

From Models and Data to Proofs

For Improving Cyberphysical Systems

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TENTH CARNEGIE MELLON CONFERENCE ON THE ELECTRICITY INDUSTRY

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# Collaborators



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August 2003. Thousands of New Yorkers crossing the Brooklyn Bridge as the NE experienced the biggest power outage.

Blackout's primary cause was a software bug in the alarm system at a control room of the FirstEnergy Corporation, in Ohio.



22 M Cars recalled in 2013

50% cost of 787 attributed to software

2M medical devices recalled, 24% for software bugs

"Program testing can best show the presence of errors but never their absence." Edsger W. Dijkstra

# Verification as a Search Problem

**Sound and Complete  
always gives correct answer**

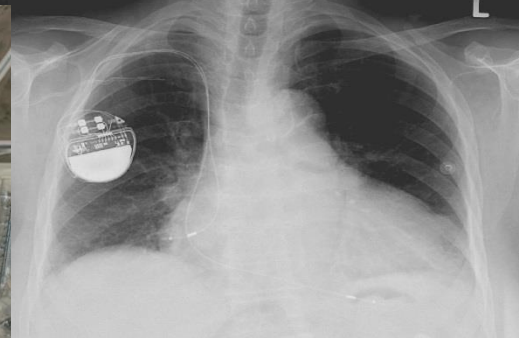
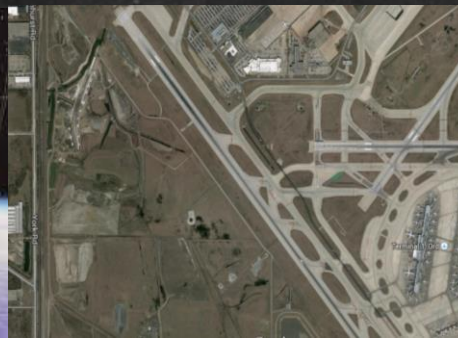


Given a system model and some requirements, find a behavior of the system that violates those requirements.

Yes (Bug-trace)

There is no such behavior (Safety certificate)

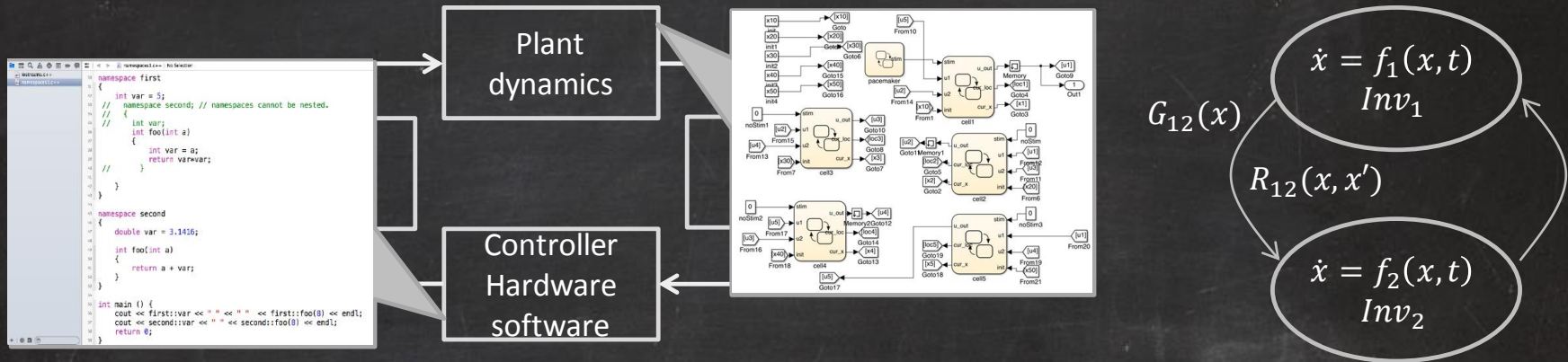
**Model + Trace Data → Proof**



# Outline

- Overview of Trace-based Verification
- Three recent case studies on
  - Alerting protocol (NASA/FAA)
  - Powertrain control system (Toyota)
  - Cardiac cells and Pacemaker Network
- Conclusions

# Our Tools Handle a Class of Simulink/Stateflow Models



A generic hybrid systems with two modes

**Early 90's:** Exact unbounded verification: Decidable for  $\dot{x} = 1$  [Alur Dill 92]

Undecidable even for  $\dot{x}=1 \dot{y}=2$  [Henzinger 95]

**Late 90'-00':** Approximate, bounded, mostly linear: Hamilton-Jacobi-Bellman [Tomlin et al. 02], Polytopes [Henzinger 97], ellipsoids [Kurzanski] zonotopes [Girard 05], support functions [Frehse 08], CEGAR [Clarke 03]

**Today:** Scalable, nonlinear: trace-based methods [Mitra 10-13][Donze 07]

# Core Idea: Trace-based Verification

Given start  $S$  and target  $T$

Compute finite cover of initial set

Execute/simulate from the center  $x_0$  of each cover

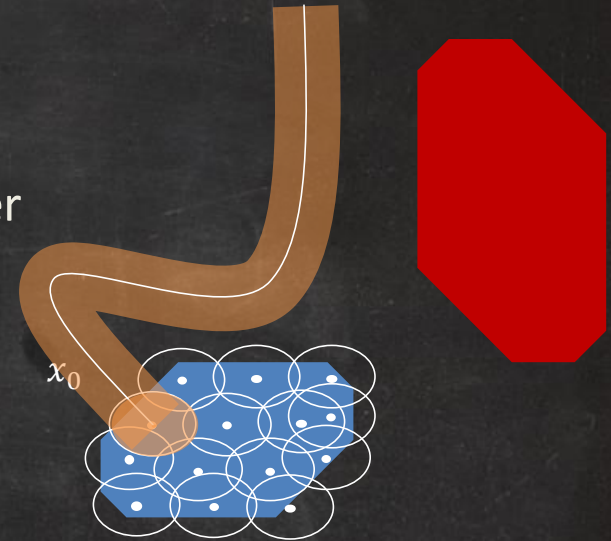
**Bloat** execution to contain all trajectories from the cover

If contained in  $T$  then UNSAFE

Union is an over-approximation of reach set

If Union is disjoint from  $T$  then SAFE

Otherwise, refine cover

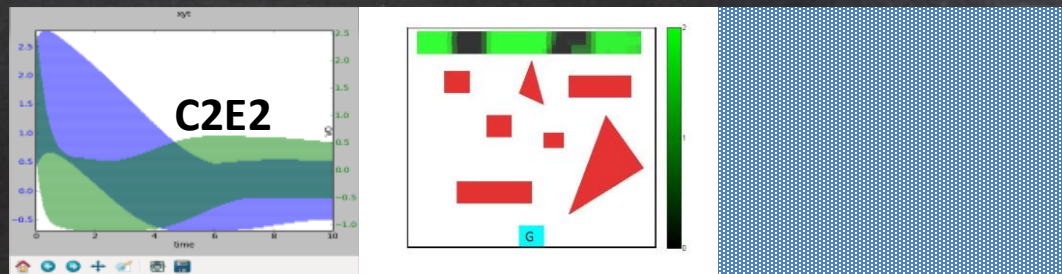


- How much to bloat? Use static analysis of model [EmSoft2013, FM 2014]
- How to handle mode switches? May-must analysis [TACAS 2015]
- How to handle large models? Compositional analysis [HSCC 2014, CAV 2014]



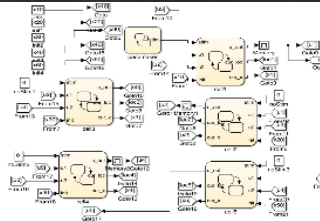
# Discrepancy: a Layer Between Algorithms for (Verification | Synthesis | Monitoring) and (Models | Testbeds | Simulators)

verification ◦ synthesis ◦ monitoring



Discrepancy  $\beta$

$$\begin{aligned} \dot{x} &= f(x, u) \\ x &\in Inv \\ y &= g(x, u) \end{aligned}$$

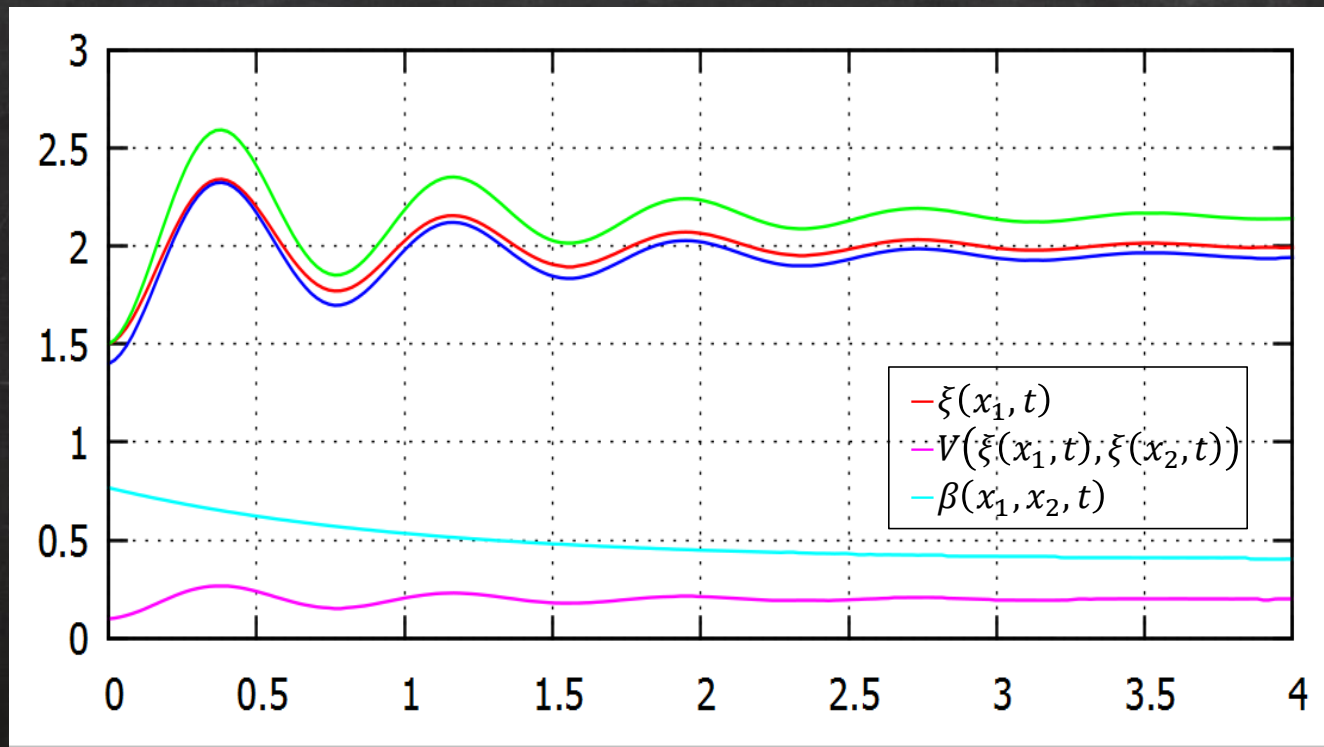


math model ◦ code ◦ hardware

# A model characteristic extracted using static analysis: Discrepancy

**Definition.**  $\beta: \mathbb{R}^{2n} \times \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$  defines a **discrepancy** of the system if for any two states  $x_1$  and  $x_2 \in X$ , For any  $t$ ,

1.  $|\xi(x_1, t) - \xi(x_2, t)| \leq \beta(x_1, x_2, t)$  and
2.  $\beta \rightarrow 0$  as  $x_1 \rightarrow x_2$



# Algorithms are Sound & Relatively Complete

**Theorem.** (Soundness). If Algorithm returns safe or unsafe, then  $A$  is safe or unsafe.

**Definition** Given any HA  $A = \langle V, Loc, A, D, T \rangle$ , an  $\epsilon$ -perturbation of  $A$  is a new HA  $A'$  that is identical except,  $\Theta' = B_\epsilon(\Theta)$ ,  $\forall \ell \in Loc, Inv' = B_\epsilon(Inv)$   
(b)  $a \in A, Guard_a = B_\epsilon(Guard_a)$ .

$A$  is **robustly safe** iff  $\exists \epsilon > 0$ , such that  $A'$  is safe for  $U_\epsilon$  upto time bound  $T$ , and transition bound  $N$ . Robustly unsafe iff  $\exists \epsilon < 0$  such that  $A'$  is safe for  $U_\epsilon$ .

**Theorem.** (Relative Completeness) Algorithm always terminates whenever the  $A$  is either robustly safe or robustly unsafe.

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# SAPA-ALAS Parallel Landing Protocol

Air traffic is going to double in the next 20-25 years

Strong need to improve airport throughput

Cost of new runways: ~ \$USD 15B+



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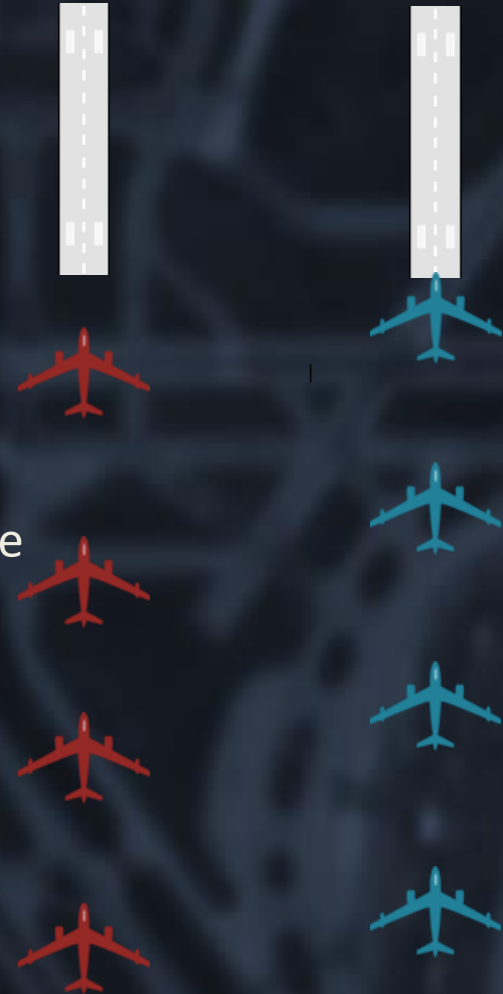
Cost of new runways: ~ \$USD 15B+

Alternatively, pack more planes in shorter space & time

There are physical limits, e.g., wake vortices

But there is also human (co-pilot) in the loop

Solution: software!



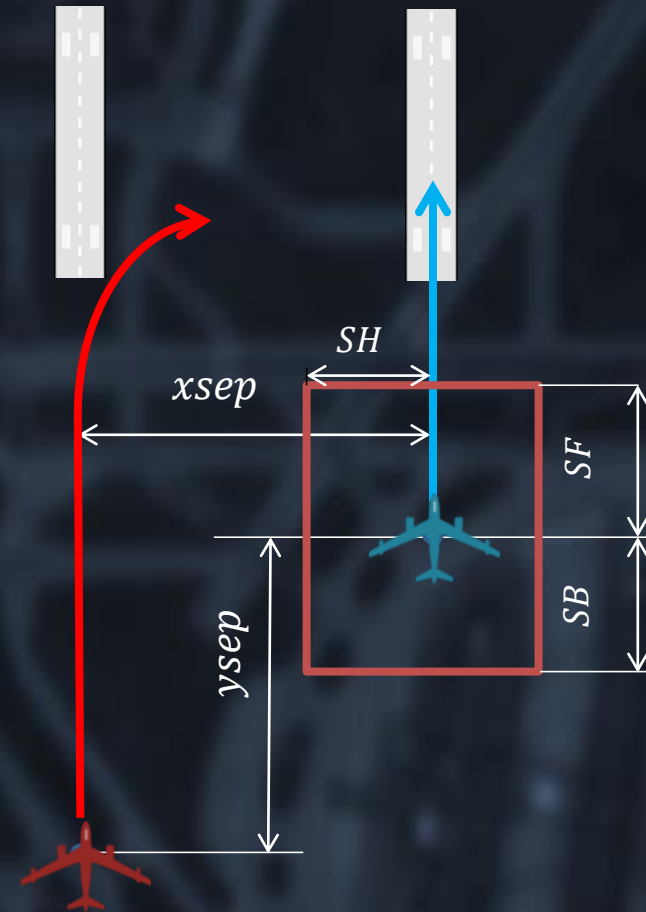
# SAPA-ALAS Parallel Landing Protocol

*Ownship* and *Intruder* approaching parallel runways with small separation

ALAS (at ownship) NASA's protocol supposed to **raise an alarm** if within  $T$  time units the *Intruder* can violate safe separation

Can we trust ALAS? *Alert*  $\prec_b$  *Unsafe* ?

Uncertainty:  $xsep \in [.11, .12]$  Nm  $ysep \in [.1, .21]$  Nm,  $\phi \in [30^\circ, 45^\circ]$   $vy_o = 136$  Nmph,  $vy_i = 155$  Nmph



# C2E2 Verifies Alerting Protocol in Minutes

Our verification tool computes **increasingly more precise over-approximations** of the reachable states of the system and automatically proves *Alert*  $<$  *Unsafe* properties for different scenarios in reasonable time

Shows that false alarms are possible

Finds scenarios where alarm may be missed

Scenario	Alert $\leq_4$ Unsafe	Running time (mins:sec)	Alert $\leq_7$ Unsafe
6	False	3:27	2.16
7	True	1:13	—
8	True	2:21	—
6.1	False	7:18	1.54
7.1	True	2:34	—
8.1	True	4:55	—
9	False	2:18	1.8
10	False	3:04	2.4
9.1	False	4:30	1.8
10.1	False	6:11	2.4

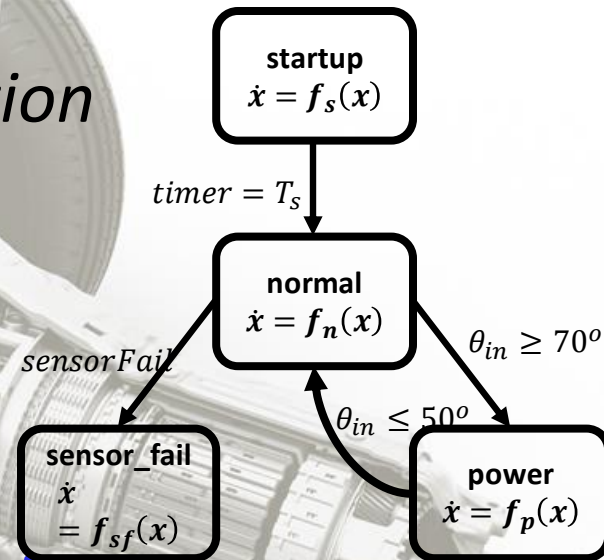


## 2. Powertrain Control System

*Simulink model of a powertrain control system provided by **Toyota** as a verification challenge. **Highly nonlinear polynomial differential equations**; **discrete mode switches***

**First to verify properties**, e.g., that the **air-fuel ratio** remains within a given range for a set of driver behaviors

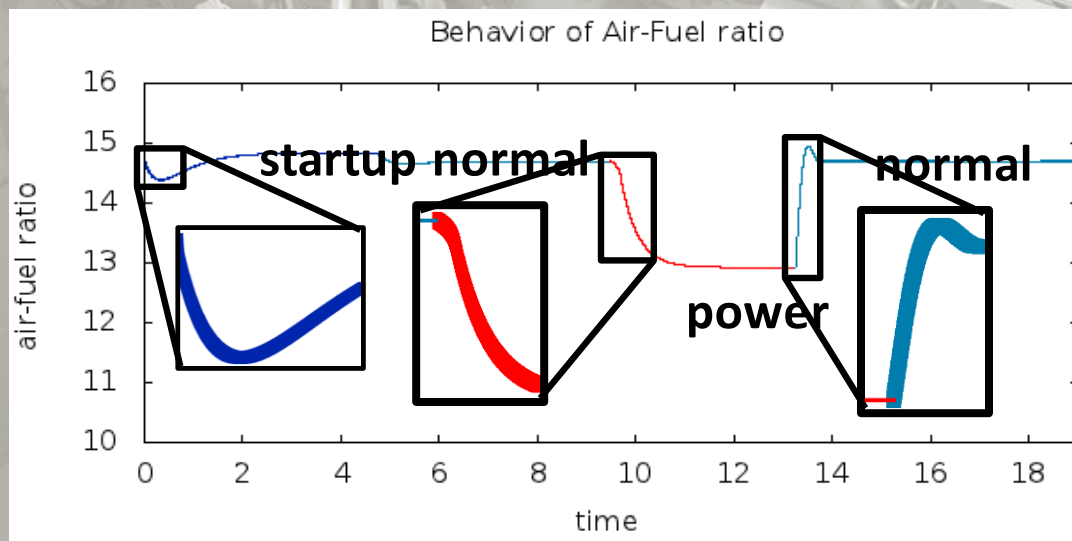
Discrepancy function  $\beta$  computed automatically using the local algorithm



## 2. Powertrain Control System

*Simulink model of a powertrain control system provided by Toyota as a verification challenge. Highly nonlinear polynomial differential equations; discrete mode switches*

We converted the model to Stateflow that can be processed by our tool; rest of the analysis was **completely automatic**. The **whole exercise took less than a month**





# APPLICATION 3: A NETWORK OF CARDIAC CELLS AND PACEMAKER

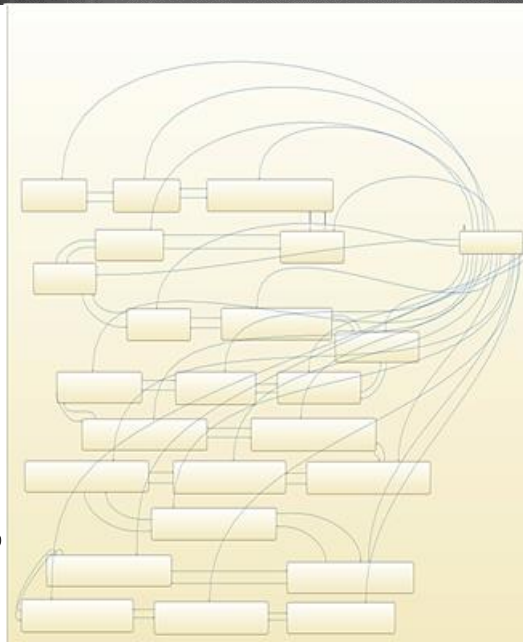
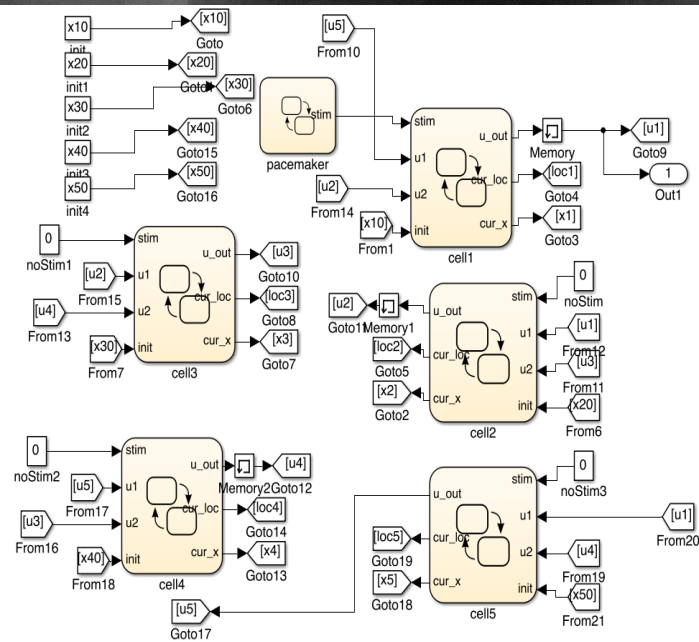
Huang ◦ Mitra (HSCC 2013)

Huang ◦ Fan ◦ Meracre ◦ Mitra ◦ Kiwatkowska (CAV 2014)

# 3. Pacemaker + Cardiac Network

Simulink model of a **network of cardiac cells** and a pacemaker; nonlinear differential equations; **30+ continuous variables**; many interacting components; uncertainty in timing and initial voltages

Key property: voltage range action potentials remain in specific interval and has periodicity



```
state1
du:
u_dot=-0.0025000000000000*u+D*(u1+u2-2*u)/(h*h)+stim;
v_dot=-0.016666666666667*v+0.016666666666667;
w_dot=-0.0726392130750601*u-0.0050000000000000*w+0.0050000000000000;
s_dot=0.0325954614796371*u-0.3657376929266330*s+0.0078827602517302;
u_out=u;
cur_x[0] = u;
cur_x[1] = v;
cur_x[2] = w;
cur_x[3] = s;
cur_loc=1;
```

[u<0.0032252252252252] [u>=0.0032252252252252]

```
state2
du:
u_dot=-0.0025934648787471*u+0.0000003014452846+D*(u1+u2-2*u)/(h*h)+stim;
v_dot=-0.016666666666667*v+0.016666666666667;
w_dot=-0.0726392130750601*u-0.0050000000000000*w+0.0050000000000000;
s_dot=0.0342238163406254*u-0.3657376929266330*s+0.0078775084405570;
u_out=u;
cur_x[0] = u;
cur_x[1] = v;
cur_x[2] = w;
cur_x[3] = s;
cur_loc=2;
```

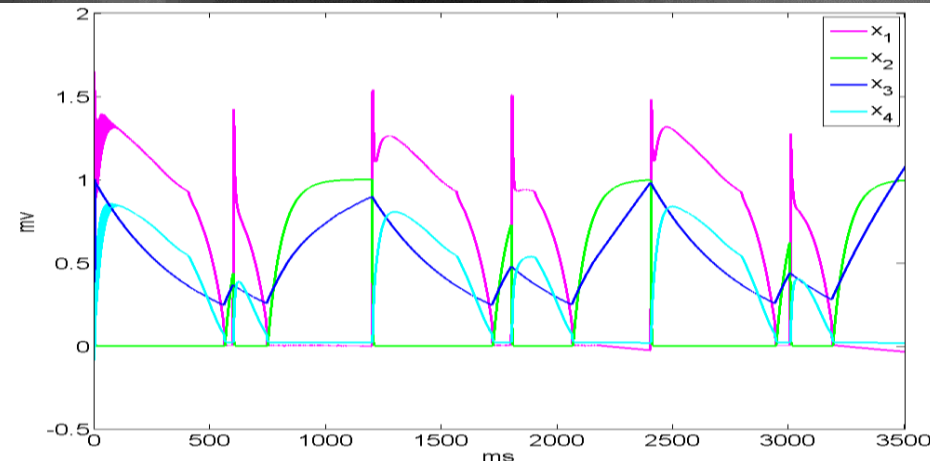
[u>=0.0059] [u<0.0059]

```
state3
du:
u_dot=-99.8500002249997323*u+0.5891000013299984+D*(u1+u2-2*u)/(h*h)+stim;
v_dot=-166.666666666666003*u-0.9486956521739127*v+157.9710144927535680*u+v+0.999999999999999;
w_dot=1.2857135714285342*u-0.0050000000000000*w-0.0030142814285712;
s_dot=0.0000003317449342*u-0.3657376929266330*s+0.0080794269996715;
```

# 3. Pacemaker + Cardiac Network

Our tool **first** to verify properties of this model (running times shown below)

**Compositional or modular analysis** for computing the discrepancy



Variables	Thresh	Sims	Run time (s)	Property
15	2	16	104.8	TRUE
15	1.65	16	103.8	TRUE
25	2	3	208	TRUE
25	1.65	5	281.6	TRUE
25	1.5	NA	63.4	FALSE
40	2	3	240.1	TRUE
40	1.65	73	2376.5	TRUE

# Conclusion

We have developed new algorithms and tools for analyzing complex, nonlinear hybrid models of control systems and software;

- Use Traces + Discrepancy → algorithms
- Sound (guarantees coverage): Gives proof of correctness or finds a bug
- Relatively complete: Always gives an answer<sup>1</sup>
- Effective: Appears to work for large & interesting examples<sup>2</sup>

Can this technology be used in design of Smart Grids

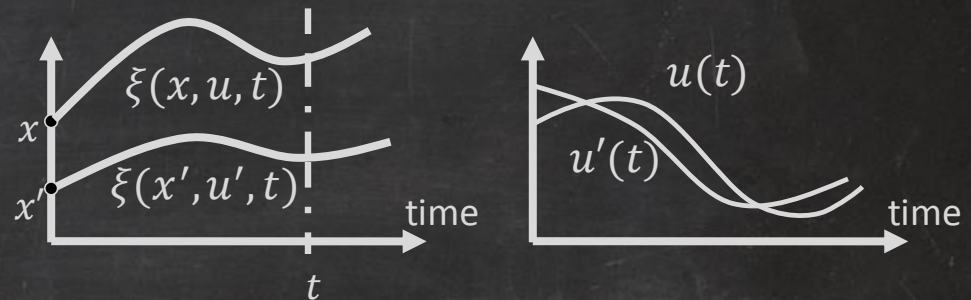
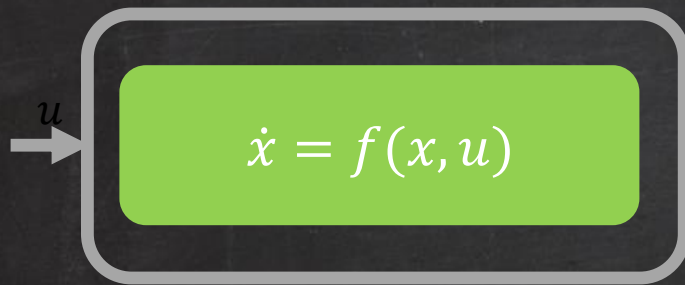
- Generating tests
- Finding parameters that satisfy properties
- Online monitoring
- Designing controllers

1: Unless the system is fragile with respect to the property in question

2: Exploiting parallelism will make it scale to even larger models



# Input-to-State (IS) Discrepancy



Definition. **IS discrepancy** is defined by  $\beta$  and  $\gamma$  such that for any initial states  $x, x'$  and any inputs  $u, u'$ ,

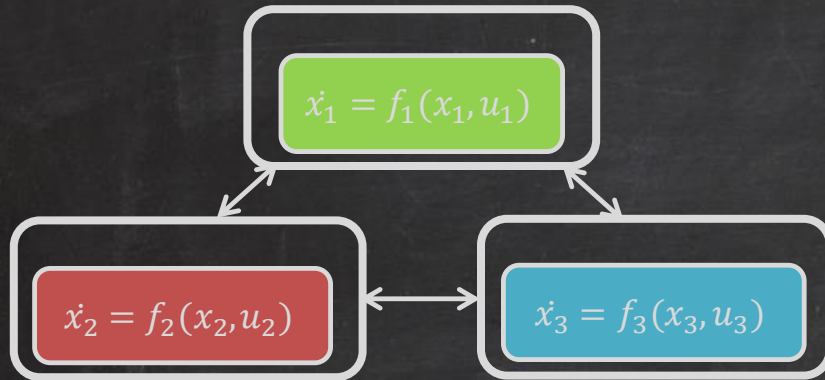
$$|\xi(x, u, t) - \xi(x', u', t)| \leq \beta(x, x', t) + \int_0^t \gamma(|u(s) - u'(s)|) ds$$

$$\beta \rightarrow 0 \text{ as } x \rightarrow x', \text{ and } \gamma \rightarrow 0 \text{ as } u \rightarrow u'$$

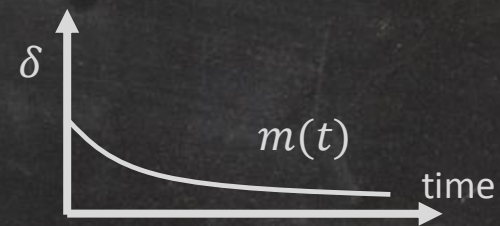
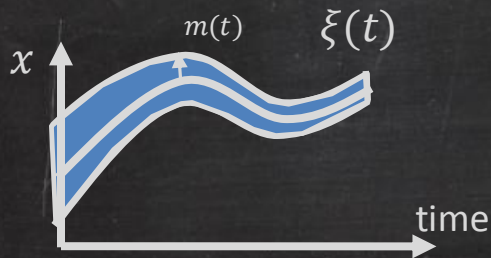
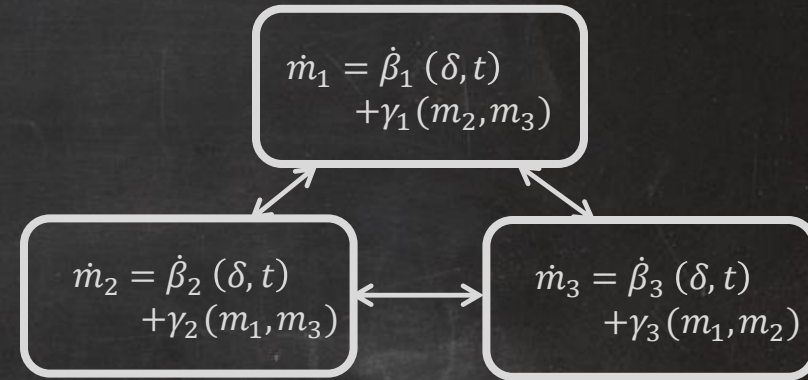


# Bloating with Reduced Model

$n \times N$  dimensional



$n \times 1$  dimensional



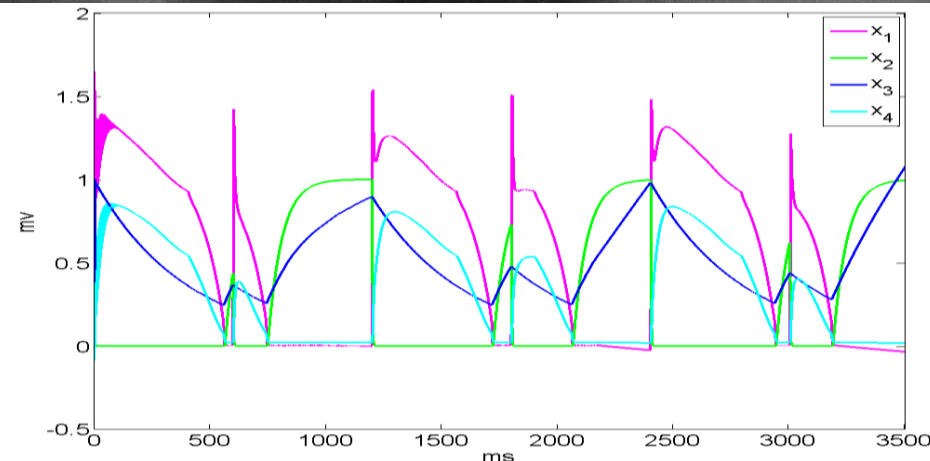
The bloated tube contains all trajectories start from the  $\delta$ -ball of  $x$ .

The over-approximation can be computed arbitrarily precise.

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