Carnegie Mellon
Electrical & Computer ENGINEERING

Intelligent Partitioning in Distributed Optimization of Electric Power Systems

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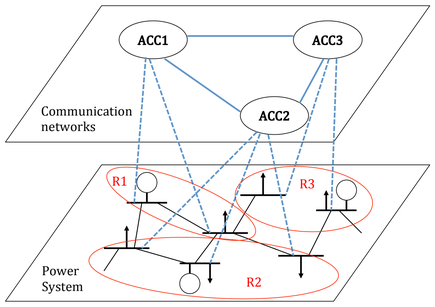
TENTH ANNUAL CARNEGIE MELLON CONFERENCE ON THE ELECTRICITY INDUSTRY

I. Motivation



❖ **Motivation for Distributed Optimization**

- Increasing distributed generation and flexible loads
- Increasing scale and complexity of optimization problems

❖ **Partitioning in Distributed Optimization**

- Unclear impact on distributed optimization
- Indispensable decision for implementation of distributed optimization methods

❖ ***Our Contributions***

- ***Evaluated how partitioning affects the convergence speed of distributed methods***
- ***Proposed an Intelligent Partitioning method that finds the best partition to speed up distributed optimization***

II. Decomposition Method

❖ **Optimization Problem**

- AC Optimal Power Flow (OPF)

❖ **Decomposition Method**

- Optimality Condition Decomposition (OCD) [1]

$$L = \sum_{a=1}^{A} [f_a(\boldsymbol{x}_a) + \boldsymbol{\lambda}_a^T c_a(\boldsymbol{x}_1, ..., \boldsymbol{x}_A) + \boldsymbol{\gamma}_a^T s_a(\boldsymbol{x}_a)]$$

$$\boldsymbol{H} \begin{pmatrix} \Delta\tilde{\boldsymbol{x}}_1 \\ \vdots \\ \Delta\tilde{\boldsymbol{x}}_A \end{pmatrix} = - \begin{pmatrix} \nabla_{\tilde{\boldsymbol{x}}_1} L \\ \vdots \\ \nabla_{\tilde{\boldsymbol{x}}_A} L \end{pmatrix}$$

$$\boldsymbol{H} = \begin{pmatrix} H_{11} & \dots & H_{1A} \\ \vdots & \ddots & \vdots \\ H_{A1} & \dots & H_{AA} \end{pmatrix}$$

III. Intelligent Partitioning

III.A Impact of partitioning on convergence time[2]

❖ **How to evaluate a specific partition of the system?**

- ***Coupling parameter*** $c = \rho(\boldsymbol{I} - \bar{\boldsymbol{H}}^{-1}\boldsymbol{H})$
  - $\boldsymbol{H}$ - Hessian matrix of the Lagrangian function of the partitioned system
  - $\bar{\boldsymbol{H}}$ - Hessian matrix of the ideally decoupled system
  - ρ - Spectral radius

❖ ***Speed of OCD Using Different Partitions of IEEE-30 System***

Partition	Regions	Coupling parameter	Iteration	Time (s)
None	1	N/A	30	0.09
a	2	0.6117	93	0.09
b	2	0.7647	158	0.15
c	3	0.8386	210	0.14
d	3	0.8550	251	0.25
e	4	0.8465	242	0.17
f	4	0.8893	321	0.21

* Convergence time: $t = k \cdot (\max\{t_1, t_2, ..., t_A\})$

❖ **Partitioning greatly affects convergence speed**

- Coupling parameter can measure computational coupling between partitioned areas
- **The smaller the *c*, the faster the convergence**
- ***H* is closer to block diagonal→*c* is smaller**

III.B An intelligent partitioning method

Group the strongly computationally coupled buses!

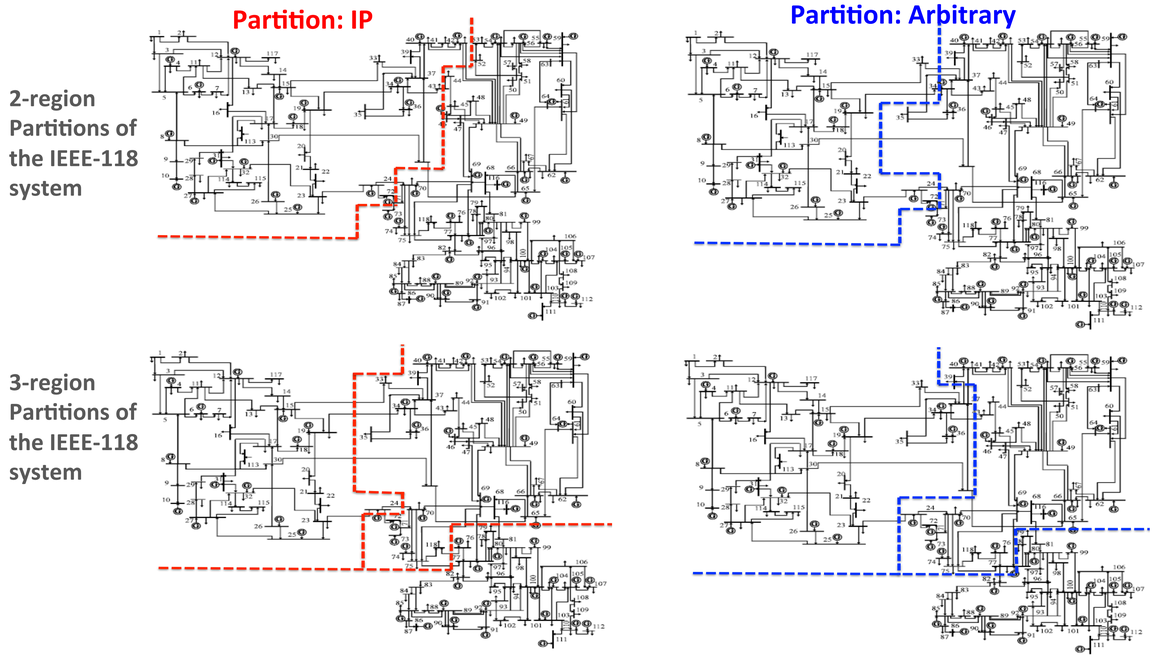
❖ **Affinity between buses**

- The entry H_{ij} in H_{sys} is non-zero→Variables with indices *i* and *j* are computationally coupled
- More than two variables associated with any two buses *i* and *j*

$$A_{i,j} = \sum_{m \in \Omega_i} \sum_{n \in \Omega j} |H_{m,n}|$$

❖ **Implementation**

- ***Step 1 Initialization***
  - Derive the Hessian matrix of the Lagrangian function of the unpartitioned power system at optimality
- ***Step 2 Calculation of the affinity matrix***
- ***Step 3 Clustering***
  - Cluster the buses into subregions using spectral clustering [2] based on the affinity matrix
  - More than one possible solutions
- ***Step 4 Selection***
  - Compute the coupling parameter of all partitions
  - Select the partition with minimum *c*



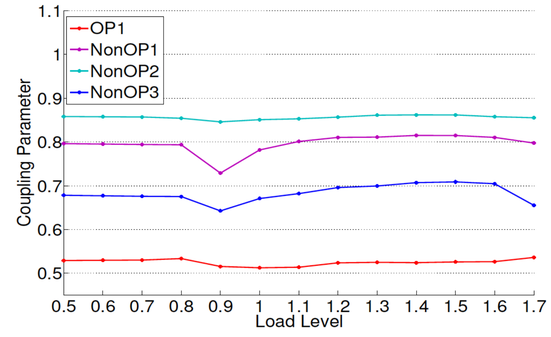
IV. Simulation Results

❖ ***Convergence Speed-up Using Different Partitions of IEEE-118 System***

#	Partition: IP				Partition: Arbitrary			
	c	*k*	*t*(*s*)	Speed-up	*c*	*k*	*t*(*s*)	Speed-up
2	0.51	105	3.4	11%	0.72	197	8.9	-134%
3	0.79	225	2.3	39%	0.91	454	4.3	-13%
4	0.79	246	1.4	63%	0.89	378	3.0	21%
5	0.82	250	1.3	66%	0.87	330	2.1	45%

- Centralized Approach: Iterations *k*=33, Convergence Time *t*=3.8*s*

❖ ***Robustness of the Best Partition to Operating Points***



V. Conclusions

❖ **Conclusions**

- Partitioning significantly affect the convergence speed of the decomposition method
- Convergence is considerably sped-up using the partition computed by the proposed method

❖ **Future Works**

- Evaluate the impact of partitioning on other performances of distributed methods such as information exchange
- Generalize the Intelligent Partitioning method for other decompositions methods

References

[1] A. J. Conejo, F. J. Nogales, and F. J. Prieto, "A decomposition procedure based on approximate Newton directions," *Mathematical Programming*, vol. 93, pp. 495-515, 2002.

[2] J. Guo, G. Hug, and O. Tonguz, "Impact of partitioning on performance of decomposition methods for AC optimal power flow," in IEEE Innovative Smart Grid Technologies (ISGT) Conference, 2015.

[3] A. Y. Ng, M. I. Jordan, and Y. Weiss, "On spectral clustering: Analysis and an algorithm," *Advances in Neural Information Processing Systems*, vol.2, pp. 849-856, 2002.