A Synergistic Combination of Surrogate Lagrangian Relaxation and Branch-and-Cut for MIP Problems in Power Systems

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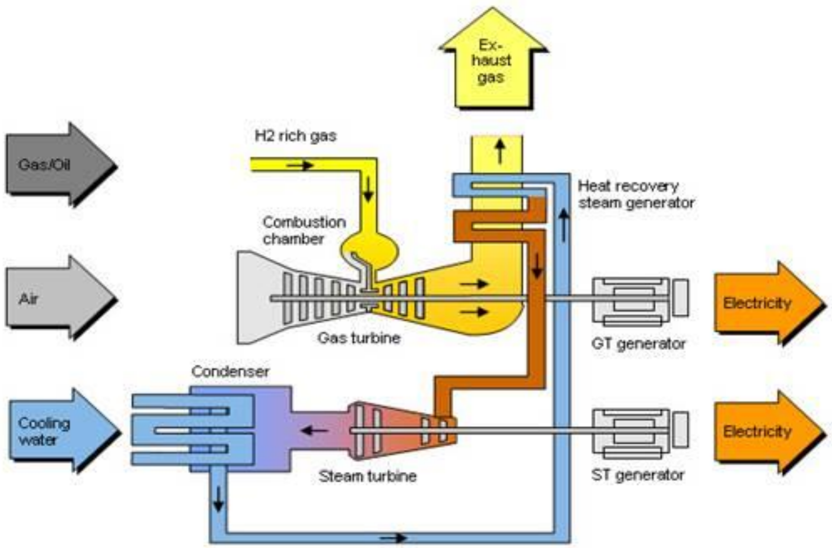
Motivation

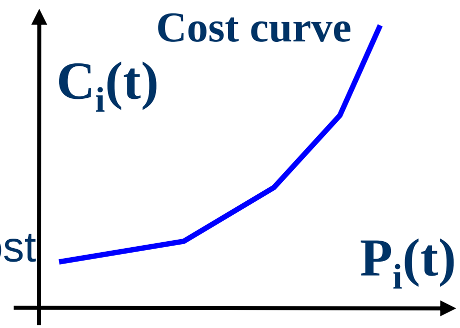
- Unit commitment and economic dispatch (UCED) is to commit units and decide their generation levels to satisfy demand and reserve requirements
- It is modeled as MIP and is computationally intensive especially when:
  - considering combined cycle units



  - considering uncertainties introduced by renewables

Formulation of UCED problem

- How to formulate the problem?



$$\min_{\{p_i(t)\}} J, \text{ with } J \equiv \sum_{t=1}^{T} \sum_{i=1}^{I} \{\underbrace{\boxed{C_i(p_i(t),t)}}_{\text{Generation cost}} + \underbrace{\boxed{S_i(t)}}_{\text{Start-up cost}}\}$$

s.t.:

  - System demand $\sum_{i=1}^{I} p_i(t) = P_d(t)$ and max/min power level constraints
  - Reserve requirements and transmission constraints are ignored for simplicity

- In view of separability, the problem can be decomposed by Lagrangian relaxation
- Since C_i can be converted to a linear function, start-up cost is linear and constraints are linear, the problem can also be solved by branch-and-cut

Solution by Lagrangian Relaxation

- Lagrangian relaxation

~ subproblem

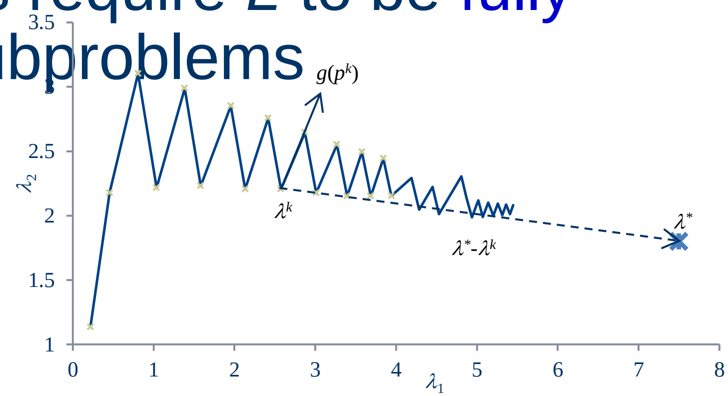
$$L(\lambda, p) \equiv \sum_{t=1}^{T}\sum_{i=1}^{I}\{C_i(p_i(t),t)) + S_i(t)\} + \sum_{t=1}^{T}\left\{\lambda(t)\left(\underbrace{\boxed{P_d(t) - \sum_{i=1}^{I} p_i(t)}}_{\sim\, g(p)}\right)\right\}$$

$$L = \sum_{i=1}^{I}\left\{\underbrace{\boxed{\sum_{t=1}^{T}\left(C_i(p_i(t),t) + S_i(t) - \lambda(t)p_i(t)\right)}}_{\sim\, \text{subproblem}}\right\} + \sum_{t=1}^{T}\lambda(t)P_d(t)$$

- Lagrangian relaxation decomposes *L* into *I* subproblems, and updates λ based on levels of constraints violation

$$\lambda^{k+1}(t) = \lambda^k(t) + c^k g\left(p^k\right); \quad c^k < \frac{q^* - L\left(\lambda^k, p^k\right)}{\left\|g\left(p^k\right)\right\|^2}$$

- Standard subgradient methods require *L* to be fully optimized with respect to all subproblems
  - *L* is difficult to fully optimize
  - λ can suffer from zigzagging
  - Convergence proof requires the optimal dual value *q**



Overview of the Presentation

- Motivation: Unit commitment and economic dispatch
- Surrogate Lagrangian relaxation (SLR)
  - Lagrangian relaxation with surrogate subgradient methods
  - Surrogate Lagrangian relaxation
- Synergistic combination of SLR and Branch-and-cut
  - A brief introduction of branch-and-cut (B&C)
  - Synergistic combination of SLR and Branch-and-cut
- Numerical examples
  - A small illustrative example
  - Large-scale generalized assignment problems
  - UCED with combined cycle units
- Conclusion

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The Surrogate Subgradient Method (1999)

- The surrogate subgradient method allows approximate optimization of *L* s.t. the surrogate optimality condition guaranteeing an acute angle with the direction toward λ^*:

$$L\left(\lambda^{k+1}, x^{k+1}\right) < L\left(\lambda^{k+1}, x^{k}\right)$$

- For UCED, it is sufficient to solve one subproblem:

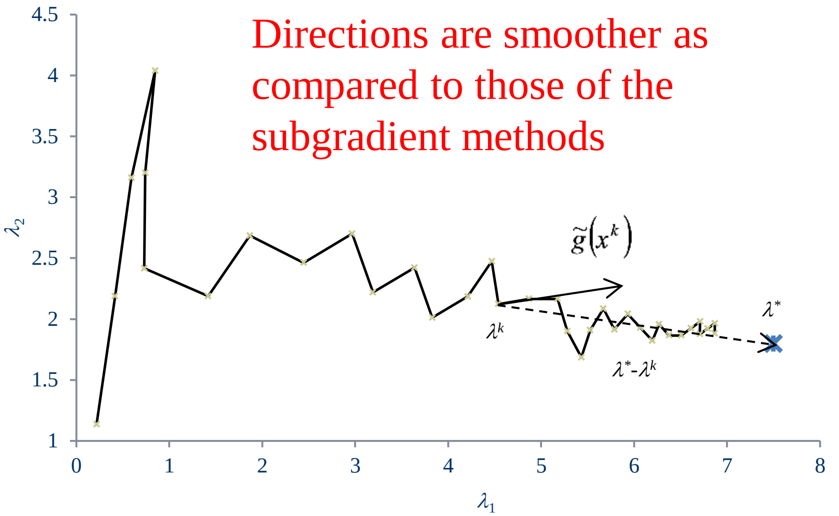
$$\min\left\{\sum_{t=1}^{T}\left(C_i(p_i(t),t) + S_i(t) - \lambda(t)\, p_i(t)\right)\right\}$$

- Multipliers are updated:

$$\lambda^{k+1}(t) = \lambda^{k}(t) + c^{k}\tilde{g}_{\lambda}^{k}$$

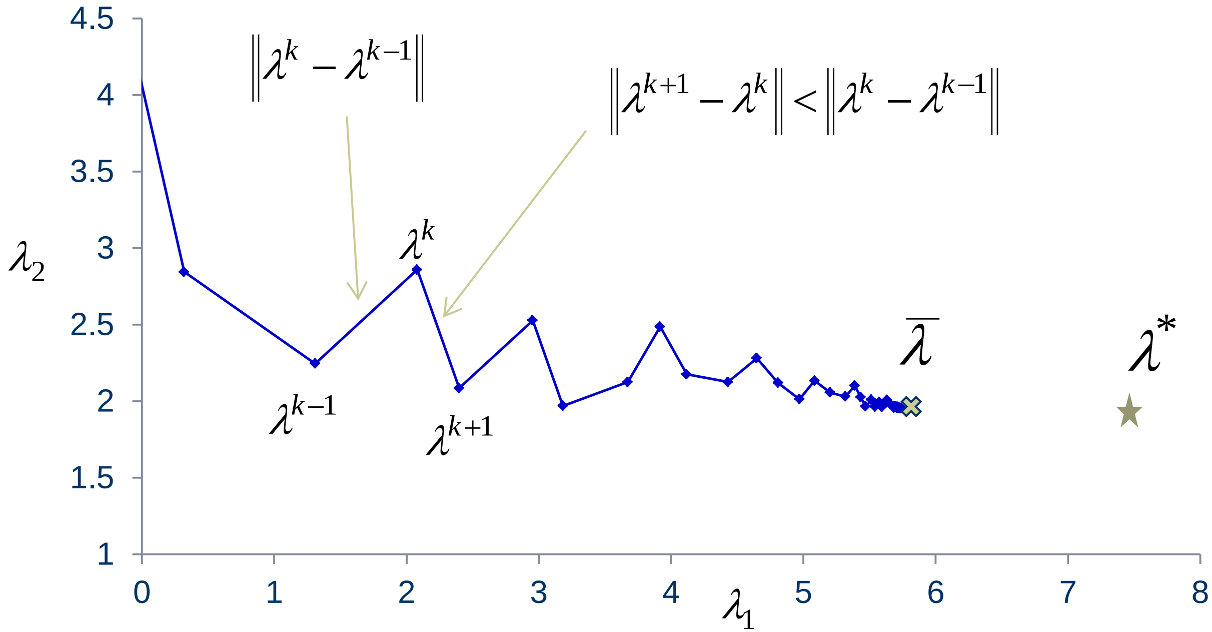
$$\tilde{g}_{\lambda}^{k} = (P_d(t) - \sum_{i=1}^{I} p_i^{k}(t))$$

- Surrogate directions are smooth because now we change only one $p_i(t)$
- Critically needs q^*



Surrogate Lagrangian Relaxation (SLR)

- Main Contribution: Develop a new method, prove convergence, and guarantee practical implementability
  - Without fully optimizing the relaxed problem (s.t. the surrogate optimality condition)
  - Without requiring q^*
- Main Idea 1: Decrease distances between multipliers at consecutive iterations ($\|\lambda^{k+1}-\lambda^k\|$ decreases)
- $\|\lambda^{k+1}-\lambda^k\|$ decreases $\Rightarrow$ fixed-point mapping $\Rightarrow$ $\lambda^k \to \bar{\lambda}$



Surrogate Lagrangian Relaxation

- $$\left.\begin{aligned} \lambda^{k+1} &= \lambda^k + c^k \tilde{g}(x^k) \\ \left\|\lambda^{k+1} - \lambda^k\right\| &= \alpha_k \left\|\lambda^k - \lambda^{k-1}\right\|,\ 0 < \alpha_k < 1, \\ \left\|c^k \tilde{g}(x^k)\right\| &= \alpha_k \left\|c^{k-1} \tilde{g}(x^{k-1})\right\| \end{aligned}\right\} \Rightarrow \boxed{c^k = \alpha_k \frac{c^{k-1}\left\|\tilde{g}(x^{k-1})\right\|}{\left\|\tilde{g}(x^k)\right\|}} \quad (1)$$
- Parameters α_k should satisfy $c^k \sim \boxed{\prod_{i=1}^{k} \alpha_i \to 0}$ (2)
- If α_k are small, $c^k \to 0$ too fast $\Rightarrow$ premature convergence
- Main Idea 2:
  - To avoid premature convergence, c_k should not decrease too fast
  - This can be achieved by keeping α_k sufficiently close to 1

$$\boxed{\lim_{k\to\infty} \frac{1-\alpha_k}{c^k} = 0}$$

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Main Theorem

- Multipliers converge to the optimum λ^* without requiring q^* provided α_k satisfy:

  1) $c^k \sim \prod_{i=1}^{k} \alpha_i \to 0$ (Main idea 1)

  2) $\lim_{k\to\infty} \frac{1-\alpha_k}{c^k} = 0$ (Main idea 2)

  Without requiring q^*!

- One possible example of α_k that satisfies conditions 1) and 2): $\alpha_k = 1 - \frac{1}{M \cdot k^p},\ 0 < p < 1,\ M > 1,\ k = 1, 2, \ldots$
- At convergence, the surrogate dual value approaches the (optimal) dual value q^* ~ valid lower bound on the feasible cost
  - Lower bound is guaranteed before convergence by fully optimizing the relaxed problem to obtain a dual value

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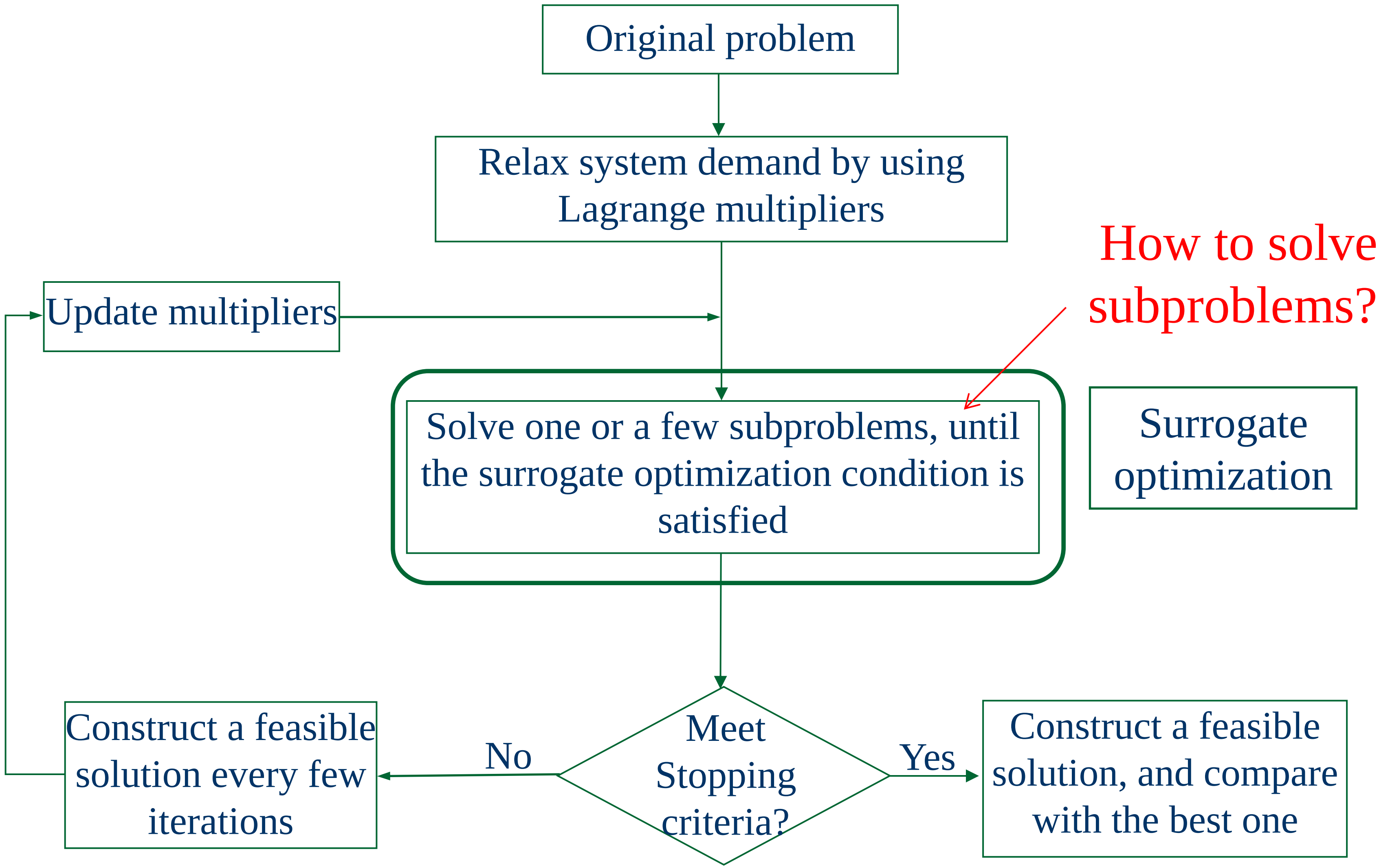
Schematic Flow chart of SLR



Difficulties of Standard Branch-and-Cut

- Branch-and-cut (B&C) can suffer from slow convergence because
  - Facet-defining cuts and even valid inequalities that cut areas outside the convex hull are problem-dependent and are frequently difficult to obtain
  - When facet-defining cuts are not available, a large number of branching operations will be performed
  - No “local” concept $\Rightarrow$ Constraints associated with one subproblem are treated as global constraints and affect the entire problem

Synergistic Combination with Branch-and-cut

- To overcome difficulties, SLR relaxation and B&C are synergistically combined to simultaneously exploit problem separability and linearity:
  - Relax coupling constraints (e.g., system demand)
  - Solve each subproblem by using branch-and-cut with warm start
    - The complexity of each subproblem is much lower than the complexity of the original problem
  - Updating multiplies by using SLR – convergence without requiring q^*
- Why is the new method effective?
  - Cuts for subproblems are more effective as compared to cuts for the original problem
  - Feasible solutions can be effectively obtained
  - The overall algorithm is efficient

Synergistic Combination with Branch-and-cut

- Why cuts for subproblems are more effective?
  - An example of important cuts is Gomory cuts
    - Constraints are linearly combined into one constraint
    - Cuts are then generated by retaining fractional parts of the coefficients (*Gomory's fractional cut*)
  - Cuts for subproblems aggregate much fewer constraints $\Rightarrow$ cut larger regions as compared to regions obtained by original Gormry cuts
  - Other cuts using aggregation follow similar logic
  - Cuts that require no aggregation (e.g., clique and cover cuts) are as efficient for solving subproblems
- Can feasible solutions be efficiently obtained?
  - Linearity of coupling constraints can be exploited to obtain feasible solutions

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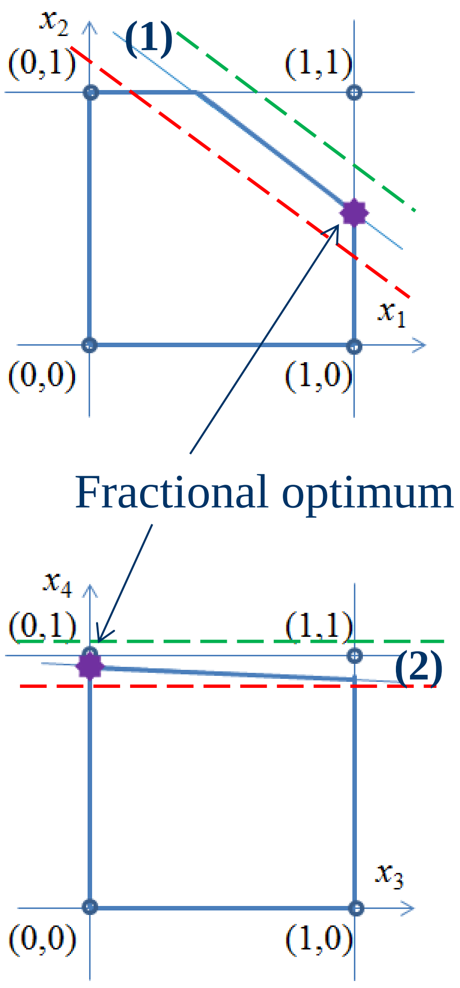
Example Illustrating the Combination of SLR and Branch-and-cut

- Consider a toy problem:
  - *min*$(79x_1+70x_2+108x_3+41x_4)$
    - *s.t.* $35x_1+51x_2\leq 64$ (1), $3x_3+65x_4\leq 64$ (2)
    - $x_1+x_3 = 1$, $x_2+x_4 = 1$
    - $x_j\in\{0,1\}$, $j = 1, 2, 3, 4$
- Gomory cut: $7x_1+10x_2+13x_4 \leq 26$
- Consider subproblems:
  - *min*$(79x_1+70x_2)-\lambda_1 x_1-\lambda_2 x_2$,
    - *s.t.* $35x_1+51x_2\leq 64$, $x_i\in\{0,1\}$, $i = 1, 2$
  - *min*$(108x_3+41x_4)-\lambda_1 x_3-\lambda_2 x_4$
    - *s.t.* $3x_3+65x_4\leq 64$, $x_i\in\{0,1\}$, $i = 3, 4$
- Gomory cuts:
  - $7x_1+10x_2 \leq 12$
  - $13x_4 \leq 12$



Flow-Chart of the Synergistic Approach

Relax coupling constraints to exploit separability (e.g., separate into subproblems)

Use branch-and-cut + warm start to solve each subproblem

Guarantee convergence without requiring q^* by updating stepsizes using Main Theorem:

$$c^k = \alpha_k \frac{c^{k-1}\left\|\tilde{g}\left(x^{k-1}\right)\right\|}{\left\|\tilde{g}\left(x^k\right)\right\|} \quad (1)$$

Update multipliers by using (1) and smooth surrogate subgradients

Are stopping criteria satisfied?

No

Yes

Search for feasible solutions

Stop

Convergence without requiring q^* enables the synergistic combination

Cuts generated for subproblem cut off large areas of sub-convex hull

Zigzagging is alleviated thereby reducing the number of iterations required for convergence

Example Illustrating the Combination of SLR and Branch-and-cut

- Consider the Generalized Assignment Problem:

$$\min_{x_{i,j}} \sum_{i=1}^{I} \sum_{j=1}^{J} c_{i,j} x_{i,j}$$ (Cost of assigning *I* jobs to *J* machines)

$$s.t. \quad \sum_{i=1}^{I} a_{i,j} x_{i,j} \le b_j, \; j = 1, \ldots, J$$ (1) (Time required by the jobs does not exceed the machine's time available)

$$\sum_{j=1}^{J} x_{i,j} = 1, i = 1, \ldots, I$$ (2) (Each job is to be performed on one and one machine only)

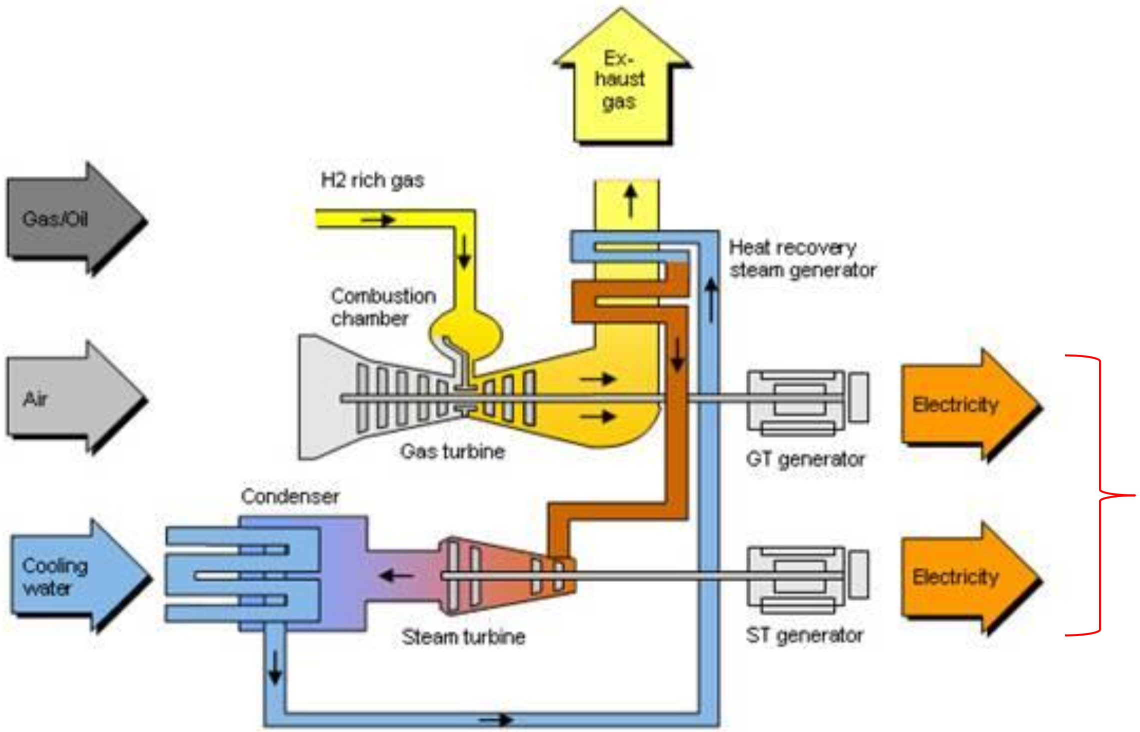
- Constraints (2) can be viewed as constraints coupling "machine subproblems"

Results on Generalized Assignment Problems

Problem Size	Synergistic Combination of SLR and B&C				Branch-and-cut			
	LB	Feasible Cost	GAP (%)	CPU Time (sec)	LB	Feasible Cost	GAP (%)	CPU Time (sec)
15 machines, 900 jobs	55403	55411	0.014438	1380	55401.5	55429	0.049613	1380
40 machines, 400 jobs	24348	24394	0.188571	2100	24348.5	24465	0.47619	2100
20 machines, 1600 jobs	97823	97834	0.011244	1860	97822.1	97895	0.074468	1860

Multi-Stage Combined Cycle Units

- Combined cycle (CC) units are efficient because
  - Heat from gas turbines (GT) is not wasted but is used to for steam turbines (ST)
- However, UCED problem with CC units is difficult
  - ST cannot be turned on if there is not enough heat from GT ⇒ Complicated state transitions causing major challenges



Multi-Stage Combined Cycle Units

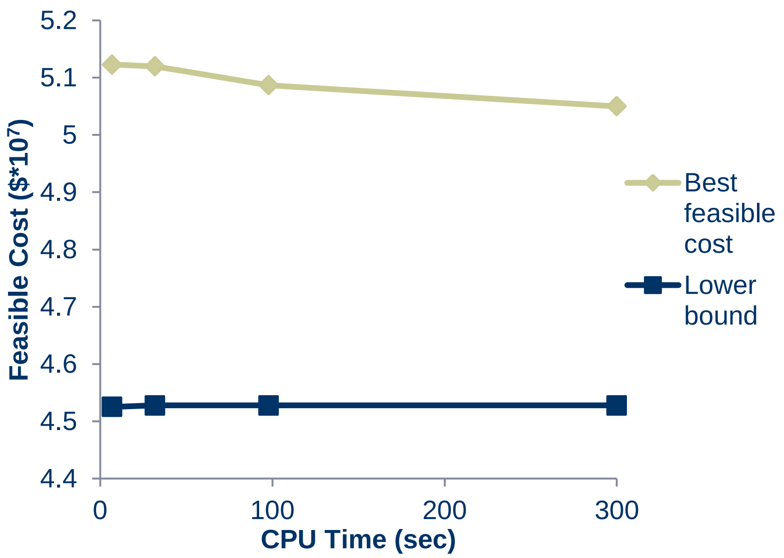
- Difficulty:
  - Constraints modeling transitions between configurations of generators are logical and complex
  - Complex transitions in one such unit affect the entire problem
  - Corresponding convex hull is difficult to obtain
- By using our new method:
  - Transitions of a combined cycle unit are handled locally and no longer affect the entire problem
  - Certain cuts generated for subproblems cut off large areas outside the sub-convex hull
  - Branch-and-cut efficiently optimizes subproblems
  - SLR efficiently coordinate subproblem solutions

Multi-Stage Combined Cycle Units

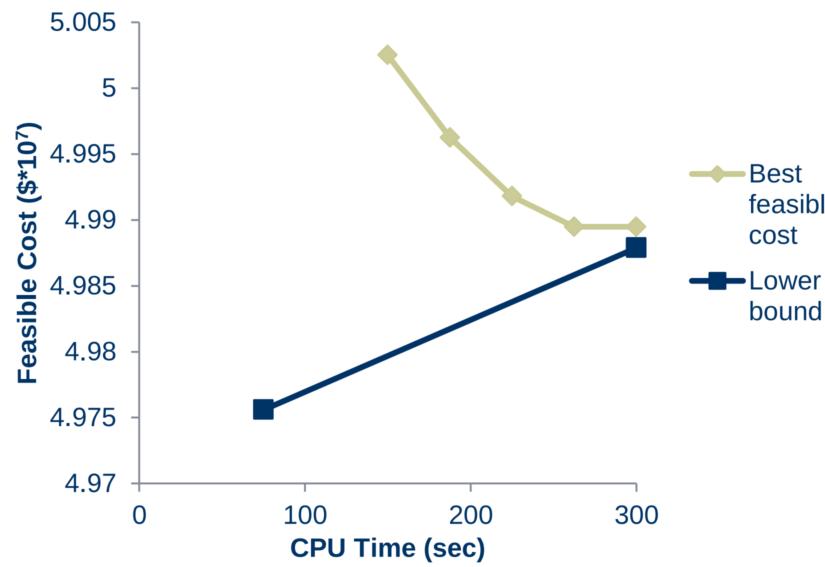
- To demonstrate the efficiency of surrogate Lagrangian relaxation, a problem with 10 CC plants and 300 conventional units is considered

Method	Feasible Cost	Lower Bound	Gap (%)	CPU Time (min)
Branch-and-cut	50,260,500	45,305,200	9.859	30
Our new method	49,894,806	49,879,027	0.032	5

Branch-and-cut



Surrogate Lagrangian relaxation



Conclusion

- Major theoretical result: Within the surrogate Lagrangian relaxation framework, multipliers converge to the optimum without requiring q^*
- SLR has been synergistically combined with B&C to solve mixed-integer programming problems efficiently
  - Subproblem constraints no longer affect the entire problem
  - Gomory cuts generated for subproblems cut off large areas outside the sub-convex hull
- Numerical results demonstrate that the innovative approach is powerful and efficient for solving mixed-integer programming problems
- Broad Impact: The novel methodology opens new directions to efficiently solve mixed-integer programming problems such as Stochastic Unit Commitment and beyond

Related Publications

Thank You!

- X. Zhao, P. B. Luh, and J. Wang, “The Surrogate Gradient Algorithm for Lagrangian Relaxation Method,” *Journal of Optimization Theory and Applications*, Vol. 100, No. 3, March 1999, pp. 699-712.

- M. A. Bragin, X. Han, P. B. Luh, and J. H. Yan, “Payment Cost Minimization Using Lagrangian Relaxation and Modified Surrogate Optimization Approach,” *Proceedings of the IEEE Power and Energy Society 2011 General Meeting*, Detroit, Michigan, July 2011.

- M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, X. Han and G. A. Stern, “An Efficient Surrogate Subgradient Method within Lagrangian Relaxation for the Payment Cost Minimization Problem,” *Proceedings of the IEEE Power and Energy Society 2012 General Meeting*, San Diego, California, July 2012, pp. 4055-4060.

- M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu and G. A. Stern, “Efficient Surrogate Optimization for Payment Cost Co-Optimization with Transmission Capacity Constraints,” *Proceedings of the IEEE Power and Energy Society 2013 General Meeting*, Vancouver, Canada, July 2013.

- P. B. Luh, M. A. Bragin, Y. Yu, J. H. Yan, G. A. Stern and N. Yu, “A Synergistic Combination of Surrogate Lagrangian Relaxation and Branch-and-Cut for MIP Problems in Power Systems,” Federal Energy Regulatory Commission Conference - Increasing Real-Time and Day-Ahead Market Efficiency through Improved Software, June 2013, Washington D.C.