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# DISTRIBUTED GENERATORS AND VOLTAGE STABILITY

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# MOTIVATION

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- **Distributed generation (PV, Wind, Gas):**
  - **Increased reliability**
  - **Unconventional operating states**
- **Distributed control of active and reactive power:**
  - **New generation of OPF-type algorithms**
  - **Mostly static analysis, implicit assumption of static and transient stability**



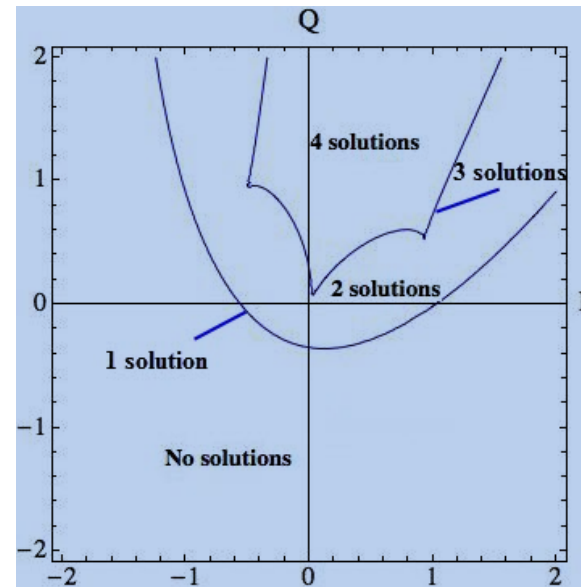
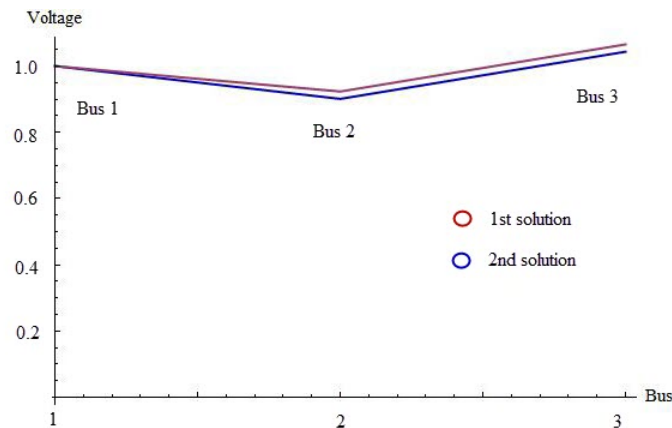
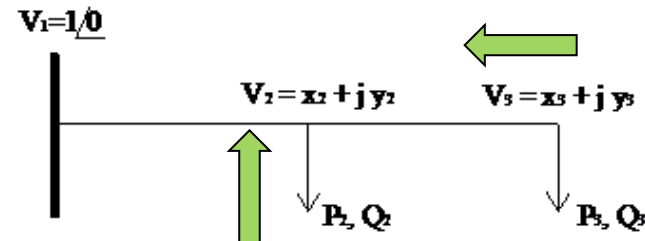
# OUTLINE

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- **Distributed generators' effects on distributed networks**
- **System equations and dynamic load modeling**
- **Dynamic stability criterion based on static solutions**
- **Dynamic simulations**
  - Trapped at the lower branch

# Power reversal creates new regimes

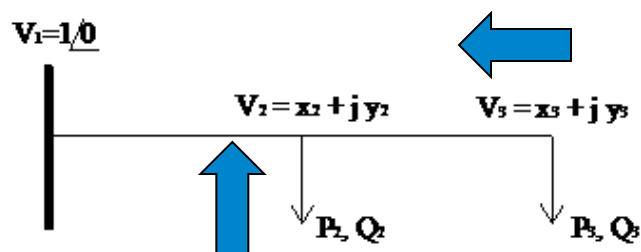
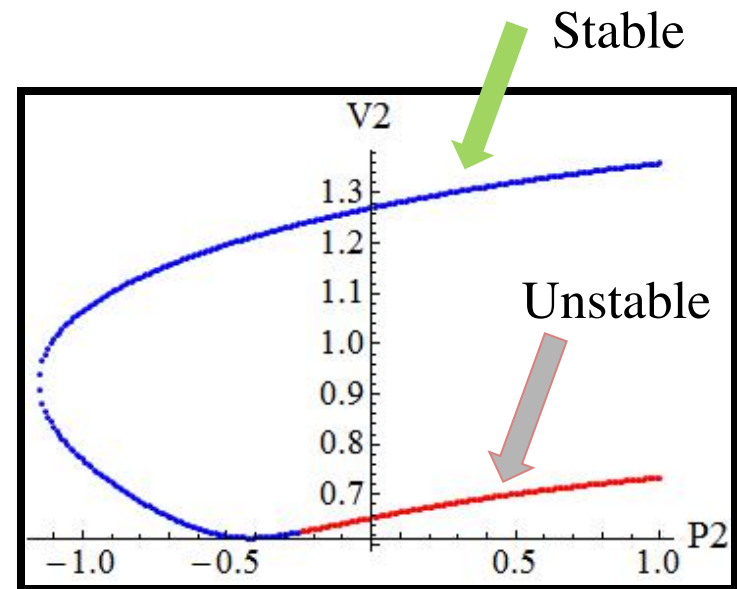
- Multiple power flow solutions
  - High voltage solutions complies with voltage standards
- New protection and control systems are required



unit: p.u.  
 + : consuming  
 - : injecting

# Power reversal and load dynamics affect the stability properties

- Unstable equilibrium may appear on the upper branch of the nose curve; whereas, new stable ones may exist at the lower branch.
- The diversity of load dynamics with the presence of DGs may jeopardize the voltage security of the system.



# DAE of the system

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$$\begin{aligned}\dot{x} &= f(x, y, p) \\ 0 &= g(x, y, p)\end{aligned}$$

x: states  
y: algebraic variables  
p: parameters

- Slow dynamics vs. fast dynamics
- Algebraic equations
  - Load flow equation vs. Kirchhoff laws: typically, the algebraic equations are considered as power flow equations.

# Dynamic load modeling

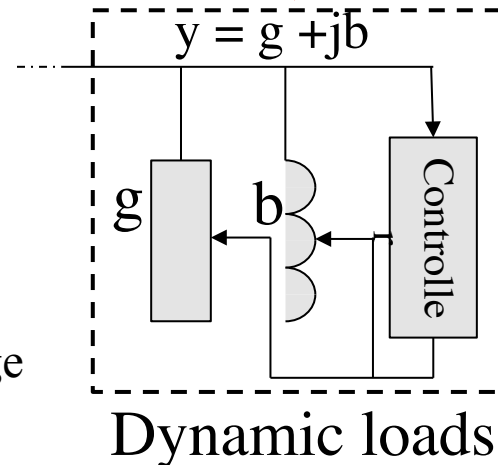
$$\begin{aligned} \dot{g} &= f_g(g, P, U) \\ \dot{b} &= f_b(b, Q, U) \end{aligned}$$



$$\begin{aligned} \tau_1 \frac{\dot{g}}{g} &= -f_1(|V|^2) \frac{p - P^0}{P^0} \\ \tau_2 \frac{\dot{b}}{b} &= -f_2(|V|^2) \frac{q - Q^0}{Q^0} \end{aligned}$$



- Various electric loads and regulators are designed to consume a fixed level of powers in order to achieve the desired performance.
  - Voltage regulators
  - Tap changing transformers
  - Thermostats ...
- Load dynamics are the driving force for voltage instability [1].



# Dynamic stability criterion based on static solutions

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- The system stability depends on:
  - Individual component stability, e.g. individual load
  - Connective stability
- Under normal fixed voltage condition, the loads are stable. Therefore, the connective stability determines the system stability.



# Dynamic stability criterion based on static solutions

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- Steady state

$$U_s = U_s(y) \quad \text{where } U = V^2$$

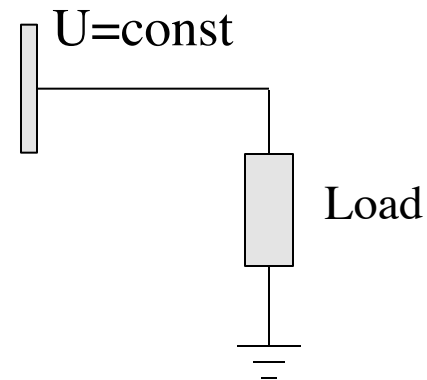
- Equilibria of individual load

$$y = y_s(U)$$

- Z constant:  $y_s = 1/Z$
- I constant:  $y_s = I/\sqrt{U} = I/V$
- P constant:  $y_s = P/U$

- Load dynamics

$$\dot{y} = F(y, U_s(y))$$



# Dynamic stability criterion based on static solutions

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- Load dynamics

$$\dot{y} = F(y, U_s(y))$$

- Equilibrium:  $y_0$

- Linearize about the equilibrium

- $$\begin{aligned}\delta\dot{y} &= \nabla_y F \delta y + \nabla_U F \cdot \nabla_y U_s \delta y \\ &= \nabla_y F (1 + \underbrace{(\nabla_y F)^{-1} \nabla_U F \cdot \nabla_y U_s}_{\nabla_U y_s}) \delta y\end{aligned}$$



$$\delta\dot{y} = \nabla_y F (1 - \nabla_U y_s \cdot \nabla_y U_s) \delta y$$



# Dynamic stability criterion based on static solutions

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$$\delta \dot{y} = \nabla_y F (1 - \nabla_U y_s \cdot \nabla_y U_s) \delta y$$

Load flow solutions:

$$y = y_s(U_s(y))$$

$$h(y) = y - y_s(U_s(y)) = 0$$

Jacobian matrix:

$$J_s = \nabla_y h = 1 - \nabla_U y_s \cdot \nabla_y U_s$$



$$\delta \dot{y} = \nabla_y F \cdot J_s \cdot \delta y$$

# Dynamic stability criterion based on static solutions

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$$\delta \dot{y} = \nabla_y F \cdot J_s \cdot \delta y$$

Each load is stable:  $\nabla_y F = -\Lambda$  where  $\Lambda > 0$ .

Let:  $\Lambda = R^T R$ . then  $\delta y = \bar{R}^T \bar{X}$

Lyapunov stability condition: the system is stable iff there exists  $K = K^T > 0$  s.t.

$$KRJR^T + RJ^T R^T K > 0$$

$$\Rightarrow \begin{aligned} R(R^{-1}KRJ + J^T R^T KR^{-1})R^T &> 0 \\ R^{-1}KRJ + J^T R^T KR^{-1} &> 0 \end{aligned}$$



# Dynamic stability criterion based on static solutions

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$$\underbrace{R^{-1}KRJ + J^T R^T K R^{-T}}_{\tilde{K}} > 0$$

$\tilde{K}$

$$\Rightarrow \tilde{K}J_s + J^T \tilde{K}^T > 0 \quad (*)$$

Statement: if  $J_s + J_s^T > 0$  then (\*) is true with  $\tilde{K} = 1$ ,



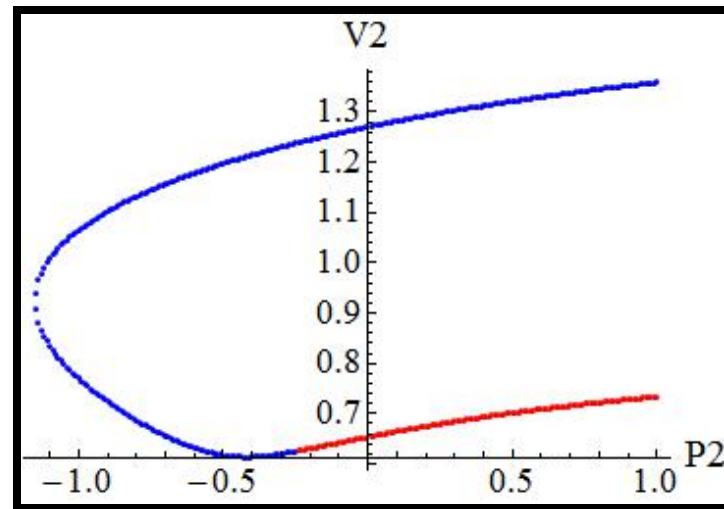
Stability criterion relies only on load flow solutions.



# Dynamic stability criterion based on static solutions

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- Simulation results



# Trapped at the lower branch

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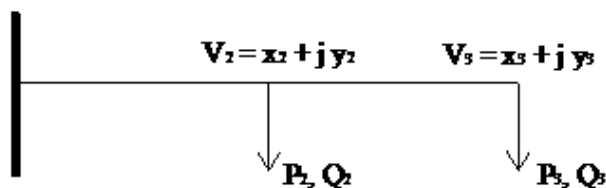
- Under typical scenarios such as when subjected to disturbances or shedding loads, the system may be trapped at a equilibrium on the lower branch without violating any voltage constraints.
- New policies for DGs should be introduced to prevent the system gets stuck at the lower branch.

# Trapped at the lower branch

## A 3-bus network case

- Consider a 3-bus network having two dynamic loads based on the IEEE Standard 4-Node Test Feeder [2].

$$\mathbf{V}_1 = 1 \angle 0$$



$$G = \begin{bmatrix} 2.04 & -1.02 \\ -1.02 & 1.02 \end{bmatrix} \quad B = \begin{bmatrix} -7.17 & 3.59 \\ 3.59 & -3.59 \end{bmatrix}$$

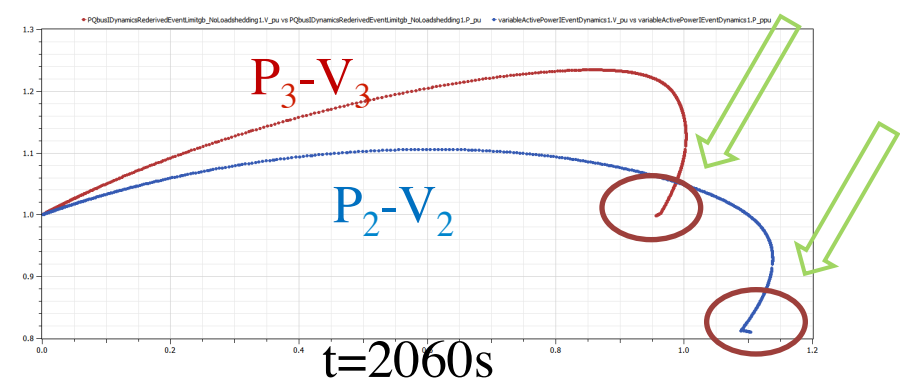
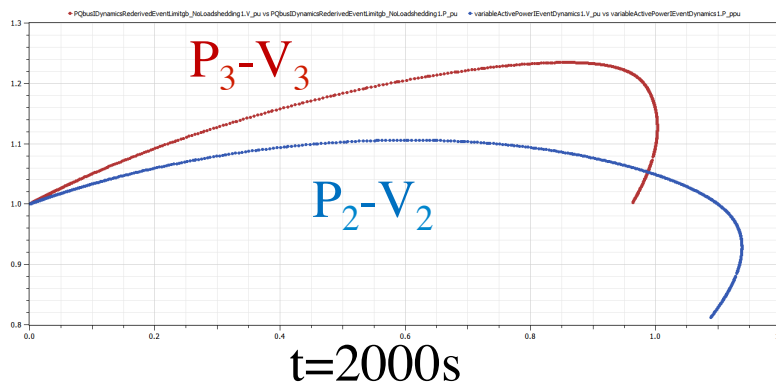
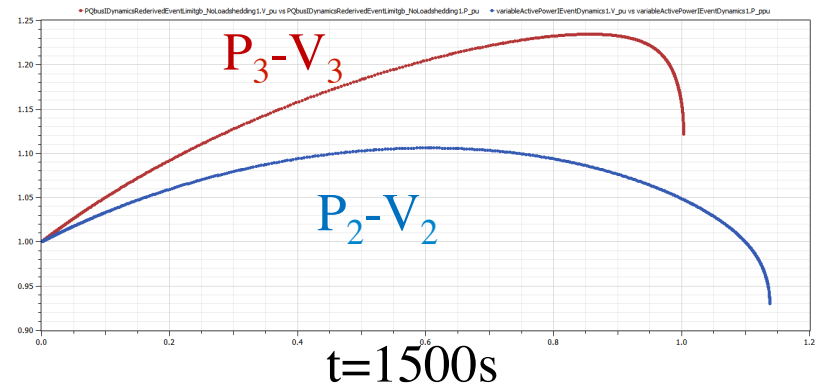
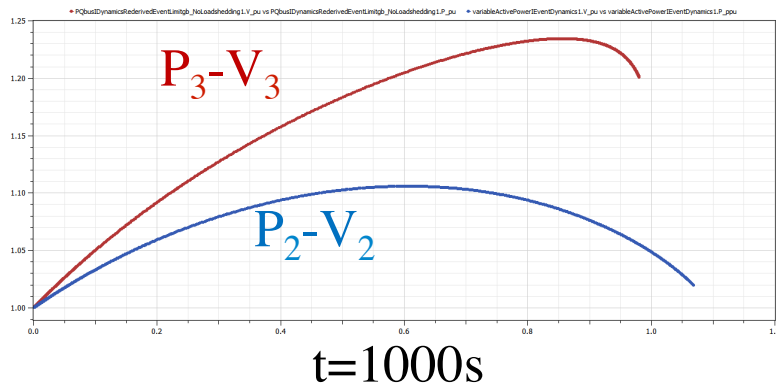
$$\left[ \begin{array}{l} P_2 = -1.095 \\ P_3 = -0.984 \\ Q_2 = -1.328 \\ Q_3 = -1 \\ \tau_1 = \tau_2 = 100\text{s} \\ f_1(|V|^2) = f_2(|V|^2) = 1 \end{array} \right.$$

Voltage	Bus #2	Bus #3
1 <sup>st</sup> solution (high voltage)	1.080	1.200
2 <sup>nd</sup> solution (low voltage)	0.821	1.013

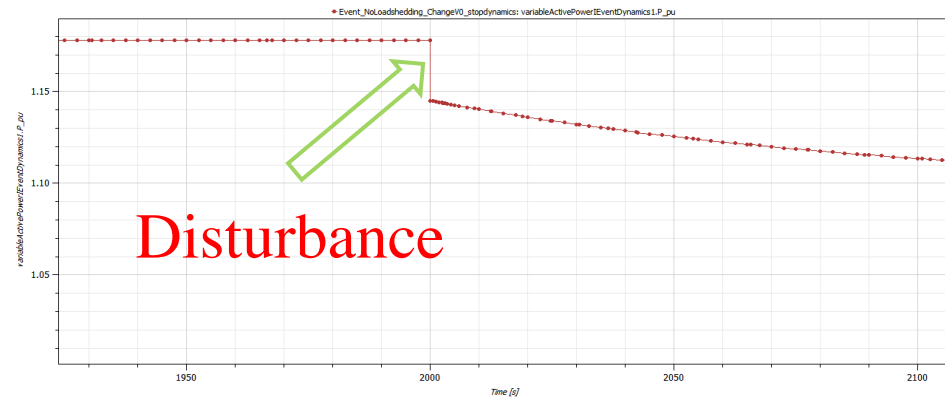
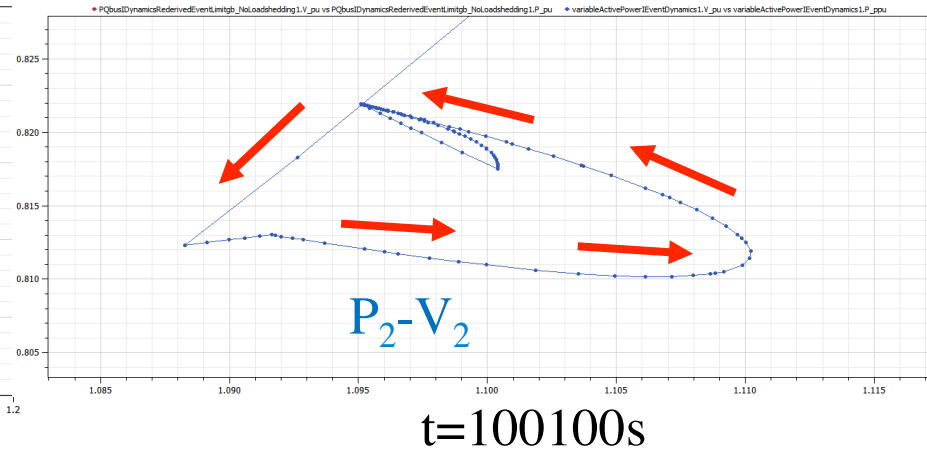
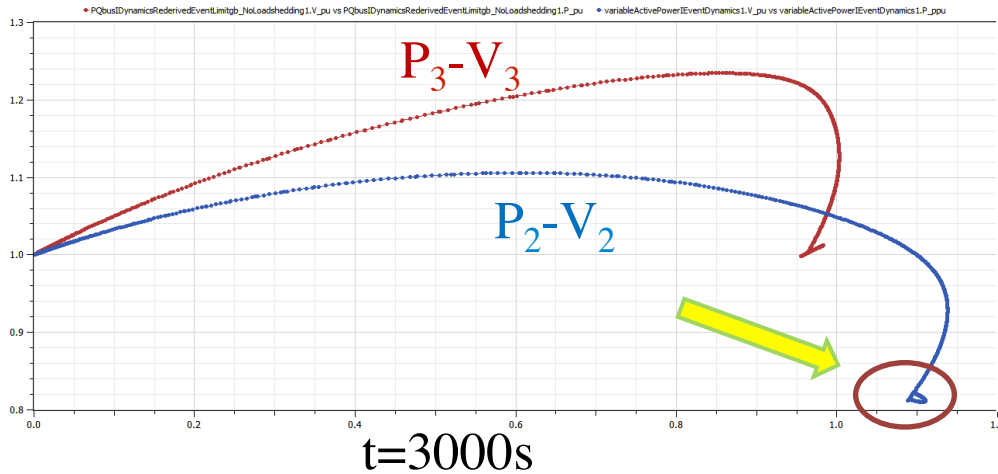


# Trapped at the lower branch

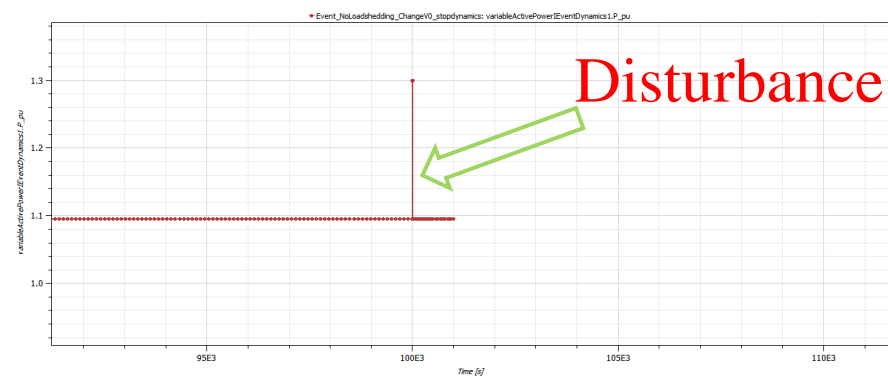
- Dynamic simulations: The nose curves at load bus #2 & #3



# Trapped at the lower branch



Demand at bus #2 at  $t=2000s$

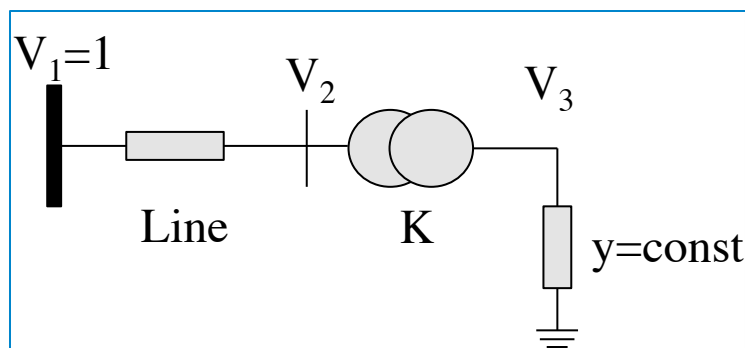


Demand at bus #2 at  $t=100100s$

# Trapped at the lower branch

## A 2-bus network case with a transformer

- Consider a 2-bus network a transformer to control the voltage at bus #2.



The transformer equation:

$$V_3 = K V_2$$

$$\tau_T \dot{K} = -(V_3 - 1) \quad \tau_T = 10 \text{ s}$$

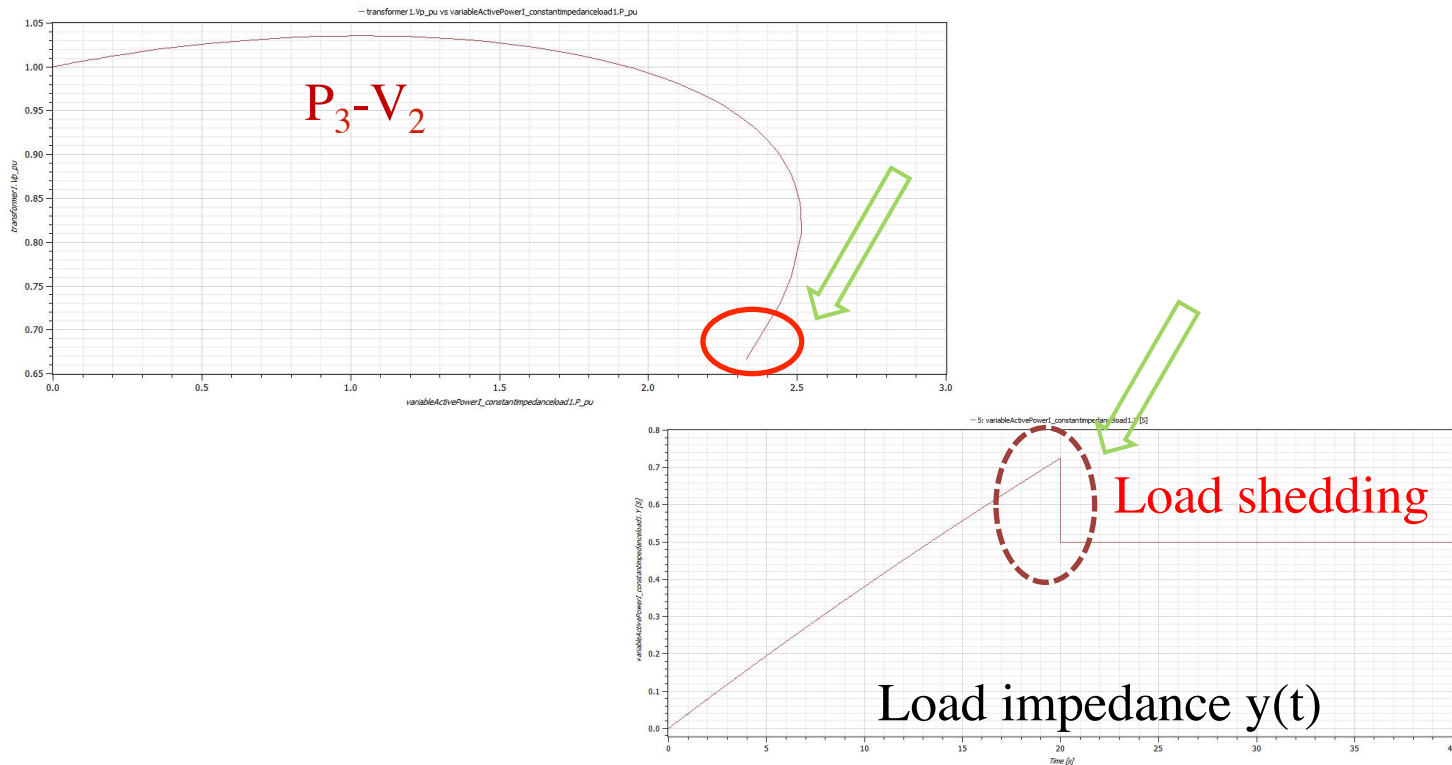
$$\left\{ \begin{array}{l} R=0.072 \\ X=0.258 \\ P_3 = -2.327 \\ Q_3 = 0.033 \end{array} \right.$$

Voltage	Bus #2
1 <sup>st</sup> solution (high voltage)	0.940
2 <sup>nd</sup> solution (low voltage)	0.667

# Trapped at the lower branch

## A 2-bus network case with a transformer

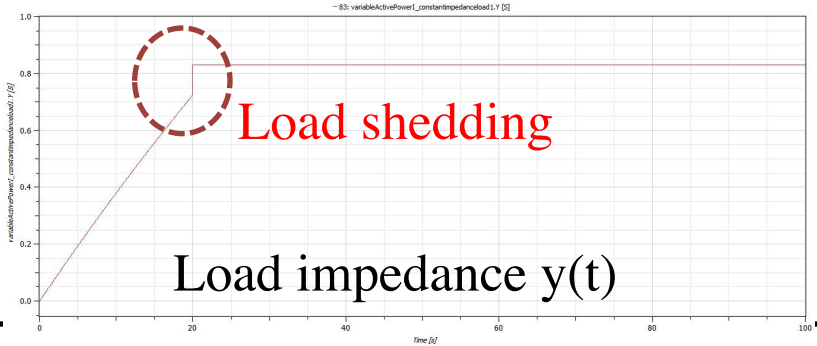
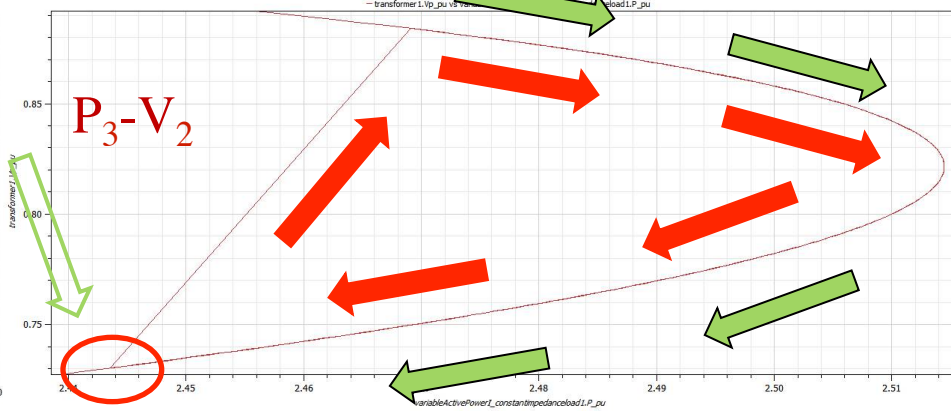
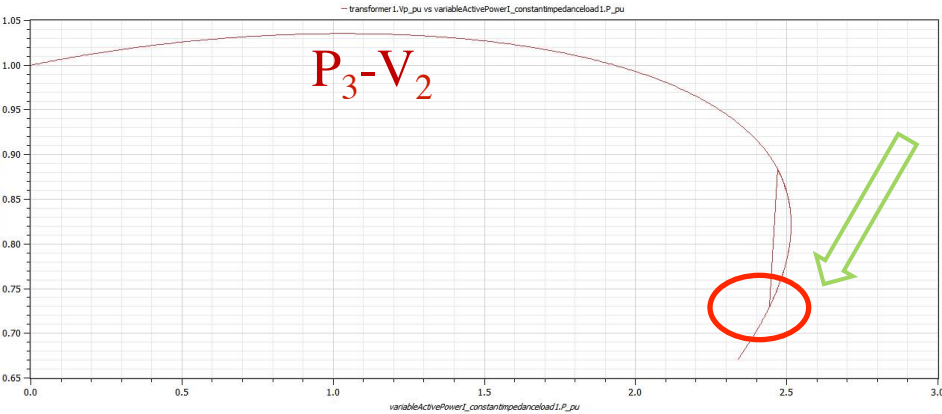
- Dynamic simulations: The nose curves at the load bus



# Trapped at the lower branch

## A 2-bus network case with a transformer

- Dynamic simulations: The nose curves at load bus



# Conclusions

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- The nose curve characteristics and the system stability can change considerably due to power reversal and load dynamics.
- The new stability criterion can be helpful in planning and assessment of the system stability when the load dynamics are unknown in advance.
- Since low voltage solution may be stable, the system may be trapped at the lower branch of the nose curve. In order to prevent such situations, new policies such as to standardize power factors or to regulate reactive power compensation should be introduced.

# Skoltech University

- **Graduate University built by MIT outside of Moscow.**
- **Energy Systems CREI:**
  - **Collaboration between MIT, Caltech, LANL, PNNL and others**
  - **Led by Janusz Bialek**
  - **Multiple faculty and postdoctoral openings**

