{ik\left|\underline{r}-\underline{r}'\right|}}{4\pi\left|\underline{r}-\underline{r}'\right|} - \frac{e^{ik\left|\underline{r}''-\underline{r}'\right|}}{4\pi\left|\underline{r}''-\underline{r}'\right|} \qquad (1)$$

where the G is the Greens function, $\mathbf{r}'$ points to the integration boundary, and $\mathbf{r}$ and $\mathbf{r}''$ point to the base station antenna and its image, respectively. In (1) the integration is over the horizontal plane boundary. Both the field U and the Green's function G satisfy the Dirichlet boundary condition at the top of the large-scale clutter: U=0, G=0. This is approximately true for both horizontal and vertical polarizations for plane wave reflection from a dielectric half space when the grazing angles are small. This is the case of interest in terrestrial communications, where the height of the base station is small compared to the distance to the remote.

As the Greens function G is zero over the entire boundary, the second term in the integrand of (1) drops out and the integration in (1) is then over the horizontal half-plane to the left of the gray region in Figure 1. The field at the base may then be calculated as:

$$U(\underline{r}) = \frac{ize^{ikx}}{\lambda x^2} e^{ik\frac{(z^2+y^2)}{2x}} \int_{-\infty}^{\infty} dy' \int_{-\infty}^{0} dx' U(x',y',0) \exp(-ikx' + \frac{iky'^2}{2x} - \frac{ikyy'}{x}) \quad (2)$$

where λ is the wavelength. Using the Fresnel approximation for the distance R between the base antenna (or its image), and the high frequency asymptote for the end-point contribution from the neighborhood of $x'=0$, the equation (2) may be simplified by evaluating the integral over the x' coordinate by means of the initial value theorem:

$$U(\underline{r}) = \frac{ze^{ikx}}{\lambda x^2 k} e^{ik\frac{(z^2+y^2)}{2x}} \int_{-\infty}^{\infty} dy' U(0,y',0) \exp(\frac{iky'^2}{2x} - \frac{ikyy'}{x}) \quad (3)$$

Now the integral is only over the y' axis, which coincides with the roof edge. The equation (3) expresses the fact that the field U measured at the base results from the diffraction of the remote field at the roof edge.

The edge field $U(0,y',0)$ in (3) is :

$$U(0,y',0) = G_1(0,y',0)s_1 + G_2(0,y',0)s_2 + \ldots = \begin{pmatrix} G_1(0,y',0) & G_2(0,y',0) & \bullet \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ \bullet \end{pmatrix} \quad (4)$$

where $G_m(0,y',0)$ is the Green's function due to the source m, which is evaluated at the roof edge, and includes all the street-level scattering, and s_m is the signal transmitted from the source m.

For an array of base antennas that are all located at the same distance x from the roof edge the channel transfer function $\mathbf{H}$ may be written as:

$$\mathbf{H} = \frac{e^{ikx}}{\lambda x^2 k} \begin{pmatrix} z_1 e^{ik\frac{z_1^2}{2x}} \int_{-\infty}^{\infty} dy' G_1(0,y',0) e^{\frac{ik(y_1-y')^2}{2x}} & z_1 e^{ik\frac{z_1^2}{2x}} \int_{-\infty}^{\infty} dy' G_2(0,y',0) e^{\frac{ik(y_1-y')^2}{2x}} & \bullet \\ z_2 e^{ik\frac{z_2^2}{2x}} \int_{-\infty}^{\infty} dy' G_1(0,y',0) e^{\frac{ik(y_2-y')^2}{2x}} & z_2 e^{ik\frac{z_2^2}{2x}} \int_{-\infty}^{\infty} dy' G_2(0,y',0) e^{\frac{ik(y_2-y')^2}{2x}} & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} \quad (5)$$

B. Vertical base array:

When the base antennas are arranged in a vertical array, $(x_n,y_n,z_n)=(x,y,z_n)$ for all n, where n is the base antenna index. Using these coordinates in (5) the channel transfer matrix may then be immediately written as:

$$\mathbf{H} = \begin{pmatrix} z_1 e^{ik\frac{z_1^2}{2x}} \\ z_2 e^{ik\frac{z_2^2}{2x}} \\ \bullet \end{pmatrix} \frac{e^{ikx}}{\lambda x^2 k} \begin{pmatrix} \int_{-\infty}^{\infty} dy' G_1(0,y',0) e^{\frac{ik(y-y')^2}{2x}} & \int_{-\infty}^{\infty} dy' G_2(0,y',0) e^{\frac{ik(y-y')^2}{2x}} & \bullet \end{pmatrix} \quad (6)$$

Note that the **H** matrix is a dyad, providing only a single degree of freedom. This degenerate behavior results from applying the initial value theorem to (2) to get (3). The resulting field as measured at the base station has lost all richness in the vertical direction.

C. Horizontal base array

When the base antennas are arranged in a horizontal linear array, the x and z coordinates are the same for all the elements: $(x_n, y_n, z_n)=(x, y_n, z)$. It may be observed that it is not possible to factor the **H** matrix in the same way as was done in the case of the vertical array in (6). The integration carried out in (5) may be seen to be a linear (and unitary) transformation of the roof edge field U(0,y′,0) and the Greens function *G*, respectively. As the field at the roof edge has undergone extensive scattering in the vicinity of the remote, the field at the roof edge is actually a sum of many multipath arrivals, and may be modeled as a complex Gaussian process, following the Central Limit Theorem. Received field $U(\underline{\mathbf{r}})$ is a linear functional of aperture field $U(\underline{\mathbf{r}}')$, as seen in (2). If $U(\underline{\mathbf{r}}')$ is a Gaussian process, so is $U(\underline{\mathbf{r}})$, because a linear transformation of a Gaussian process is also Gaussian. Gaussian processes are completely characterized by their mean and covariance.

The antenna elements at the remote are assumed to be adequately separated to produce no correlation between the remote antennas (antenna separation on the order of λ/2 for isotropic scattering). Thus, the Greens functions $G_m(0,y',0)$ and $G_p(0,y',0)$ are not correlated for any $m \neq p$, and, alternatively, entries of **H** which are in different columns are uncorrelated. The only correlations that need be computed are between the entries in different rows but the same column of **H**.

Now the mean of the field at the base due to remote antenna *m* is computed by taking the average of any element of (5), and is found to be zero, provided the mean field at the roof edge is zero, $\langle G_m(0,y',0)\rangle = 0$

The field of the remote antenna *m* at the roof edge may be represented by an incoherent line source of intensity *I(y')*. The correlation of the roof edge field is

$$\left\langle G_m(0,y',0)G_m(0,y'',0)^*\right\rangle = \frac{4\pi^2}{k^2} I\left(\frac{y'+y''}{2}\right)\delta(y'-y'') \quad (7)$$

where the fields at two distinct points are assumed to be uncorrelated. This is an approximation which is thought to be valid as the field is expected to be decorrelated on a scale of λ/2, which is much smaller than other spatial scales. Using (7), the correlation of the field due to source *m*, measured at the two base antennas $\left\langle U_m(\underline{r_1})U_m(\underline{r_2})^*\right\rangle$ may be determined from the correlation of the corresponding entries of (5) that are in the same column *m*:

$$\left\langle U_m(\underline{r_1})U_m(\underline{r_2})^*\right\rangle = \frac{z_1 z_2 e^{i\frac{k}{2x}(z_1^2-z_2^2)} e^{\frac{ik}{x} y_c y_d}}{\lambda^2 k^2 x^4} \int_{-\infty}^{\infty} dy_c' \frac{4\pi^2}{k^2} I(y_c') e^{\frac{-ik}{x} y_c' y_d} \quad (8)$$

If we assume the incoherent intensity to be of Gaussian form with the spatial scale σ_y of the order of street width, $I(y_c') = \sqrt{\frac{1}{2\pi\sigma_y^2}} \exp(-\frac{y_c'^2}{2\sigma_y^2})$, the correlation coefficient, obtained by normalizing (8), is also of Gaussian form:

$$\rho(y_d) = \frac{\left\langle U_m(\underline{r_1})U_m(\underline{r_2})^*\right\rangle}{\sqrt{\left\langle |U_m(\underline{r_1})|^2\right\rangle \left\langle |U_m(\underline{r_2})|^2\right\rangle}} = \exp(-\frac{k^2\sigma_y^2 y_d^2}{2x^2}) \quad (9)$$

where y_d is the distance between base antennas.

Now the stochastic process determining the channel transfer matrix **H** is completely specified. Given the assumptions in this section, and for the case of the horizontal base array, the **H** process is seen to be a complex Gaussian process with matrix mean zero, and zero covariance between distinct remote antennas. The correlation coefficient between antennas has been derived in [7] based on the assumption that the angular spectrum of signals received at the base is of width 2^0 at 1 km, which is equivalent to (9), if σ_y is set to 30 meters.. The results in [7] may therefore be applied here without reservation. It was found that, when the base antennas are separated by 4λ, the capacity at 10% outage is 80% of that achievable in completely uncorrelated Gaussian channels. This appears to be a reasonable compromise between base antenna array size and achievable capacity.

III. Conclusion

It was shown here that degenerate channel phenomena, called "keyholes" may arise under realistic assumptions which have zero correlation between the entries of the channel matrix **H** and yet only a single degree of freedom. Decorrelation is therefore not a guarantee of BLAST performance. Of most relevance to outdoor propagation, a diffraction-induced keyhole that is perceived by a vertical base array has been discussed and modeled in detail. A remedy for this degeneracy is to use a horizontal array, where the antenna elements are separated enough to resolve the scattering region around the remote. It is found to be sufficient to separate the elements by 4λ, provided the scattering region is about 30 meters in diameter (about a street width), and the remote is less than 1 km away from the base.

IV. References

[1] G. J. Foschini and M. J. Gans, "On Limits of Wireless Communications in a Fading Environment When Using Multiple Antennas", *Wireless Personal Commun.*, V. 6, No. 3, Mar. 1998, p. 311.

[2] G. J. Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment When Using Multiple Antennas", *Bell Labs Tech. J.*, V. 1, No. 2, Autumn 1996, pp. 41-59.

[3] G. D. Golden, G. J. Foschini, R. A. Valenzuela, P. W. Wolniansky, "Detection Algorithm and Initial Laboratory Results using the V-BLAST Space-Time Communication Architecture", *Electronics Lett.*, Vol. 35, No. 1, Jan. 7, 1999, pp. 14-15.

[4] P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel", *Proc. ISSSE-98*, Sept., 1998.

[5] P. F. Driessen and G. J. Foschini, "On the Capacity Formula for Multiple Input–Multiple Output Wireless Channels: A Geometric Interpretation", *IEEE Trans. on Commun.*, vol. 47, no. 2, Feb. 1999, pp. 173-176.

[6] D.-S. Shiu, G.J. Foschini, M.J. Gans, and J. M. Kahn, "Fading Correlation and Its Effect on the Capacity of Multielement Antenna Systems", *IEEE Trans. On Commun.*, vol. 48(2), March, 2000, pp. 502-513.

[7] D. Chizhik, F. Rashid-Farrokhi, J. Ling, A. Lozano, "Effect of antenna separation on the capacity of BLAST in correlated channels", accepted for publication in *IEEE Commun. Lett.*

[8] D. Chizhik, G. J. Foschini and R. A. Valenzuela, "Capacities of Multi-element Transmit and Receive Antennas: Correlations and Keyholes", *IEE Electronic Lett.*, June 22, 2000, V. 36, No. 13, p. 1099-1100.

[9] D. Gesbert, H. Bolcskei, D. Gore, A. Paulraj, "MIMO Wireless Channels: Capacity and Performance Prediction", CT10-5, *IEEE Globecom 2000*, Nov. 27-Dec. 1, 2000, San Fran., CA.

[10] W.C. Jakes, Ed., *Microwave Mobile Communications*, New York, Wiley, 1974, reprinted by IEEE, 1994.

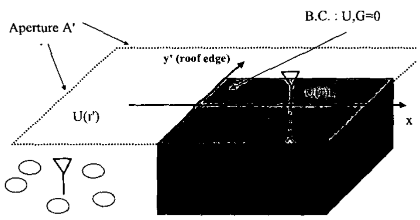

Figure 1. Canonical outdoor environment.