# Reference Transmission Network: A Game Theory Approach

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***Abstract*—The transmission network plays a key role in an oligopolistic electricity market. In fact, the capacity of a transmission network determines the degree to which the generators in different locations compete with others and could also greatly influence the strategic behaviors of market participants. In such an oligopolistic framework, different agents may have distinct and sometimes opposite interests in urging or hindering certain transmission expansions. Therefore, the regulatory authority, starting from the existing grid, faces the challenge of defining an optimal network upgrade to be used as benchmark for approval or rejection of a given transmission expansion.**

**The aim of this paper is to define the concept of reference transmission network (RTN) from an economic point of view and to provide a tool for the RTN assessment in a deregulated framework where strategic behaviors are likely to appear. A general game-theoretic model for the RTN evaluation is presented, and the solution procedure is discussed. The strategic behavior of market agents in the spot market is modeled according to a Supply Function Equilibrium approach. The impact of transmission capacity expansion on market participants' strategic behavior is studied on a three-bus test network. The RTN is computed and compared with the optimal expansion found when perfect competition among power producers is assumed.**

***Index Terms*—Game theory, optimal network, strategic bidding, transmission planning.**

## NOMENCLATURE

The main mathematical symbols used throughout this paper are listed (in alphabetical order) in the following. Note that if a single generic element is defined using two indexes (for example, $x_{i,j}$), the notation $x_i$ is used to refer to the vector whose elements are $x_{i,j}$, for $j = 1, 2, \ldots$, and the notation $x$ is used to refer to the vector composed by the vectors $x_i$, for $i = 1, 2, \ldots$. Analogously, when a generic element is defined using a single index $x_i$, the notation $x$ refers to the vector whose elements are $x_i$, for $i = 1, 2, \ldots$.

| | |
|---|---|
| $a_{n,g}$ | Intercept of the $G_{n,g}$ supply function. |
| $b_{n,g}$ | Slope of the $G_{n,g}$ supply (marginal cost) function. |
| $B$ | Susceptance matrix. |
| $\beta_{n,g}$ | Strategic bid coefficient of $G_{n,g}$. |
| $\beta_{-(n,g)}$ | Strategic bid coefficients of all the power producers except $G_{n,g}$. |
| $d_{n,\ell}$ | Intercept of the $L_{n,\ell}$ demand function. |
| $\delta$ | Discount factor. |
| $\eta_{n,\ell}$ | Marginal value of power withdrawn by $L_{n,\ell}$. |
| $F$ | Total number of $TE^f$'s in $\mathcal{TE}$ (i.e., $F = \lvert\mathcal{TE}\rvert$). |
| $F_{m,n}^{max}$ | Transmission capacity of the line between buses $m$ and $n$. |
| $g$ | Generator index. |
| $\mathcal{G}_n$ | Set of generators $g$ at bus $n$. |
| $G_{n,g}$ | Generator $g$ at bus $n$. |
| $IC(TE^f)$ | Investment cost of $TE^f$. |
| $\ell$ | Load index. |
| $\lambda_n$ | Locational marginal price at bus $n$. |
| $\mathcal{L}_n$ | Set of loads $\ell$ at bus $n$. |
| $L_{n,\ell}$ | Load $\ell$ at bus $n$. |
| $m$, $n$ | Bus indexes. |
| $\mu_{m,n}$, $\nu_{m,n}$ | Marginal values of transmission capacity of the line between buses $m$ and $n$. |
| $\mathcal{N}$ | Set of power system buses. |
| $\Pi_{G_{n,g}}$ | Payoff of $G_{n,g}$. |
| $\Pi_{\text{planner}}$ | Planner payoff. |
| $q_{G_{n,g}}$ | Power produced by $G_{n,g}$. |
| $q_{G_{n,g}}^{max}$ | Capacity limit for $G_{n,g}$ power plant. |
| $Q_G$ | Vector whose generic element is $Q_{G_n}$. |
| $Q_{G_n}$ | Corresponds to the total power injected at bus $n$, and it coincides with the sum of elements of vector $q_{G_n}$ (if the vector $q_{G_n}$ exists, otherwise to 0). |
| $q_{L_{n,\ell}}$ | Power withdrawn by $L_{n,\ell}$. |
| $q_{L_{n,\ell}}^{max}$ | Maximum power withdrawal by $L_{n,\ell}$. |
| $Q_L$ | Vector whose generic element is $Q_{L_n}$. |
| $Q_{L_n}$ | Corresponds to the total power withdrawal at bus $n$ and coincides with the sum of elements of vector $q_{L_n}$ (if the vector $q_{L_n}$ exists, otherwise to 0). |
| $s_{n,\ell}$ | Slope of the $L_{n,\ell}$ demand function. |
| $t$ | Period index. |
| $\vartheta_n$ | Voltage angle at bus $n$. |
| $\tau$ | Spot market session index. |
| $T$ | Total number of time periods. |
| $\mathcal{TE}$ | Set of Transmission Expansion candidate plans. |

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|---|---|
| $TE^f$ | $f$th transmission expansion *candidate* plan. |
| $x^t$ | Refers to the generic $x$ during period $t$. |
| $x^{t,\tau}$ | Refers to the generic $x$ at session $\tau$ during period $t$. |
| $\|\mathcal{X}\|$ | Dimension of the generic set $\mathcal{X}$. |

**Acronyms**

| | |
|---|---|
| CS | Consumer surplus. |
| ISO | Independent system operator. |
| MK | Market session. |
| MSG | Multistage game. |
| NE | Nash equilibrium. |
| PF | Performance function. |
| PS | Producer surplus. |
| RTN | Reference transmission network. |
| SFE | Supply function equilibrium. |
| SG | Stage game. |
| SPNE | Subgame perfect Nash equilibrium. |
| SW | Social welfare. |

## I. Introduction

THE structure of the electricity industry has dramatically changed in the last two decades. As a matter of fact, the power sector has been privatized and restructured in many countries around the world. The main reason for such a change lies in the expectation that competition could lead to a reduction in electricity prices and could stimulate the emergence of new technologies. However, in quite a few cases, strategic behaviors have emerged, and prices considerably higher than marginal costs have been observed [1].

In this new deregulated environment, the transmission network plays a key role when the analysis of power market and of market power issues are addressed. In fact, the capacity of transmission lines determines the degree to which generators in different locations could compete [2].

The existing transmission network has been designed, planned, and built in a vertically integrated environment, following the traditional planning criteria. Since the system was operated by a unique entity, the transmission planning was required to achieve a certain level of efficiency in a framework where only centralized and coordinated decisions were made. In [3], an extensive classification of transmission expansion planning models and of different synthesis algorithms for transmission planning is presented. The synthesis planning models can be classified as mathematical optimization or heuristic methods. The former type includes linear programming, dynamic programming, nonlinear programming, mixed integer programming, and optimization techniques such as Bender decomposition and hierarchical decomposition [4]–[6]. The latter one includes simulated annealing, tabu search, expert systems, fuzzy set theory, and greedy randomized adaptive search procedures [7]–[10].

In the context of competitive electricity markets, several new transmission planning issues have emerged [11], [12]. They originate from new uncertainties related, for example, to the impact of market transactions on physical flows, to decisions of new generation investments and old power plant stripping made independently by market participants, and also from the increasing requirements of flexibility and robustness of the network needed in the competitive environment.

There is an ongoing debate concerning the best approach for the management of transmission investments (monopolistic or market-driven approach) and the most suitable way to recover them, e.g., congestion and variable charges, fixed costs' allocation [13]–[17].

Furthermore, various ways to attract investments are described in the literature [18]. In particular, the merchant approach, relying on auctions of long-term Financial Transmission Rights (FTRs), and the related critiques can be found in [19] and [20]. An alternative approach, built upon a regulatory mechanism for Transco and based on a price cap formula, is developed in [21], whereas in [22], an approach that defines optimal transmission capacity, according to the strategic behavior of generators in a discriminatory auction market structure, is proposed. Interesting models of decentralized coalition formation for transmission expansion purposes, based on concepts from cooperative game theory and distributed artificial intelligence, are studied in [23] and [24].

Nevertheless, without restricting ourselves to a particular transmission investor or a specific method for recovering the transmission investment costs, we believe that an independent authority should at least monitor and approve the proposed expansion plans.

Indeed, the independent authority, starting from the existing grid, faces the challenge of defining a benchmark network to which any expansion proposal should be compared. The benchmark network should foster competition and reduce, when possible, the exercise of market power by market agents.

To address this challenge, the authority needs to be able to quantify the benefits accruing from network capacity expansion. This can be realized through the modeling of strategic spatial interactions among agents in the market [25].

The analysis of agents' strategic interactions in electricity networks and the market results forecasts, both for single and for multiperiod electricity market sessions, have been extensively studied in the last years. For example, models based on game theory concepts such as Bertrand, Cournot, Supply Function Equilibrium, and Conjectural Variation as well as models based on price-quota concept have emerged ([26]–[36] are among the most recent examples, whereas [26] and [28], besides proposing new approaches, provide a review of the previous most significant oligopolistic electricity market models). New market structures where energy and transmission service are traded have also been proposed [37], [38].

The focus of this paper is on the interrelationship between investment in transmission capacity and strategic behavior of market participants.

First, the concept of Reference Transmission Network is introduced, from an economic point of view. Then, a tool for the estimation of the RTN is developed using a game theory approach. The RTN assessment is based on the analysis of a dynamic noncooperative game.

The first level of the game corresponds to the choice of optimal network topology, whereas the remaining part of the game represents an oligopolistic framework in which power producers interact for a certain time horizon. In particular, it includes the modeling of the bilevel problem faced by each power producer that needs to make a strategic bid aimed to maximize its own payoff. In fact, when making the strategic choice, the producer should take into account that: 1) all the other generators are simultaneously making analogous choices and that 2) an ISO will clear the market on the basis of the bids he received.

The strategic behavior of power producers is modeled using an SFE approach. In particular, the single-period approach proposed in [39] is extended in order to address a multisession framework in which there exists an obligation for firms to bid consistently over a certain time horizon.

The RTN estimation is derived from the subgame perfect Nash equilibrium (SPNE) of the game.

The organization of this paper is as follows. Section II elaborates on the principles upon which a RTN should be defined. The proposed RTN simulation tool is illustrated in Section III. More precisely, Section III-A describes the spot market clearing mechanism, Section III-B elaborates on the model used for power producers' strategic behavior simulation, together with the algorithm used for the NE calculation, and Section III-C shows the procedure used for evaluating the SPNE of the game and determining the RTN. Test results are presented and discussed in Section IV, and finally, Section V concludes.

## II. Reference Transmission Network

In a deregulated market, it is of fundamental importance, both for regulation purposes and for a utility's decision-making purposes, to be able to rely on quantitative measures of system performances, as well as to know how those system performances are affected by changes of decision-makers' operation strategies, and by system reinforcements and expansions.

The necessity of defining performance benchmarks for the existing transmission network is addressed in [40]. Furthermore, in [41], the concept of performance indexes for a distribution network is linked with the concept of reference distribution network.

These two studies focus on system performances concerned with reliability issues. Moreover, both methodologies are developed taking the utility point of view. In particular, in [40], reliability performance indexes are aimed to provide the transmission utility with measures of the quality of service to be given to the customers, as well as to provide measures to be used by the utility when evaluating different planning and design procedures; whereas, in [41], the distribution network operator could benefit from the definition of the reference distribution network to justify future plans and charges, since this reference distribution network specifies the best tradeoff between costs and benefits delivered in terms of performance indexes.

In this paper, the definition of reference distribution network of [41] is modified and extended to the transmission network. In particular, the proposed definition relies on economic performances evaluated by taking the point of view of a centralized "planner," where the word "planner" is taken in its broadest sense. We assume that the "planner" is the entity designated to judge and, when appropriate, to approve transmission expansion plans proposed by investors.

The decision of the "planner" of approving or rejecting a specific transmission expansion proposal should be founded on a fair comparison between the same proposal and a benchmark defined in a transparent and objective manner. The aim of the RTN is to represent such a benchmark.

The RTN, from an economic point of view, is a network obtained by an optimal upgrade of the existing transmission network. In particular, the RTN is characterized by an optimal network topology aimed to achieve the best tradeoff between transmission expansion costs and benefits deriving from the expansion, where the benefits are calculated over the whole time horizon in which the expansion plan is supposed to be in place.

An appropriate Performance Function, used by the "planner" for measuring the tradeoff between expansion costs and of actual economic advantages of market participants, is introduced. While the value of transmission expansion costs is supposed to be estimated by the "planner," the estimation of *actual* economic benefits of market participants is supposed to be calculated using a simulation tool that mimics the strategic behaviors of market participants. The transmission expansion that maximizes the PF value corresponds to the RTN.

The PF to be chosen for the RTN definition should certainly be fair and objective. However, depending on the policy objectives of the "planner," or depending on the particular market environment, different PFs can be plausible.

In this paper, we assume that the "planner" considers as economic PF the long-run social welfare function shown in the following:

$$PF(TE^f) = \sum_{t=1}^{T} [\delta^\wedge(t-1)]\, SW^t - IC(TE^f). \quad (1)$$

The performance function corresponds to the difference between the overall value of social welfare over the whole temporal horizon in which the transmission expansion is supposed to be in place and the estimated transmission expansion costs.[1,2]

A long-run social welfare function, similar to the one we use as PF, has been traditionally adopted in the vertically integrated environment, when the Peak Load Pricing problem has been studied with reference to generation planning and price tariffs [42], [43]. More recently, it has been used in the context of perfect competitive market, for the definition of an appropriate transmission access charge to allow transmission investment recovery [44].

It should be noted that the RTN solution strongly depends on the set $\mathcal{TE}$ of candidate network upgrades that will be considered for PF assessment. Given the combinatorial nature of the transmission expansion problem, it is very hard to analyze all the possible transmission expansion alternatives, especially for a real-size network. Therefore, the "planner," beyond to analyze the transmission expansion proposals—if any—proposed by independent investors, should consider a set of solutions obtained, for example, from classical transmission expansion programs [4]–[10]. Additionally, transmission upgrades suggested by the system operator or by the authority in charge of monitoring market power abuse should be considered.

[1]Note that $\delta = e^{-r\omega}$, where $r$ is the instantaneous interest rate, and $\omega$ is the "real" time between two periods. Furthermore, $[\delta^\wedge(t-1)]$ refers to $\delta$ to the power of $(t-1)$.

[2]Section III illustrates the simulation tool proposed for estimating the $SW^t$ for the specific market structure analyzed in this paper.

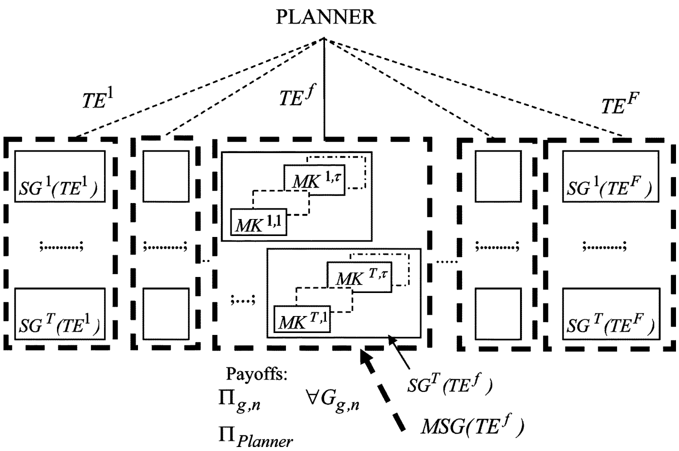


Fig. 1. Sketch of the whole game.

Since the RTN definition is quite general, different market structures and different "planner" concerns can be conveniently addressed by an appropriate simulation tool, including market rules and market participants' strategic behaviors modeling and by a suitable choice of PF. For example, if we suppose that market agents, beyond to participate to energy spot market, trade also (short- or long-term) FTRs, the modeling of those additional trades needs to be considered when social welfare estimation is addressed; whereas, if we assume that the "planner" is concerned with equity issues, regarding the share of welfare between producers and consumers, then the PF used to determine the RTN could be the long-run consumer surplus.

## III. RTN Simulation Tool

The RTN is evaluated as the SPNE of a dynamic noncooperative game [45], [46]. An SPNE is a refinement of NE, i.e., it is an NE equilibrium that is more credible than others, since the correspondent players' strategies constitute an NE in *every* subgame of the original game [47]. In context of electricity market equilibrium analysis, the notion of SPNE has already been used by [48].

In Fig. 1, a sketch of the game is drawn.

The players of the game are the $planner$ and all the power producers $G_{n,g}\ \forall n \in \mathcal{N}$ and $\forall g \in \mathcal{G}_n$.

The game is organized in the following way.

1) The $planner$ chooses a possible investment expansion plan $TE^f$ from the set of feasible transmission expansion plans $\mathcal{TE} = \{TE^1, TE^2, \ldots, TE^f, \ldots, TE^F\}$.
2) The chosen investment $TE^f$ is observed, and a $MSG(TE^f)$—composed by $T$ stage games $SG^t(TE^f)$, $\forall t = 1, 2, \ldots, T$—is played by the power producers only. Each stage game is composed by several spot market sessions.
3) The players' payoffs, both for the power producers, $\Pi_{G_{n,g}} \forall G_{n,g}$ and for the $planner$, $\Pi_{\text{planner}}$ occur.

The stage games that follow a given $TE^f$ are in general all different, even if it is assumed that the market participants are the same in every stage game (we do not assume any entry or exit from the market).

When analyzing the $MSG(TE^f)$, we do not consider a physical link between the stage games composing the multistage game, and so, the result of a $SG^t(TE^f)$ does not influence the $SG^j(TE^f) \forall j = (t+1), \ldots, T$.

The proposed simulation tool allows computing an SPNE of the game by simulating strategic behaviors of power producers during every $SG$ and so evaluating an NE sample for each $MSG$ that follows a particular transmission investment expansion.

In the following, assumptions and mathematical models for spot market sessions, for stage games and for multistage games, are described.

### A. Spot Market Session $MK^t$

*1) Spot Market Assumptions:* The model of a single spot market session considered in this paper is based on a pure pool model. The spot market is cleared by an ISO. The nodal price scheme, also called Locational Marginal Price scheme, is adopted and no *ex ante* transmission charges, or active trade of transmission rights, are considered. The spot market is organized as an auction in which both power producers and consumers are allowed to participate. Moreover, suppliers and customers are supposed to bid, respectively, affine nondecreasing curves and affine nonincreasing curves.

We assume that the generator marginal cost function and the load demand function are as in the following:

$$p\left(q^{t,\tau}_{G_{n,g}}\right) = a_{n,g} + b_{n,g} q^{t,\tau}_{G_{n,g}} \quad (2)$$

$$p\left(q^{t,\tau}_{L_{n,\ell}}\right) = d^{t,\tau}_{n,\ell} - s^{t,\tau}_{n,\ell} q^{t,\tau}_{L_{n,\ell}} \quad (3)$$

where $p$ is the price, and $a_{n,g}$, $b_{n,g}$, $d^{t,\tau}_{n,\ell}$, and $s^{t,\tau}_{n,\ell}$ are all positive coefficients.

*2) Market Clearing Mechanism:* The market clearing mechanism is based on the maximization of the hourly *declared* social welfare subject to constraints. It can be formulated as a convex quadratic programming problem, as in the following:[3]

$$\max_{q_L^{t,\tau}, q_G^{t,\tau}, \vartheta^{t,\tau}} \left( d^{t,\tau\prime} q_L^{t,\tau} - \frac{1}{2} q_L^{t,\tau\prime} \text{diag}(s^{t,\tau}) q_L^{t,\tau} \right) - \left( a' q_G^{t,\tau} + \frac{1}{2} q_G^{t,\tau\prime} \text{diag}(\beta^t) q_G^{t,\tau} \right) \quad (4)$$

subject to

$$B\vartheta^{t,\tau} = Q_G^{t,\tau} - Q_L^{t,\tau} \quad (5)$$

$$-F^{max} \leq A\vartheta^{t,\tau} \leq F^{max} \quad (6)$$

$$0 \leq q_G^{t,\tau} \leq q_G^{max} \quad (7)$$

$$0 \leq q_L^{t,\tau} \leq q_L^{max,t,\tau} \quad (8)$$

where (4) represents the social welfare function and depends on the bid submitted by market participants. Equation (5) enforces the power balance at each node (dc load flow formulation), since $Q_G^{t,\tau}$ and $Q_L^{t,\tau}$ are the vectors of nodal injections and nodal withdrawals, respectively. Equation (6) imposes restrictions over the line flows to comply with line limits. The line flows $A\vartheta^{t,\tau}$ are calculated on the basis of line reactances. Finally, (7) and (8) specify the market participant's maximum production and maximum demand, respectively.

Note that the matrices $A$ and $B$ are, in fact, function of the network topology $TE^f$, i.e., $B = B(TE^f)$ and

[3]The notation "diag($w$)" is used to indicate a diagonal matrix whose entries are the components of the vector $w$, and the apex "$\prime$" means transpose.

$A = A(TE^f)$. Also note that all the variables (primal and dual variables) are functions of the $TE^f$ under analysis. The maximum transmission capacities of the lines $F^{max}$ depend on transmission network topology. In particular, $F^{max}(TE^f) = F^{Ex} + F^{\text{New}}(TE^f)$, where $F^{Ex}$ is the existing transmission capacity, and $F^{\text{New}}$ is the new transmission capacity added to the system because of the transmission investment plan $TE^f$ considered to be in place.

By collecting the Karush–Kuhn–Tucker (KKT) optimality conditions for the primal problem (4)–(8), and using the appropriate dual variables, we can write the following linear complementary formulation of the market clearing mechanism [49]:

$$0 \le \left(q_G^{max} - q_G^{t,\tau}\right) \quad \perp \zeta^{t,\tau} \ge 0 \tag{9}$$

$$0 \le q_G^{t,\tau} \quad \perp a + \text{diag}(\beta^t) q_G^{t,\tau} - \lambda^{t,\tau} + \zeta^{t,\tau} \ge 0 \tag{10}$$

$$0 \le \left(q_L^{max,t,\tau} - q_L^{t,\tau}\right) \quad \perp \eta^{t,\tau} \ge 0 \tag{11}$$

$$0 \le q_L^{t,\tau} \quad \perp -d^{t,\tau} + \text{diag}(s^{t,\tau}) q_L^{t,\tau} + \lambda^{t,\tau} + \eta^{t,\tau} \ge 0 \tag{12}$$

$$0 \le \mu^{t,\tau} \quad \perp F^{max} - A\vartheta^{t,\tau} \ge 0 \tag{13}$$

$$0 \le \nu^{t,\tau} \quad \perp F^{max} + A\vartheta^{t,\tau} \ge 0 \tag{14}$$

$$B'\lambda^{t,\tau} + A'\mu^{t,\tau} - A'\nu^{t,\tau} = 0 \tag{15}$$

$$B\vartheta^{t,\tau} = Q_G^{t,\tau} - Q_L^{t,\tau}. \tag{16}$$

Note that the notation $u \perp v$ means that the two vectors are perpendicular. Moreover, the two formulations (4)–(8) and (9)–(16) are equivalent.

*3) Spot Market Session Results:* When the spot market session under analysis is solved, the market prices $\lambda_n^{t,\tau} \forall n$ and dispatched quantities $q_{G_{n,g}}^{t,\tau} \forall G_{n,g}$ and withdrawn quantities $q_{L_{n,\ell}}^{t,\tau} \forall L_{n,\ell}$ are obtained.

Therefore, for a single session $MK^{t,\tau}(TE^f)$, of the $SG^t(TE^f)$ stage game, the generator's payoff is

$$\Pi_{G_{n,g}}^{t,\tau}\left(TE^f, \beta^t(TE^f)\right) = \left(\lambda_n^{t,\tau} q_{G_{n,g}}^{t,\tau} - \left(a_{n,g} q_{G_{n,g}}^{t,\tau} + \frac{1}{2} b_{n,g}\left(q_{G_{n,g}}^{t,\tau}\right)^2\right)\right) \tag{17}$$

the benefit of each load is

$$CS_{L_{n,\ell}}^{t,\tau}\left(TE^f, \beta^t(TE^f)\right) = \left(d_{n,\ell}^{t,\tau} q_{L_{n,\ell}}^{t,\tau} - \frac{1}{2} s_{n,\ell}^{t,\tau}\left(q_{L_{n,\ell}}^{t,\tau}\right)^2 - \lambda_n^{t,\tau} q_{L_{n,\ell}}^{t,\tau}\right) \tag{18}$$

and the "true" social welfare, computed taking into account the demand bid functions, and the producer cost functions, is

$$SW^{t,\tau}\left(TE^f, \beta^t(TE^f)\right) = \sum_{L_{n,\ell}} \left(d_{n,\ell}^{t,\tau} q_{L_{n,\ell}}^{t,\tau} - \frac{1}{2} s_{n,\ell}^{t,\tau}\left(q_{L_{n,\ell}}^{t,\tau}\right)^2\right) - \sum_{G_{n,g}} \left(a_{n,g} q_{G_{n,g}}^{t,\tau} + \frac{1}{2} b_{n,g}\left(q_{G_{n,g}}^{t,\tau}\right)^2\right). \tag{19}$$

The total producers' payoff is equal to $PS^{t,\tau}(TE^f, \beta^t(TE^f)) = \sum_{G_{n,g}} \Pi_{G_{n,g}}^{t,\tau}(TE^f, \beta^t(TE^f))$, and the total consumers' payoff is $CS^{t,\tau}(TE^f, \beta^t(TE^f)) = \sum_{L_{n,\ell}} CS_{L_{n,\ell}}^{t,\tau}(TE^f, \beta^t(TE^f))$.

### B. Stage Game: Strategic Behavior and Market Equilibrium

The aim of this section is to illustrate the stage game model and its equilibrium. In particular, first, a detailed discussion concerning the reasons of assuming the producers' strategic variable to be the slope of the supply function and then concerning the reasons of choosing to group a certain number of spot market sessions in a unique stage game are provided. Finally, the procedure adopted for evaluating a sample of NE of $SG^t(TE^f)$ is presented.

*1) $SG^t$ Assumptions and Producer's Strategic Variable:* A number of short-term models—proper of industrial economics literature and based on game-theory concepts—have been adopted for simulating strategic behaviors of market participants and electricity market equilibria (see [28] for a review). All those game theory models rely on the concept of Nash equilibrium, i.e., they aim to determine an equilibrium condition in which none of the power producers has the incentive to unilaterally change its strategy.

The difference among those models depends on the strategic variable the agents are supposed to manipulate and on the beliefs regarding rivals' reactions.

In this paper, an SFE model, introduced by [50], is adopted to represent the strategic behavior of power producers. The SFE model is considered one of the most appropriate to be used when dealing with electricity market auctions, since it allows representing more realistically the bidding procedure through the strategic variation of supply bid [32].

However, when an SFE model is chosen, there exist different ways the bid function could be manipulated, and depending on the supply function parametrization adopted, different results can arise. The most frequently used models for supply bid parametrization are [32] as follows:

- manipulation of the *sole* intercept $a_{n,g}$ of the bid supply curve (with the prices axis) [39], [51];
- manipulation of the *sole* slope $b_{n,g}$ of the supply bid curve [51];
- manipulation of intercept and slope *both arbitrarily chosen* [52];
- manipulation of intercept and slope when a *linear relationship is supposed to exist between the two parameters* [30], [53].

Baldick demonstrates that, for a single-pricing period, the parametrization of the supply function is so critical for the equilibrium results that some of the results shown in the literature are "artifacts of assumptions about the choice of particular bid parameters," and that the manipulation of both intercept and slope both chosen arbitrarily should be adopted when the NE is studied [52]. Furthermore, he suggests that, in a multiple pricing-period model, revealing the intercept honestly could be optimal. In [32], indeed, Baldick *et al.* demonstrate that, in a market setting in which supply bid functions are supposed to be held constant over a certain time horizon, and for a certain load duration characteristic, having the strategic intercept equal to the true intercept is incentive compatible. The economic interpretation of this type of behavior is that at low output levels, the bidders have less motive to exaggerate their costs.

The model adopted in this paper, for representing the strategic interactions among the power producers in the stage game, extends the multifirm bid model proposed by Hobbs *et al.* [39] to a slightly different framework in which, for market power mitigation purposes, the suppliers are requested to bid consistently over an extended time horizon, as suggested by [52].

The multifirm model from [39] "coordinates the results of individual firm's models in an attempt to find an equilibrium for all generators' bids." In particular, the individual firm problem is formulated as a bilevel model and is solved by adopting a mathematical program with equilibrium constraints (MPEC) approach or, more specifically, a mathematical program with linear complementarity constraints.

Differently from [39] where the producer's strategic variable is the supply function intercept, in this paper, the suppliers are supposed to manipulate their bids by varying only the slope of their true marginal cost curve. Basically, instead of offering their true slope $(b_{n,g})$, they offer a strategic slope $(\beta_{n,g}^{t,\tau}(TE^f))$. Note that, when the producer's strategic variable is the slope of the supply function, the equilibrium conditions become nonlinear, whereas this is not the case when the producer's strategic variable is the intercept, as assumed in Hobbs *et al.*'s model.

Furthermore, since in the considered regulatory framework the producers are *obliged* to offer the *same bid for all the sessions* $\tau$ that compose a period $t$, the power producer strategic variable is the same for all the market session of a given stage game, i.e., $\beta_{n,g}^{t,\tau}(TE^f) = \beta_{n,g}^{t}(TE^f)\forall\tau$.[4] It is worth remarking that, from the power producers' point of view, the $MK^{t,\tau}\forall\tau$ are coupled by the same strategic variables $\beta_{n,g}^{t}(TE^f)\forall\tau$. Therefore, when each power producer is seeking its best response for the $SG^t(TE^f)$, he needs to take into account that his choice affects all its own spot market payoffs $\Pi_{G_{n,g}}^{t,\tau}\forall\tau$. Hence, when the NE profile $\beta^{t*}(TE^f) = \{\beta_{1,1}^{t\ *}(TE^f),\ldots,\beta_{n,g}^{t\ *}(TE^f),\ldots\}$ for the stage game $SG^t(TE^f)$ is computed, the spot market equilibrium results for $MK^{t,\tau}\forall\tau$ can be determined.

Finally, let us also remark that the study here concerns pure supply strategies only. In general, Nash SFE for pure strategies of the players does not necessarily exist, nor is it necessarily unique [1], [50]. As in [39] and [51], we study a case where we are able to find an equilibrium.

*2) Power Producer Problem:* For every period $t$, each producer faces the need of maximizing its profit, taking into account what the ISO does (i.e., the market clearing rules) as well as what its rivals do (i.e., how its rivals bid). The problem of the single $G_{n,g}$ constitute a bilevel model and may be modeled as a Nonlinear Problem with Equilibrium Constraints (NLPEC), as in the following:

$$\max_{\beta_{n,g}^{f,t}} \sum_{\tau} \left( \lambda_n^{t,\tau} q_{G_{n,g}}^{t,\tau} - \left( a_{n,g} q_{G_{n,g}}^{t,\tau} + \frac{1}{2} b_{n,g} \left( q_{G_{n,g}}^{t,\tau} \right)^2 \right) \right) \tag{20}$$

subject to

$$b_{n,g} \le \beta_{n,g}^{f,t} \tag{21}$$

$$\text{and constraints } [(9)-(16)], \quad \forall\tau \tag{22}$$

[4]Note that sometimes, for sake of compactness, we use the notation $\beta_{n,g}^{f,t}$ for $\beta_{n,g}^{t}(TE^f)$.

where the function to be maximized in (20) corresponds to the total profit $\Pi_{G_{n,g}}^{t}$ of producer $G_{n,g}$ obtained as the sum of the profits $\Pi_{G_{n,g}}^{t,\tau}$ accruing to him from the various $MK^{t,\tau}$ that compose the $SG^t$ under analysis. Equation (21) enforces the strategic variable to be at least equal to the true slope of the marginal supply bid. Finally, the impact of ISO actions and the rivals' actions, in any market session composing the $SG^t$, are taken into account through the constraints in (22).

*3) NE Calculation:* An equilibrium, in pure strategies, for the $SG^t$ will exist when no producer will have the incentive to change its bid unilaterally. Therefore, we need to find a strategy profile $(\beta_{n,g}^{f,t*},\beta_{-(n,g)}^{f,t*})$ such that $\beta_{n,g}^{f,t*}$ is the best response of player $G_{n,g}$ to the best response of any of the other market participants [45].

The *diagonalization algorithm* has been used to compute the NE. This is an iterative algorithm for the NE calculation, used also in the power market context (see, e.g., [30], [39], and [54]).

The solution of the diagonalization algorithm $\beta^{t*}(TE^f) = \{\beta_{1,1}^{t\ *}(TE^f),\ldots,\beta_{n,g}^{t\ *}(TE^f),\ldots\}$ corresponds to the sample of NE of the $SG^t(TE^f)$.

*4) $SG^t$ Market Equilibrium Results:* Given the calculated NE strategy profile of the stage game $SG^t(TE^f)$, the market equilibrium results of each $MK^{t,\tau}(TE^f)$ composing the stage game can be calculated.

The producer payoff for the period $t$ is calculated as the sum of the payoffs obtained in each of the spot market sessions $MK^{t,\tau}(TE^f)$ that compose the $SG^t(TE^f)$, as shown in the following:

$$\Pi_{G_{n,g}}^{t}\left(TE^f,\beta^{t*}(TE^f)\right) = \sum_{\tau} \Pi_{G_{n,g}}^{t,\tau}\left(TE^f,\beta^{t*}(TE^f)\right). \tag{23}$$

Analogously, the consumer payoff for the stage game is

$$CS_{L_{n,\ell}}^{t}\left(TE^f,\beta^{t*}(TE^f)\right) = \sum_{\tau} CS_{L_{n,\ell}}^{t,\tau}\left(TE^f,\beta^{t*}(TE^f)\right). \tag{24}$$

Finally, the true social welfare for the stage game is as follows:

$$SW^t\left(TE^f,\beta^{t*}(TE^f)\right) = \sum_{\tau} SW^{t,\tau}\left(TE^f,\beta^{t*}(TE^f)\right). \tag{25}$$

### C. Game Solution

In order to evaluate a sample of SPNE of the game, the generalized backward induction procedure can be adopted [46].

The process for the SPNE's sample computation works as follows. An NE for any stage game $SG^t(TE^f)$, i.e., a strategy profile, is calculated as illustrated in Section III-B

$$\beta^{t*}(TE^f) = \left\{\beta_{1,1}^{t\ *}(TE^f),\ldots,\beta_{n,g}^{t\ *}(TE^f),\ldots\right\}, \quad \forall t, \ \forall f.$$

The corresponding static payoff of the player $G_{n,g}$ is $\Pi_{G_{n,g}}^{t}(TE^f,\beta^{t*}(TE^f))$ [see (23)].

Then, from the knowledge of these NEs, a sample of SPNE of the $MSG(TE^f)$ is determined as follows:

$$\beta^{*}(TE^f) = \left\{\beta^{1*}(TE^f),\ldots,\beta^{t*}(TE^f),\ldots,\beta^{T*}(TE^f)\right\}.$$

In particular, the sample of NE for the $MSG(TE^f)$ is evaluated as the sequence of samples of NE for $SG^t \forall t = 1, 2, \ldots, T$. In fact, since it is assumed that there is no physical link between the periods $t = 1, 2, \ldots, T$ for any given history of the game (i.e., for any given set of choices of the players in every stage game that precedes the one under analysis), each power producer chooses as optimal strategy for the $SG^t$ the one that optimizes its static payoff $\Pi^t_{G_{n,g}}(TE^f)$ [55].

It should be noticed that if for any stage game where there exists a unique equilibrium in pure strategies, then there exists a unique SPNE for the $MSG(TE^f)$. This is not necessarily the case in our framework; however, we limit ourselves to the computation of a sample of SPNE of the game.

For the generic $MSG(TE^f)$, the payoff of $G_{n,g}$, corresponding to the strategy profile $\beta^*(TE^f)$, is given by[5]

$$\Pi_{G_{n,g}}\left(TE^f, \beta^*(TE^f)\right) = \sum_{t=1}^{T} [\delta^\wedge(t-1)]\, \Pi^t_{G_{n,g}}\left(TE^f, \beta^{t*}(TE^f)\right). \quad (26)$$

The payoff of the $planner$ for the $MSG(TE^f)$ subgame is equal to

$$\Pi^{MSG(TE^f)}_{\text{planner}}\left(TE^f, \beta^*(TE^f)\right) = \sum_{t=1}^{T} [\delta^\wedge(t-1)]\, SW^t\left(TE^f, \beta^{t*}(TE^f)\right). \quad (27)$$

Finally, the reduced form of the game—obtained by substituting each $MSG(TE^f)$ with the corresponding NE strategy profile—is derived.

Given the generic $TE^f$, the "planner" payoff for the whole game is equal to the payoff of the multistage game following $TE^f$, i.e., $MSG(TE^f)$, net of investment costs

$$\Pi_{\text{planner}}\left(TE^f, \beta^*(TE^f)\right) = \Pi^{MSG(TE^f)}_{\text{planner}}\left(TE^f, \beta^*(TE^f)\right) - IC(TE^f). \quad (28)$$

Hence, the optimal investment plan $TE^{f^*}$ for the "planner" is the one that maximizes expression (28), that is

$$TE^{f^*} = \underset{TE^f \in \mathcal{TE}}{\arg\max}\, \Pi_{\text{planner}}\left(TE^f, \beta^*(TE^f)\right). \quad (29)$$

The sample of SPNE of the whole game we calculated is $(TE^{f^*}, \beta^*(TE^f))$, and the RTN corresponds to the chosen transmission investment plan $TE^{f^*}$.

Note that the "planner" subgame perfect outcome is the value of the PF, defined in (1), calculated for the $TE^{f^*}$.

Whenever, the SPNE of this game can be proved to be unique, the RTN is indeed an optimal network from an economic point of view. Otherwise, the found RTN is optimal in a weaker sense.

## IV. Test Results

A numerical example is presented below to illustrate the method proposed in this paper.

[5]To be more precise, when the producer payoff is evaluated for a $MSG^t$, also the fixed costs borne by the producer during that period should be included. However, for purpose of simplicity, these costs are not taken into account here.

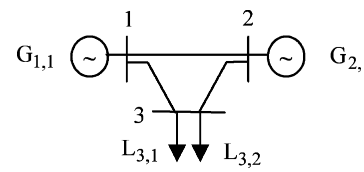


Fig. 2. Test network.

TABLE I
PARAMETERS OF MARGINAL COST FUNCTION AND DEMAND FUNCTIONS

| | $a_{n,g}$ | $b_{n,g}$ | $q^{max}_{G_{n,g}}$ |
|---|---|---|---|
| | $[\$/MWh]$ | $[\$/((MW)^2h)]$ | $[MW]$ |
| $G_{1,1}$ | 10 | 0.35 | 500 |
| $G_{2,1}$ | 12 | 0.45 | 500 |
| | $d^{t,\tau}_{n,\ell}$ | $s^{t,\tau}_{n,\ell}$ | $q^{t,\tau,\,max}_{L_{n,\ell}}$ |
| | $[\$/MWh]$ | $[\$/((MW)^2h)]$ | $[MW]$ |
| $L^{t,1}_{3,1}$ | 100 | 0.52 | 192 |
| $L^{t,1}_{3,2}$ | 100 | 0.65 | 153 |
| $L^{t,2}_{3,1}$ | 350 | 0.73 | 480 |
| $L^{t,2}_{3,2}$ | 320 | 0.80 | 400 |

The model has been implemented in GAMS. In particular, the NLPEC problem has been solved using CONOPT3, which is a nonlinear programming solver.

For purpose of clarity, all the stage games are supposed to be equal in our example, that is, the $SG^t$'s have the same inputs: market participants, production cost, and demand functions for all $t$. Moreover, the only effect of line investment is supposed to be the increase of the maximum line capacity.

We consider that any stage game is composed by 730 market sessions $MK^{t,\tau}$ (about one month). We suppose that 487 h are off-peak hours with the same characteristics (we indicate those hours with apex $\tau = 1$), and the remaining 243 h are peak hours with the same characteristics (we indicate those hours for $\tau = 2$ sessions). The whole time horizon (ten years) is composed by 120 stage games.[6] Furthermore, the discount factor $\delta$ is assumed to be equal to 0.98.

The system under study is represented in Fig. 2. It is composed of three buses $n = 1, 2, 3$, two generators $G_{1,1}$, $G_{2,1}$, two loads $L_{3,1}, L_{3,2}$, and three lines that interconnect these three buses.

The cost and the utility function parameters considered in the example are shown in Table I.

The transmission capacity of the network has been assumed as in the following. For the line connecting buses $n = 1$ and $n = 2$, $F^{Ex}_{1,2} = 1000$ MW, whereas for the remaining two lines, $F^{Ex}_{1,3} = F^{Ex}_{2,3} = 100$ MW.

[6]The example of this paper is for illustrative purposes only. When the analysis is performed for a real-size network, it should be appropriate to consider a higher number of market sessions and stage games. The appropriate number of stage games to be analyzed depends on the number of sessions $\tau$ composing each $SG^t$ and so on a regulatory constraint. If we suppose that every stage game is composed by a single market spot session, and so no regulatory constraint for mitigating market power is adopted, a discretization of the annual load duration curve in 6–10 blocks could be sufficient; whereas, if we suppose that each $SG$ is composed by 24 market sessions, then we could think about considering a representative working day and a representative weekend day, for each month, and use appropriate weight for taking into account the real number of days.

The studied investment plans alternatives $\{TE^1, \ldots, TE^f, \ldots, TE^F\}$, in this illustrative example, are chosen by considering possible combinations of investment in the two lines having a maximum transmission capacity of 100 MW. A significant number of investment alternatives $TE^f = (F_{1,2}^{\text{New}}(TE^f), F_{1,3}^{\text{New}}(TE^f), F_{2,3}^{\text{New}}(TE^f))$ belonging to the set $\mathcal{TE}$ are considered, and some of those are shown in the following: {(0,0,0),(0,0,100),(0,0,150),(0,0,200),(0,50,0),(0,50,150),(0,100,100),(0,150,100),(0,200,150),...}. An appropriate investment cost $IC(TE^f)$ for any transmission expansion alternative, has been considered.[7]

Results obtained for three particular investment plan alternatives, chosen among more than 50 different $TE^f$'s studied, are presented in the following. In particular, the results of a stage game $SG^t$ for the existing network $TE^1$ are shown, together with those of $TE^2$ and $TE^3$ cases. Investment plans $TE^2$ and $TE^3$ are the transmission alternatives that maximize the performance function $PF(TE^f)$ calculated for perfect competitive behavior and for strategic behavior, respectively, of market participants.

### A. Existing Network: Case $TE^1$

The case $TE^1$ corresponds to have "no investment in any line," i.e., $TE^1 = (0, 0, 0)$.

The following NE strategy profile for the stage game $SG^{1,t}$ is obtained: $\beta_{1,1}^{t}{}^*(TE^1) = 3.325$ \$/((MW)$^2h$) and $\beta_{2,1}^{t}{}^*(TE^1) = 1.979$ \$/((MW)$^2h$).

The market results, of the stage $SG^t(TE^1)$, for a single off-peak hour session ($\tau = 1$) and a single peak hour session ($\tau = 2$), obtained in case of Perfect Competition (PC), i.e., when each generator offers its marginal costs, and in case of Strategic Behavior (SB), i.e., when they offer their $\beta_{n,g}^{t}{}^*(TE^1)$, are shown in Table II.

It is useful to observe that for the $\tau = 1$ session, the market results for the PC and SB cases are completely different, both for the produced quantities and for the prices; whereas, for the $\tau = 2$ session, it is interesting to notice that, even if the produced/withdrawal quantities do not change, the prices at the production buses in the SB case differ significantly from the prices obtained in the PC case. Basically, this type of behavior attests that the power producers are able to extract all the transmission rent, which in case of PC was taken by the ISO.[8]

The social welfare, the producer surplus, and the consumer surplus, of the stage game $SG^t(TE^1)$, for the 730 market sessions, computed according to (23)–(25), respectively, measured in millions of dollars, are shown in Table III.

By comparing the social welfare values obtained as the results of the PC case and the SB case, it can be noted that the social welfare decreases only in the first period. In the second period, indeed, the share of social welfare corresponding to the transmission rent is taken by the power producers, but the whole amount of social welfare does not change. On the contrary, the values of producer surplus in the SB case are higher than those in the PC case for both market sessions.

[7]In this paper, the $IC(TE^f)$ is obtained as the sum of the line investment costs. The assumed line investment cost is a linear function of megawatts installed, where the unit cost is equal to \$180 000/MW. Moreover, the set of transmission investment alternatives $\mathcal{TE}$, considered in this paper, includes only line investments $F_{i,j}^{\text{New}}$ that are integer multiple of 50 MW.

[8]Similar findings have been highlighted also by [2], [51], and [56]. Note that in [2] and [56], the generators were assumed to compete according to a Cournot model, whereas in [51], various strategic bidding models are considered.

TABLE II
MARKET RESULTS OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^1)$: PC AND SB CASES

| | $\tau$ | $\lambda_1^{t,\tau}$ | $\lambda_2^{t,\tau}$ | $\lambda_3^{t,\tau}$ | $q_{L_{3,1}}^{t,\tau}$ | $q_{L_{3,2}}^{t,\tau}$ | $q_{G_{1,1}}^{t,\tau}$ | $q_{G_{2,1}}^{t,\tau}$ |
|---|---|---|---|---|---|---|---|---|
| | | [\$/$MWh$] | | | [$MWh$] | | | |
| PC | 1 | 45.53 | 46.78 | 48.35 | 99.33 | 79.46 | 101.51 | 77.29 |
| PC | 2 | 36.25 | 68.25 | 259.35 | 124.18 | 75.82 | 75.00 | 125.00 |
| SB | 1 | 83.24 | 83.24 | 83.24 | 32.24 | 25.79 | 22.03 | 36.00 |
| SB | 2 | 259.35 | 259.35 | 259.35 | 124.18 | 75.82 | 75.00 | 125.00 |

TABLE III
SOCIAL WELFARE, PRODUCER SURPLUS, AND CONSUMER SURPLUS ($10^6$\$) OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^1)$

| | $SW^{t,1}$ | $SW^{t,2}$ | $CS^{t,1}$ | $CS^{t,2}$ | $PS^{t,1}$ | $PS^{t,2}$ |
|---|---|---|---|---|---|---|
| PC | 3.980 | 12.890 | 2.249 | 1.926 | 1.533 | 1.093 |
| SB | 2.088 | 12.890 | 0.237 | 1.926 | 1.851 | 10.964 |

TABLE IV
MARKET RESULTS OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^2)$: PC AND SB CASES

| | $\tau$ | $\lambda_1^{t,\tau}$ | $\lambda_2^{t,\tau}$ | $\lambda_3^{t,\tau}$ | $q_{L_{3,1}}^{t,\tau}$ | $q_{L_{3,2}}^{t,\tau}$ | $q_{G_{1,1}}^{t,\tau}$ | $q_{G_{2,1}}^{t,\tau}$ |
|---|---|---|---|---|---|---|---|---|
| | | [\$/$MWh$] | | | [$MWh$] | | | |
| PC | 1 | 47.00 | 47.00 | 47.00 | 101.93 | 81.54 | 105.70 | 77.77 |
| PC | 2 | 114.55 | 121.05 | 129.17 | 302.50 | 238.53 | 298.71 | 242.33 |
| SB | 1 | 57.30 | 57.30 | 57.30 | 82.11 | 65.69 | 80.67 | 67.13 |
| SB | 2 | 157.45 | 157.45 | 157.45 | 263.77 | 203.19 | 251.44 | 215.52 |

TABLE V
SOCIAL WELFARE, PRODUCER SURPLUS, AND CONSUMER SURPLUS ($10^6$\$) OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^2)$

| | $SW^{t,1}$ | $SW^{t,2}$ | $CS^{t,1}$ | $CS^{t,2}$ | $PS^{t,1}$ | $PS^{t,2}$ |
|---|---|---|---|---|---|---|
| PC | 3.983 | 22.192 | 2.368 | 13.647 | 1.615 | 7.005 |
| SB | 3.828 | 21.582 | 1.537 | 10.184 | 2.291 | 11.398 |

### B. Transmission Expansion: Case $TE^2$

The transmission investment plan $TE^2$ corresponds to $TE^2 = (0, 200, 150)$. The total cost of such a transmission investment alternative is equal to \$63 million.

The values of the strategic variables that constitute an NE of the stage game $SG^t(TE^2)$ are $\beta_{1,1}^{t}{}^*(TE^2) = 0.586$\$/((MW)$^2h$) and $\beta_{2,1}^{t}{}^*(TE^2) = 0.675$\$/((MW)$^2h$). Note that these $\beta_{n,g}^{t}{}^*(TE^2)$ values are in between the values that would have been obtained if different bids could be submitted in any market session. In fact, the optimal strategy profiles in such cases are $\beta_{1,1}^{t}{}^*(TE^2) = 0.549$\$/((MW)$^2h$) and $\beta_{2,1}^{t}{}^*(TE^2) = 0.639$\$/((MW)$^2h$) for the off-peak market session $MK^{t,1}$ and $\beta_{1,1}^{t}{}^*(TE^2) = 0.595$\$/((MW)$^2h$) and $\beta_{2,1}^{t}{}^*(TE^2) = 0.682$\$/((MW)$^2h$) for the peak market session $MK^{t,2}$.

The market results, of the stage game $SG^t(TE^2)$, organized as in the previous example, are reported in Tables IV and V.

TABLE VI
MARKET RESULTS OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^3)$: PC AND SB CASES

| | $\tau$ | $\lambda_1^{t,\tau}$ | $\lambda_2^{t,\tau}$ | $\lambda_3^{t,\tau}$ | $q_{L_{3,1}}^{t,\tau}$ | $q_{L_{3,2}}^{t,\tau}$ | $q_{G_{1,1}}^{t,\tau}$ | $q_{G_{2,1}}^{t,\tau}$ |
|---|---|---|---|---|---|---|---|---|
| | | $[\$/MWh]$ | | | $[MWh]$ | | | |
| PC | 1 | 47.00 | 47.00 | 47.00 | 101.93 | 81.54 | 105.70 | 77.77 |
| PC | 2 | 97.50 | 102.00 | 163.92 | 254.90 | 195.10 | 250.00 | 200.00 |
| SB | 1 | 59.16 | 59.16 | 59.16 | 78.54 | 62.83 | 79.17 | 62.21 |
| SB | 2 | 164.23 | 164.23 | 164.23 | 254.48 | 194.71 | 248.39 | 200.80 |

TABLE VII
SOCIAL WELFARE, PRODUCER SURPLUS, AND CONSUMER SURPLUS ($10^6\$$) OF THE $MK^{t,\tau}$ SESSIONS COMPOSING $SG^t(TE^3)$

| | $SW^{t,1}$ | $SW^{t,2}$ | $CS^{t,1}$ | $CS^{t,2}$ | $PS^{t,1}$ | $PS^{t,2}$ |
|---|---|---|---|---|---|---|
| PC | 3.983 | 21.352 | 2.368 | 9.463 | 1.615 | 4.845 |
| SB | 3.772 | 21.338 | 1.406 | 9.429 | 2.366 | 11.909 |

For the $TE^2$ investment alternative, in the PC case, congestion arises only during the peak hour, when the flow in the line between buses $n = 1$ and $n = 3$ reaches 300 MW; whereas, in the SB case, both in the peak and in the off-peak hour, the line flows are lower than the maximum line transmission capacity. Therefore, there is a unique price for each session of the market (i.e., the three nodal prices for a given market session are the same). Moreover, the nodal prices in the SB case are higher than the corresponding nodal prices in the PC case, because of the strategic behavior of the suppliers.

By comparing the results from Table II with those from Table IV, the general increase of dispatched quantities in any of the corresponding cases, together with a remarkable decrease of the nodal prices in almost all the cases can be noted. Therefore, an increase of the social welfare with respect to the $TE^1$ case is observed, as shown in Table V.

By comparing the market results gathered in Table III and Table V, for the transmission expansion plans $TE^1$ and $TE^2$, it can be noted that, while for the PC case, significant differences of the results ($SW$, $CS$, and $PS$) can be observed only with regard to the peak load market session ($\tau = 2$), for the SB case, also the off-peak values of social welfare and surplus increase.

## C. Transmission Expansion: Case $TE^3$

The transmission investment plan $TE^3$ corresponds to (0, 150, 100). The total cost of such investment alternative is \$45 millions.

The values of the strategic variables that constitute the calculated NE of the stage game $SG^t(TE^3)$ are $\beta_{1,1}^{t}{}^*(TE^3) = 0.621\$/((\text{MW})^2h)$ and $\beta_{2,1}^{t}{}^*(TE^3) = 0.758\$/((\text{MW})^2h)$.

The market results are reported in Tables VI and VII. It is useful to note that, due to the limited transmission capacity of the lines, a reduction of dispatched quantities with respect to the $TE^2$ case occurs in the peak hour of the PC case; whereas, in the SB case, the limited transmission capacity allows the power producers to exercise even more of their market power. The values in Table VII show that, both in the PC and in SB case, the values of $SW^{t,2}$ and $CS^{t,2}$ are in between those obtained for the $TE^1$ and $TE^2$; whereas, the $PS^{t,2}$ in the SB case of $TE^3$ is higher than the producers' surplus obtained for the other two transmission investment alternatives $TE^1$ and $TE^2$.

TABLE VIII
POWER PRODUCERS' PAYOFFS ($10^6\$$) FOR THE STAGE GAMES $SG^t(TE^f)$

| | | $\Pi_{G_{1,1}}^{t,1}$ | $\Pi_{G_{1,1}}^{t,2}$ | $\Pi_{G_{1,1}}^{t}$ | $\Pi_{G_{2,1}}^{t,1}$ | $\Pi_{G_{2,1}}^{t,2}$ | $\Pi_{G_{2,1}}^{t}$ |
|---|---|---|---|---|---|---|---|
| $SG^t(TE^1)$ | PC | 0.878 | 0.239 | 1.117 | 0.655 | 0.854 | 1.509 |
| $SG^t(TE^1)$ | SB | 0.744 | 4.305 | 5.049 | 1.107 | 6.659 | 7.766 |
| $SG^t(TE^2)$ | PC | 0.952 | 3.794 | 4.747 | 0.663 | 3.211 | 3.873 |
| $SG^t(TE^2)$ | SB | 1.304 | 6.320 | 7.624 | 0.987 | 5.078 | 6.065 |
| $SG^t(TE^3)$ | PC | 0.952 | 2.658 | 3.610 | 0.663 | 2.187 | 2.850 |
| $SG^t(TE^3)$ | SB | 1.361 | 6.685 | 8.046 | 1.005 | 5.223 | 6.228 |

TABLE IX
PLANNER PAYOFFS ($10^6\$$) OF THE ANALYZED $TE^f$'S

| | | $\sum_{t=1}^{T}[\delta\wedge(t-1)]SW^t$ | $IC(TE^f)$ | $\Pi_{Planner}$ |
|---|---|---|---|---|
| $TE^1$ | $PC$ | 768.837 | 0 | 768.837 |
| $TE^1$ | $SB$ | 682.622 | 0 | 682.622 |
| $TE^2$ | $PC$ | 1192.869 | 63.000 | **1129.869** |
| $TE^2$ | $SB$ | 1158.006 | 63.000 | 1095.006 |
| $TE^3$ | $PC$ | 1154.593 | 45.000 | 1109.593 |
| $TE^3$ | $SB$ | 1144.330 | 45.000 | **1099.330** |

## D. Payoffs of the Players

In Table VIII, the payoffs that each of the power producers gains at NE, for the previously considered cases, are gathered. In the PC cases, the profits increase together with the increased transmission capacity available; whereas, for the SB cases, certainly the power producers benefit from the increase of transmission capacity, even though their profits decrease when the transmission expansion is too large.

Table IX shows the value of planner payoffs, i.e., the PF value, obtained when the power producers act according to the calculated NE, in the three cases under study. Moreover, in order to better analyze the results, the $IC(TE^f)$ and the total social welfare values, over the considered time horizon, are also reported.

If the power producers are supposed to offer their true marginal costs, the optimal choice is the transmission expansion plan $TE^2$, since the $\Pi_{\text{planner}}(TE^2)$ is the highest among the planner payoffs values calculated in the PC cases.

Meanwhile, if the strategic behavior of power producers is taken into account, the best value of the $PF(TE^f)$ is obtained for the $TE^3$ transmission investment alternative.

Finally, it should be noted that $(TE^3, \beta^*(TE^3))$ is the subgame perfect Nash outcome of the game and that $TE^3$ agrees with the definition of RTN given in Section II.

# V. CONCLUSION AND FUTURE WORK

The aim of this paper is to provide the social planner with a reference transmission network that can be used as a benchmark when he evaluates investment proposals. In fact, the RTN represents the network upgrade that maximizes a given performance function, over a certain time horizon, when oligopolistic behaviors that are likely to arise in a deregulated environment have been taken into account.

In this paper, a general approach based on game-theory concepts for the estimation of the RTN has been proposed. The strategic interactions among power producers that interact in a multisession framework have been modeled using an SFE approach.

Furthermore, the impact of producers' behaviors on nodal prices, dispatched quantities, and social welfare value, for various transmission expansion alternatives, has been shown and compared with analogous results obtained by assuming a perfect competitive framework.

The proposed method has been applied to the study of a sample system. The simulations show how, in the studied oligopolistic framework, the optimal network has transmission line capacities smaller than the optimal transmission line capacities obtained for a perfect competitive setting. However, it would not be surprising to find examples in which RTN transmission line capacities, greater than optimal transmission capacities estimated for a perfect competitive framework, may increase the performance function value and enhance market efficiency.

Future research can take different directions. One of the main challenges would be to frame the RTN concept in a regulatory policy for transmission expansion. In particular, an appropriate centralized transmission investment framework could be investigated. In such a case, the RTN could constitute the transmission expansion that the central planner would implement. Another research direction may be to investigate a regulatory framework in which the RTN is only evaluated by the central planner and then provided as a benchmark in a public call for transmission network upgrades. The investors interested in participating in transmission upgrade may apply to the call and may submit proposals for transmission expansions they would like to realize. In such a case, the planner could accept combinations of expansions whose PF value does not differ significantly from the PF value of the RTN.

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