Limits to Decentralized Control:
A Perspective from Stat. Mech.

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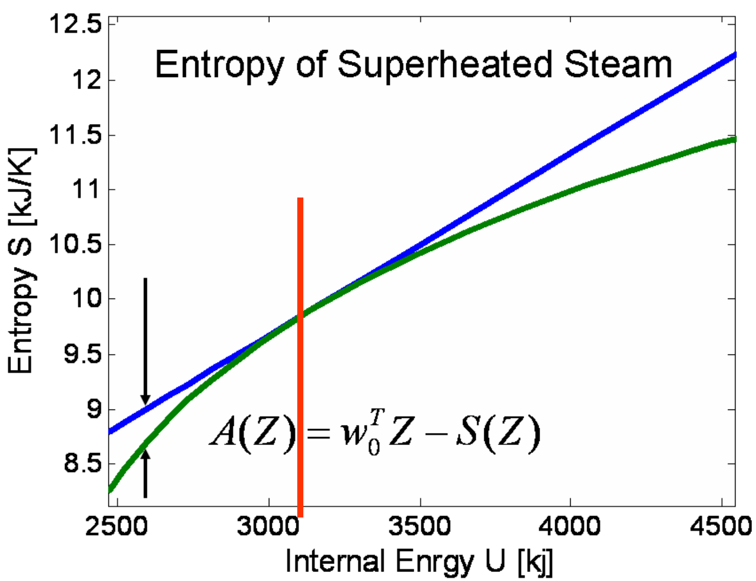
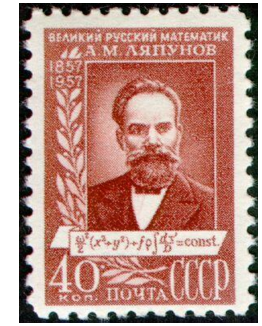
Thermodynamics

Boltzmann

$$S = k_B \ln \Omega(U, V, N)$$



Mathematical Systems

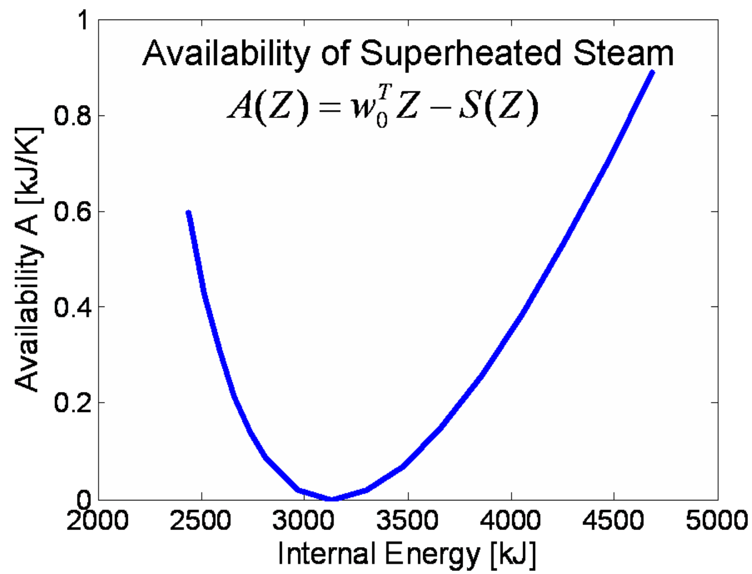


Lyapunov

$$\frac{dx}{dt} = f(x) + g(x)u$$

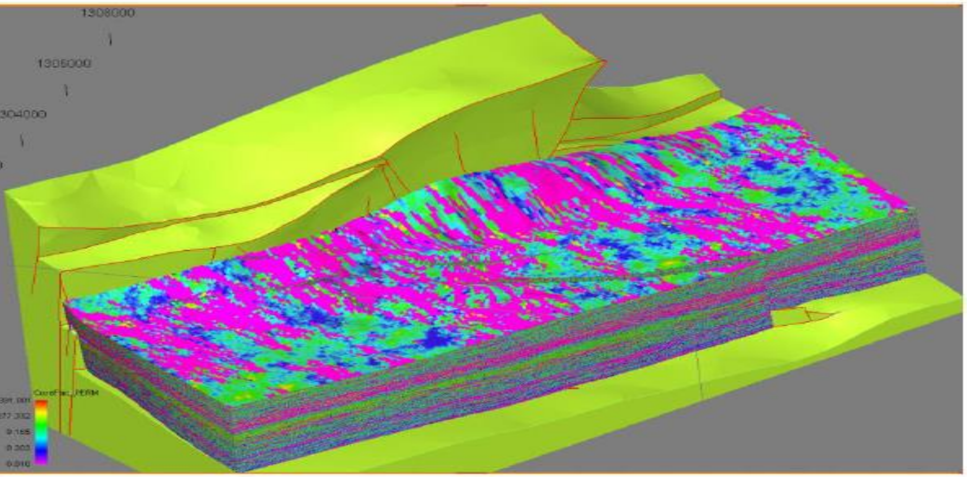
$$y = h(x)$$

Measurements



Field Theory

Applications: Chemical reaction, multi-phase fluid flow, particulate systems,…



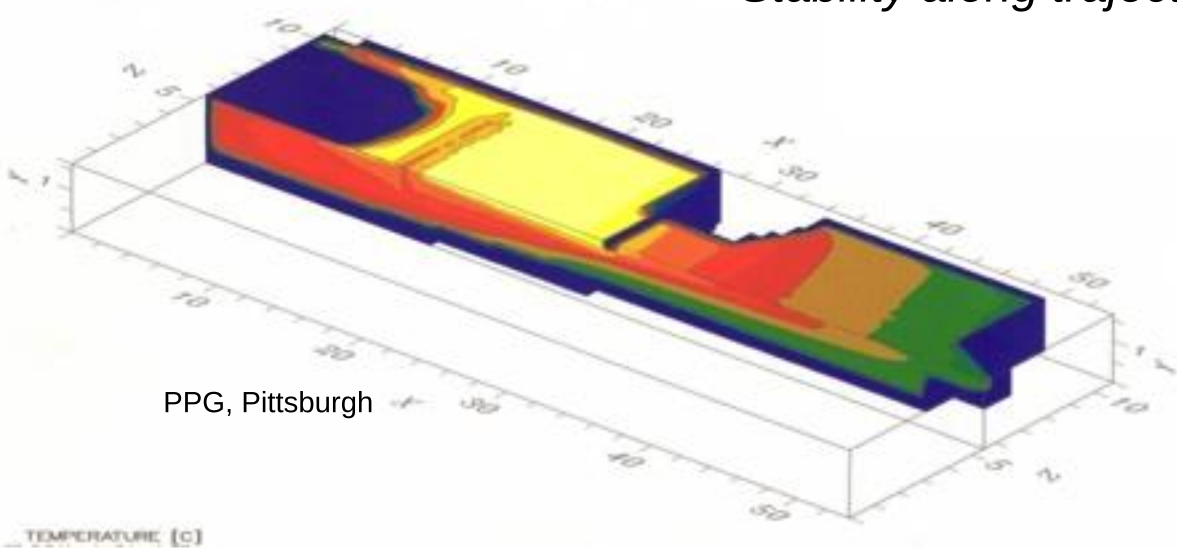
Ecopetrol, Columbia

Dissipation Inequality:

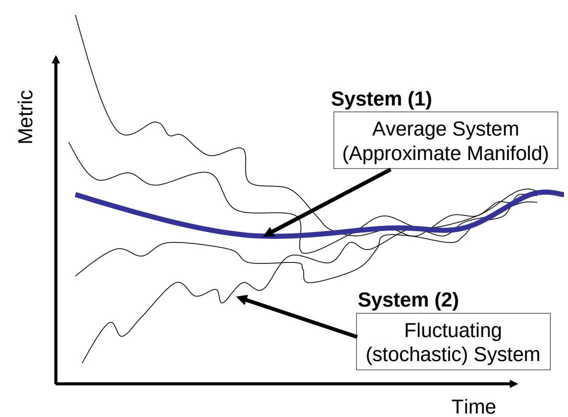
$$\dot{V}(t) + \langle \bar{w}, \bar{f}\cdot \mathbf{n}\rangle_{\mathscr{B}} - \frac{\mu_1 \varepsilon_1}{\varepsilon_0}\|\bar{w}\|^2_{L_2(\mathscr{B};\, R^n)} \leqslant -\left(\mu_0 - \frac{\mu_1}{\varepsilon_0}\right)\|\bar{X}_k\|^2_{L_2(\mathscr{V};\, R^n)}.$$

Minimization Principle (Min Entropy Production)

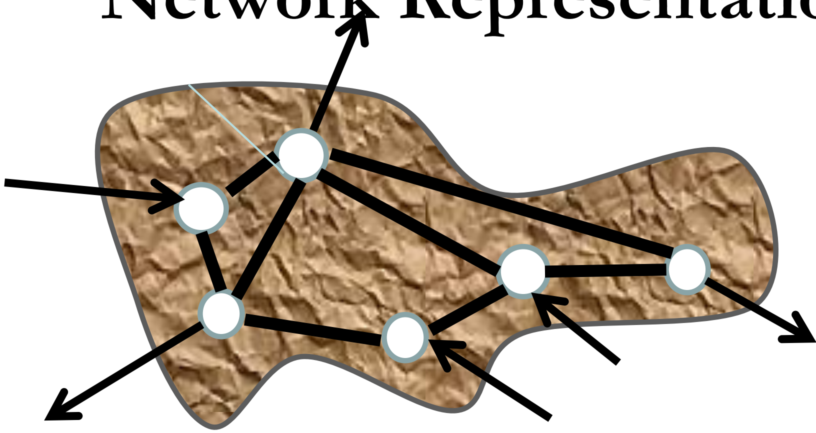
- *Distributed Sensing and Control*
- *Stability along trajectories (Poincare Lemma)*



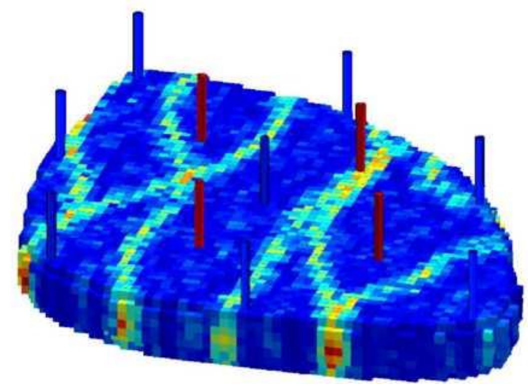
PPG, Pittsburgh



Network Representation of Petroleum Reservoirs



continuous domain



$$\partial_t z_i + \partial_{x_k} f_{ik} = \sigma_i, \qquad i = 1,\dots,n, \qquad \text{conservation laws}$$

$S(z) \in C^1$ concave homogeneous degree 1

potentials: $w = \partial_z S$

driving force: $W = \partial_{x_k} w$

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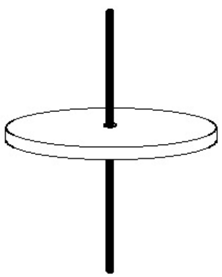
Networks of Spinning Discs

Spin glasses, phase transition, Hamiltonian systems (broken symmetry), ..

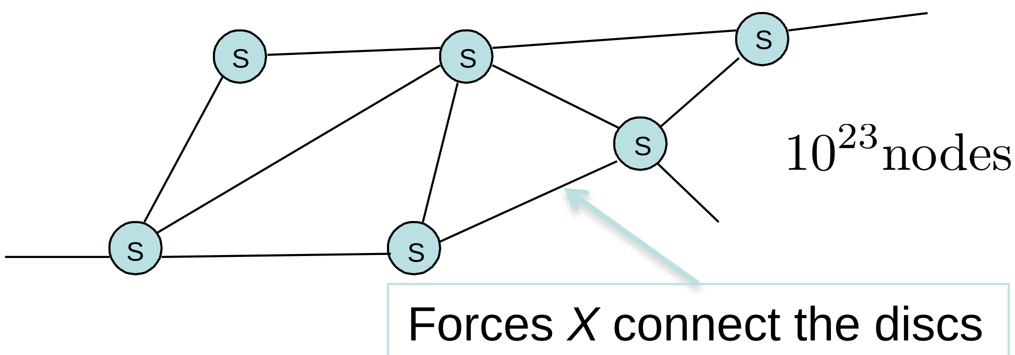
$$dS = dZ^T w, \qquad Z = (U, V, N, x)^T, \quad w = \frac{1}{T}(1, P, \mu, \xi)^T$$



$$\text{Kinetic Energy: } \frac{1}{2} J\omega^2$$

$$\text{Speed: } \omega$$

$$S = -\frac{2}{3}\sqrt{\frac{2}{J}} Z^{3/2}$$

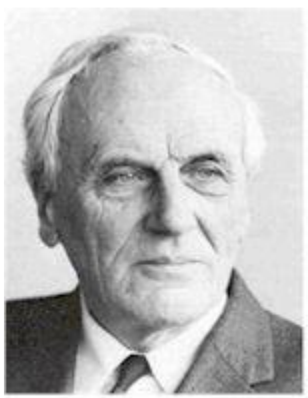


10^{23}nodes

D.H. Tellegen

1900-1990

Orthogonality of Voltage and current flow

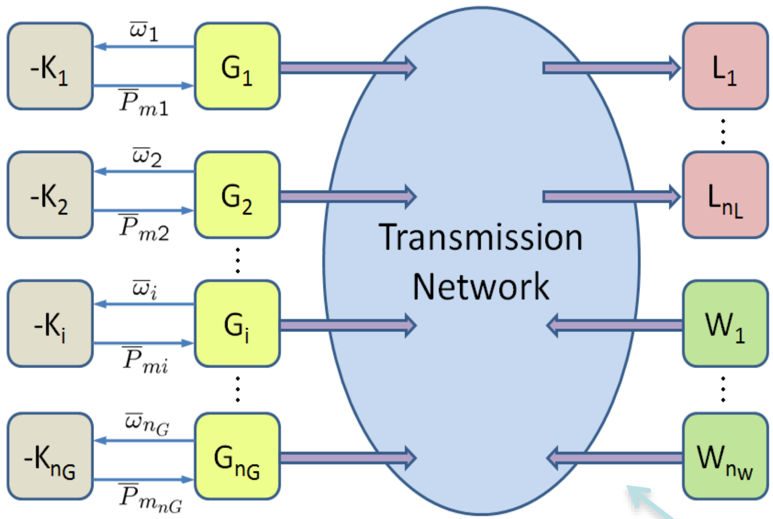


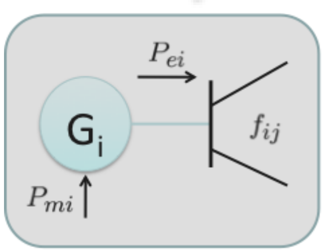
Tellegen's Theorem

$$\frac{d}{dt}A = \sum_{\text{nodes}} p^T w + \sum_{\text{connections}} f^T X + \sum_{\text{terminals}} f^T w$$

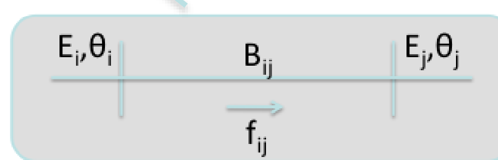
Application to the Grid

Juhua Liu (ABB), Bruce Krogh (ECE)





$$
\begin{aligned}
\dot{\theta}_i &= \omega_s(\omega_i - \omega^*) \\
J_i\dot{\omega}_i &= T_{mi} - T_{ei} - D_i\left(\omega_i - \omega^*\right) \\
P_{mi} &= T_{mi}\omega_i \qquad P_{ei} = T_{ei}\omega_i \\
P_{ei} &= \sum_{j=1, j\neq i}^{n} f_{ij}
\end{aligned}
$$

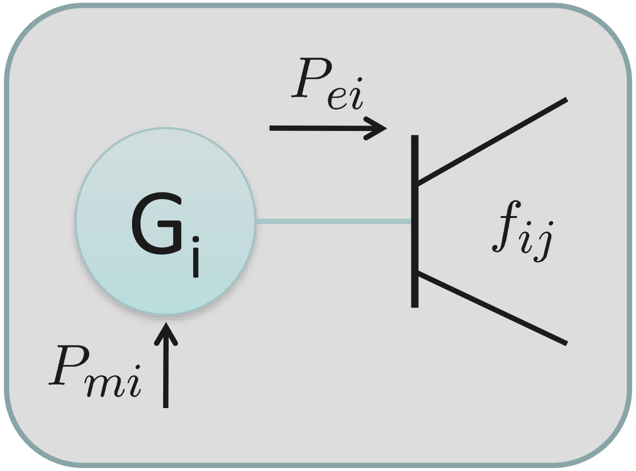


$$f_{ij} = E_i E_j B_{ij} \sin\left(\theta_i - \theta_j\right)$$

$$f_{ij} \approx E_i E_j B_{ij}\left(\theta_i - \theta_j\right)$$

$$f_{ij} = -f_{ji}$$

Generator Model: Swing Equations



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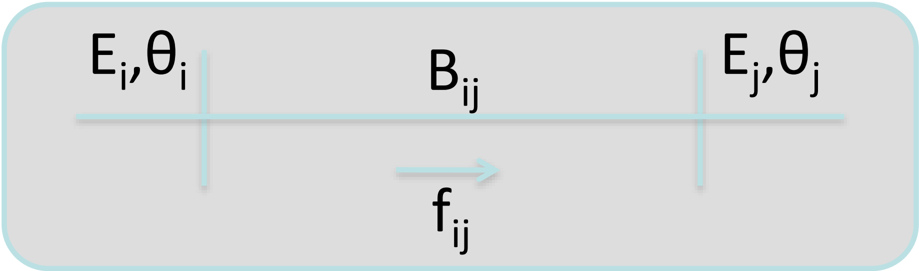
Network Model: Algebraic Equations

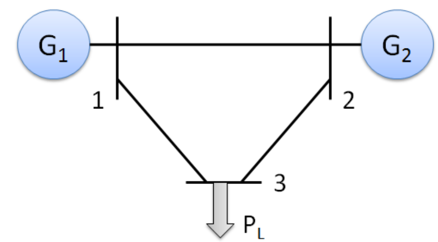


$$f_{ij} = E_i E_j B_{ij} \sin\left(\theta_i - \theta_j\right)$$

$$f_{ij} \approx E_i E_j B_{ij} \left(\theta_i - \theta_j\right)$$

$$f_{ij} = -f_{ji}$$

- KCL Conservation Laws:
  - Generator bus: $\frac{d\overline{Z}_i}{dt} = \overline{P}_{mi} - D_i\omega_i\overline{\omega}_i - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}$
  - Load bus: $\frac{d\overline{Z}_i}{dt} = -\overline{P}_{Li} - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}$
  - Wind bus: $\frac{d\overline{Z}_i}{dt} = \overline{P}_{Wi} - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}$



- KVL: $\sum_{loop} \left(\overline{\omega}_i - \overline{\omega}_j\right) = 0$
- By *Tellegen's theorem*: $\frac{dW}{dt} = \sum_{i=1}^{n} \overline{\omega}_i \frac{d\overline{Z}_i}{dt}$

$$\frac{dW}{dt} = \sum_{i=1}^{n_G} \overline{\omega}_i \overline{P}_{mi} + \sum_{i=1}^{n_W} \overline{\omega}_i \overline{P}_{Wi} - \sum_{i=1}^{n_L} \overline{\omega}_i \overline{P}_{Li} - \sum_{i=1}^{n_G} D_i \omega_i \overline{\omega}_i^2 - \sum_{j>i}^{n} \sum_{i=1}^{n} (\overline{\omega}_i - \overline{\omega}_j) \overline{f}_{ij}$$


Ceci n'est pas une pipe.