Analysis of the Junction Between Smooth and Corrugated Cylindrical Waveguides in Mode Converters

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Abstract —**A method for the determination of the scattering matrix of the junction between smooth and corrugated cylindrical waveguides is developed based on the expansion of the modal fields for the corrugated waveguide into eigenfunctions of the transmission matrix of a waveguide unit cell. This method, used in conjunction with usual techniques for evaluation of the scattering matrix of mode converters, is here shown to improve the precision of results obtained by rendering uniform the accuracy of the models applied in the calculations. Also, the analysis is now valid for any size of corrugation depth, and the frequency band of applicability is enlarged accordingly.**

I. Introduction

The return loss and the generation of unwanted modes in corrugated horns are essentially determined by the characteristics of the mode converter located between the smooth-walled input waveguide and the horn [1], [2]. A common type of converter is composed of a nonuniform section of corrugated waveguide with gradually varying slot depths. In the typical application of conversion from the TE_{11} to the HE_{11} mode in cylindrical corrugated horns the initial slot has a depth of $\lambda/2$ and the final one a depth of approximately $\lambda/4$ [3]. Wider bandwidths and lower SWR's can be achieved using ring-loaded corrugations or a quarter-wave transformer at the input of the converter [4], [5].

An accurate performance analysis of the converter, an essential procedure to validate its design, is normally made by dividing the structure into several elementary sections, calculating the scattering matrix of each section, and progressively cascading them to obtain the overall scattering matrix of the device [1], [6].

Unless the cascading process is extended up to the aperture of the horn [7] (which requires excessive computer time), it must be stopped at a section where the remainder of the horn can be approximated by a semi-infinite uniform corrugated waveguide. In this case, the last elementary section to be computed is the junction between a smooth-walled and a corrugated waveguide. The usual method for calculating the scattering matrix of this junction is based on mode-matching techniques, the modes in the corrugated waveguide being obtained from an approximate model where the effect of the corrugations is represented by an anisotropic impedance [1], [8], [9].

In most situations, this technique produces quite good results. If the depth of the corrugations of the terminal corrugated waveguide is small in comparison with a single wavelength, however, significant errors are observed in the calculated return loss.

The problem derives from the fact that in the adopted approximate model for the corrugated waveguide, only the fundamental mode propagating into the corrugations is considered. For shallow corrugations, however, this fundamental mode has a small magnitude, and higher order evanescent modes must be taken into consideration.

In this paper, a more rigorous model, one that circumvents the problem cited above, will be presented for determining the scattering matrix of the junction between smooth and corrugated cylindrical waveguides. The model, as will be shown here, is based on the representation of the modal fields for the corrugated waveguide by an expansion into eigenfunctions of the transmission matrix of a unit cell formed by a period of the waveguide.

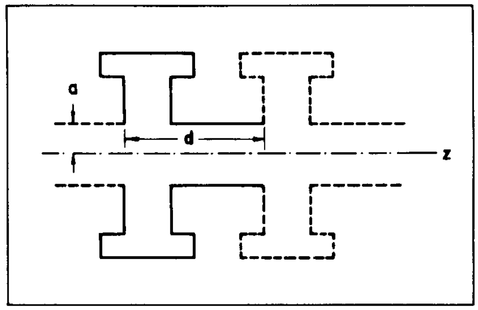

Fig. 1. The unit cell of a corrugated waveguide.

II. Modal Fields in a Corrugated Waveguide

Consider a corrugated waveguide and isolate a unit cell of this waveguide, as shown in Fig. 1. (The corrugations can be ring loaded, as shown in the figure.) The modal fields for the corrugated waveguide can be obtained from the eigenvectors of the transmission matrix of the unit cell, according to the expressions [10]

$$\vec{e}_{cn} = \sum_{j=1}^{N} \alpha_{jn} \vec{e}_j \tag{1a}$$

$$\vec{h}_{cn} = \sum_{j=1}^{N} \alpha_{j+N,n} \vec{h}_j. \tag{1b}$$

Here $\vec{e}_{cn}$ and $\vec{h}_{cn}$ are the modal fields of the nth mode for the corrugated waveguide, $\vec{e}_j$ and $\vec{h}_j$ are the modal fields of the jth mode for a smooth-walled waveguide of radius a (inner radius of the corrugated waveguide), $\alpha_{j,n}$ is the jth component of the nth eigenvector, and $2N$ is the dimension of the transmission matrix.

The transmission matrix of the unit cell can be determined, through algebraic operations, from the scattering matrix, and the scattering matrix can be calculated according to [1], [2], [6], or [7].

III. Scattering Matrix of the Smooth–Corrugated Waveguide Junction

Consider the junction between a circular smooth-walled waveguide with radius a_1 and a corrugated waveguide with inner radius a. Without loss of generality, it will be assumed that $a_1 = a$ (if not, the junction can be decomposed into a discontinuity between two smooth-walled waveguides, with radii a_1 and a, cascaded with a smooth–corrugated waveguide junction).

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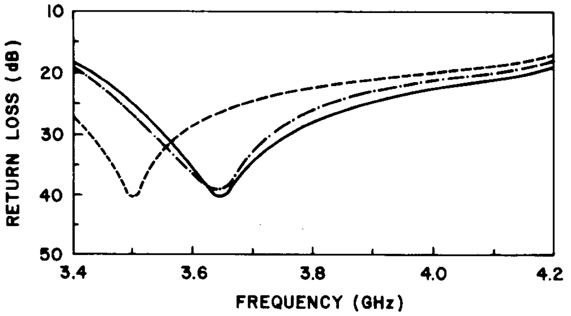

Fig. 2. Return loss as a function of frequency for the discontinuity between a smooth-walled waveguide with an inner radius of 27 mm and a corrugated waveguide with an inner radius of 28 mm, a corrugation depth of 12.6 mm, a corrugation length of 10 mm, and a distance between corrugations of 5 mm. (———) Theoretical results according to the present method; (—·—·–) experimental results; (––––) theoretical results according to [1].

The scattering matrix of the junction is given in [1]. Of particular interest here are the elements p_{ij}, q_{ii}, and r_{ii} of the matrices $\bar{\bar{P}}$, $\bar{\bar{Q}}$, and $\bar{\bar{R}}$ in [1]. Making use of (1) for the fields in the corrugated waveguide and taking into consideration the orthogonality between modal fields for the smooth-walled waveguide $(\vec{e}_i, \vec{h}_i)$, the expressions for p_{ij} and q_{ii} take the form

$$p_{ij} = \alpha_{j+N,i} r_{jj} \tag{2a}$$

$$q_{ii} = \sum_{j=1}^{N} \alpha_{ji} \alpha_{j+N,i} r_{jj} \tag{2b}$$

where it is assumed that the first N eigenvalues of the transmission matrix correspond to modes propagating in the positive z direction and the expression for r_{jj} is given by [1, eq. (13)].

Only a slight effort is necessary to implement this technique into previously developed computer programs for the analysis of mode converters, since the algorithms for the calculation of scattering matrices are already available in such programs.

IV. Experimental and Numerical Results

An example of a typical situation where the present method of analysis improves the accuracy of the results is now considered. The scattering matrix of the discontinuity between a smooth-walled waveguide with inner radius of 27 mm and a corrugated waveguide with inner radius of 28 mm, corrugation depth of 12.6 mm, corrugation length of 10 mm, and distance between corrugations of 5 mm (the corrugation depth corresponding to 0.1248λ at 3.4 GHz) was computed. The calculated return loss, as a function of frequency, is shown in Fig. 2. In the same figure are also shown measured results and theoretical results obtained according to [1] (approximate impedance model for the terminal corrugated waveguide). It is observed that the method proposed here produces good agreement with experiment, with discrepancies less than 1.5 dB over the frequency band. The method proposed in [1] introduces a significant error at the lower frequencies, amounting to 8 dB at the lowest frequency.

As a second example, the mode converter shown in Fig. 3, with the dimensions given in Table I, was considered. The calculated return loss, as a function of frequency, applying the present method of analysis is shown in Fig. 4. The scattering matrices of the sections of the converter preceding the terminal discontinuity smooth–corrugated waveguide were determined

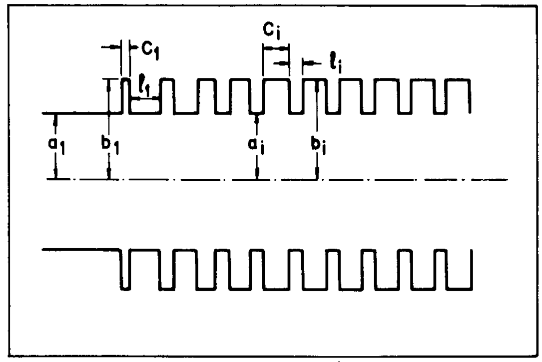

Fig. 3. Longitudinal section of the TE_{11}-to-EH_{11} mode converter.

TABLE I
DIMENSIONS (IN MM) OF THE MODE CONVERTER SHOWN IN FIG. 3

i	a_i	b_i	c_i	ℓ_i	OBS.
1	27.00	41.75	3.21	12.50	↑
2	27.50	42.00	5.71	10.00	
3	27.50	42.13	7.00	6.43	
4	27.75	42.25	8.50	5.43	
5	27.75	42.50	10.00	5.00	CONVERTER
6	28.00	42.50	10.00	5.00	
7	28.00	42.00	10.00	5.00	
8	28.00	41.60	10.00	5.00	
9	28.00	41.10	10.00	5.00	↓
10	28.00	40.60	10.0	5.00	corrugated waveguide

according to [6]. Eighteen modes in the inner waveguide and four radial modes in the corrugations were sufficient to ensure convergence of the results.

Also shown in Fig. 4 are the measured results and theoretical results obtained according to [1]. Once again, the present method of calculation produces results in good agreement with experiment (discrepancies less than 1.5 dB for return losses below 30 dB). The error introduced by the approximate impedance model reaches a value of 8.0 dB at the lower end of the frequency band.

V. Conclusions

A rigorous method for the determination of the scattering matrix of the discontinuity between smooth-walled and corrugated cylindrical waveguides was developed. The method can be easily implemented in existing computer programs for the improved analysis of the performance of cylindrical waveguide mode converters. This method is particularly useful in mode

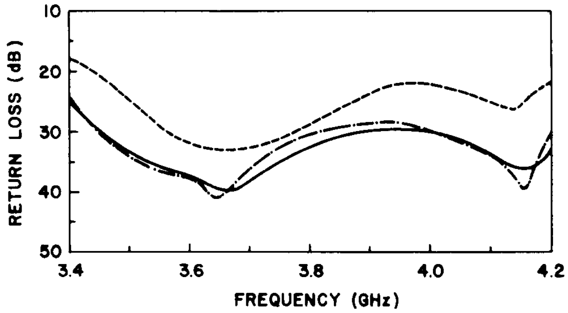

Fig. 4. Return loss, as a function of frequency, for the mode converter shown in Fig. 3. (———) Theoretical results according to the present method; (—·—·—) experimental results; (----) theoretical results according to [1].

converters terminated into corrugated waveguides with shallow corrugations.

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Resonant Frequency of Cylindrical Dielectric Resonator Placed in an MIC Environment

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***Abstract*—In this paper, an effective dielectric constant technique has been used to determine the resonant frequency of the $TE_{01\delta}$ mode of a cylindrical dielectric resonator placed in an MIC environment. A suitable expression for ϵ_{eff} has been reported which makes it possible to obtain results that compare favorably with rigorous methods. A large number of experimental results are also reported to demonstrate the validity of the method. Finally, for a given resonant frequency, closed-form expressions are given for computing the height of the resonator.**

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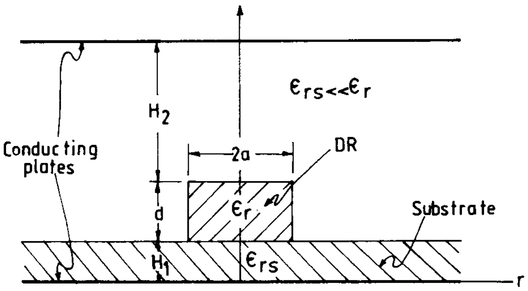

Fig. 1. Cylindrical dielectric resonator placed in an MIC environment.

I. Introduction

A typical configuration in which a cylindrical dielectric resonator is used in MIC's is shown in Fig. 1. A few rigorous methods have been reported during the last few years for determining the resonant frequency of a dielectric resonator placed in configurations of the type shown in Fig. 1 (e.g. [1] and [2]). A number of rigorous methods have also been reported for structures which are special cases of the structure shown in Fig. 1 [3]–[9]. However, all the rigorous methods are computationally quite complex, which makes their use in practical design applications almost prohibitive. On the other hand, approximate methods such as the dielectric waveguide model (DWM) method [10] are simple to use but do not offer adequate accuracy.

Today's CAD trends indicate the need for a method which offers both simplicity and accuracy. An approximate but accurate and simple effective dielectric constant technique has previously been proposed for the analysis of isolated cylindrical dielectric resonators [11], [12]. The technique is basically an improvement of the DWM method. In principle, the improvement is similar to that offered by the EDC technique [13] over Marcatili's method [14] for analyzing rectangular dielectric waveguide structures. In this paper, we use the effective dielectric constant technique to find the resonant frequency of the structure shown in Fig. 1. The technique leads to results nearly matching in accuracy those of rigorous methods. The resonant frequency of the resonator has been obtained by using a suitable approximation for ϵ_{eff}. The expression reported for ϵ_{eff} is different from the one used earlier for the case of an isolated resonator [11], [12].

II. Theory

We limit our attention to the lowest order $TE_{01\delta}$ mode of resonance, which is the most commonly employed mode of resonance in practical applications. For this mode, only three field components, i.e., E_ϕ, H_z and H_r, exist. The H_z component, from which the other field components can be derived, is assumed to be of the following form inside the resonator at resonance:

$$H_z = J_0(hr)\left[A_m \cos\{\beta(z-H_1)\} + B_m \sin\{\beta(z-H_1)\}\right]. \quad (1)$$

In the above equation J_0 denotes the Bessel function of first kind and order zero. The problem of finding the resonant frequency is one of finding wavenumbers h and β which also satisfy the separation equation

$$h^2 + \beta^2 = \epsilon_r k_0^2 \quad (2)$$

where k_0 is the free-space wavenumber corresponding to the