

# Load-side Frequency Control

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# Outline

Motivation

Dynamic network model

Load-side frequency control

Simulations

Zhao, Topcu, Li, Low, TAC 2014  
Mallada, Low, 2013





# Motivation: frequency control

## Synchronous network

- All buses synchronized to same nominal frequency (US: 60 Hz)
- Supply-demand imbalance → frequency fluctuation

## Frequency regulation

- Generator based
- Frequency sensitive (motor-type) loads

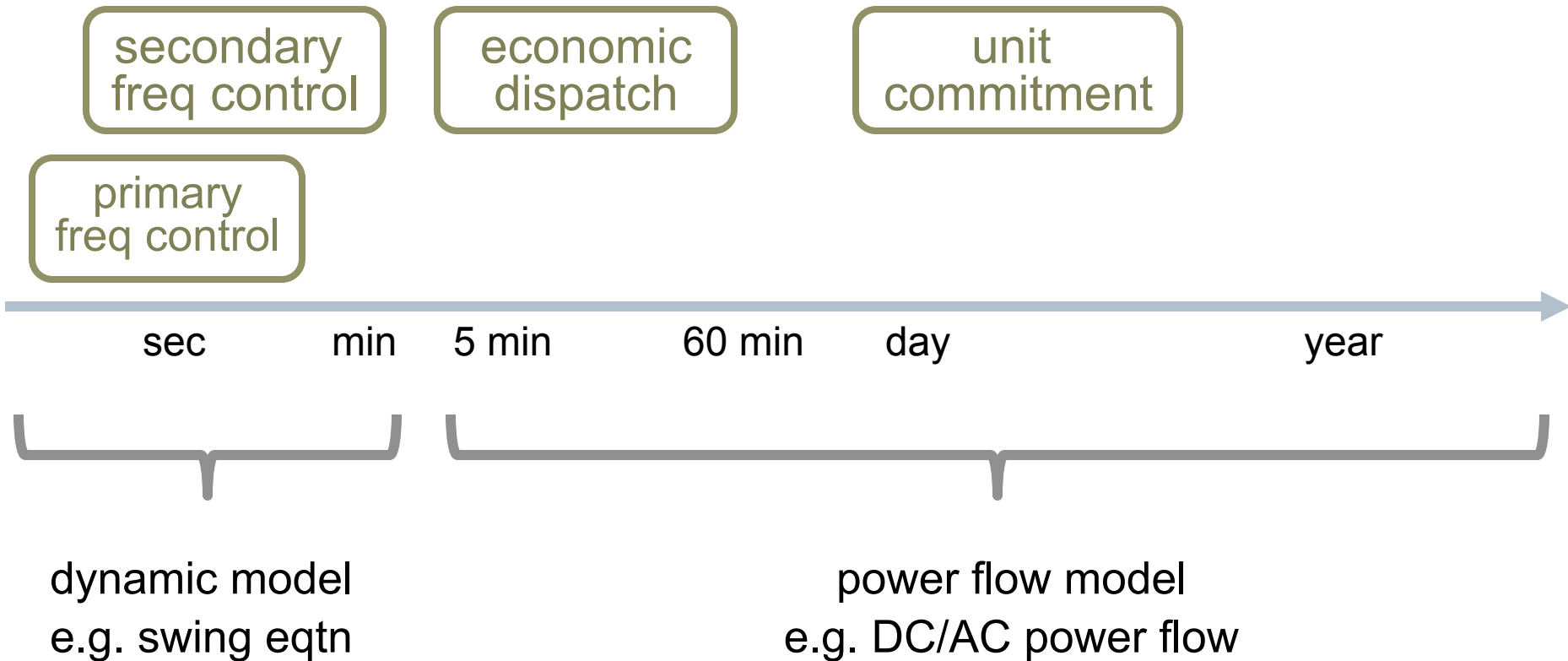
## Controllable loads

- Do not react to frequency deviation
- ... but intelligent
- Need active control – how?



# Frequency control

Frequency control is traditionally done on generation side





# Advantages of load-side control

Distributed loads can supplement generator-side control

- faster (no/low inertia!)
- no waste or emission
- more reliable (large #)
- localize disturbance

secondary  
freq control

primary  
freq control

sec

min

5 min

60 min

day

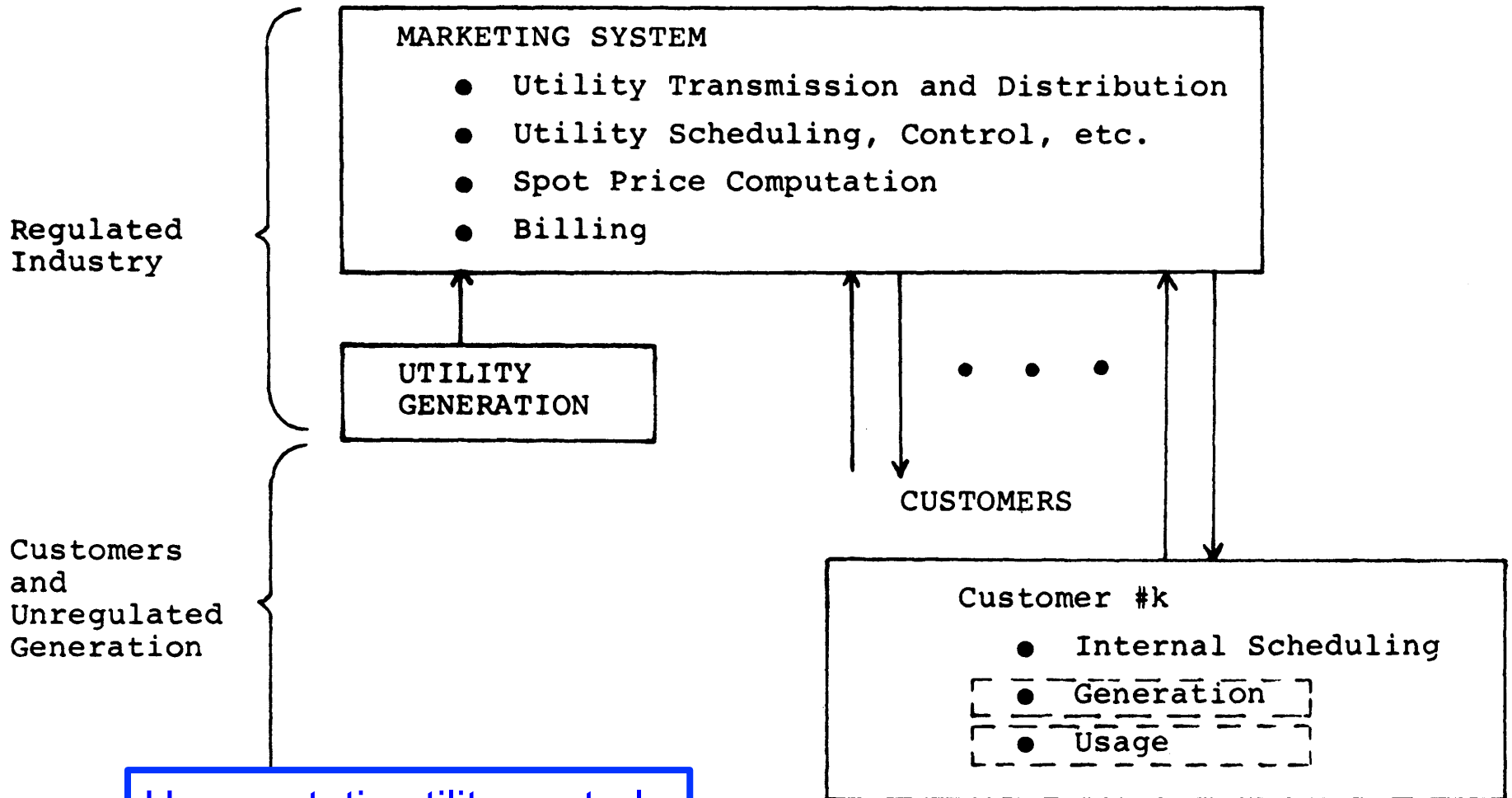
year

dynamic model  
e.g. swing eqtn

It's about supply-demand balance, but synchronous frequency helps



# Idea dates back to 1970s



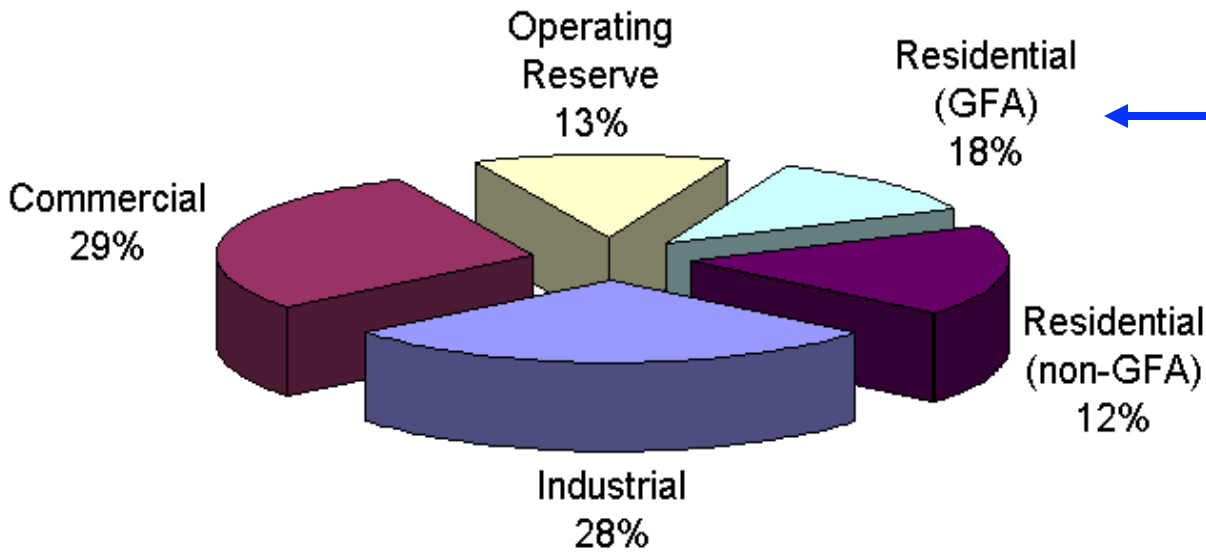
## Homeostatic utility control :

- freq adaptive loads
- spot prices
- IT infrastructure

Schweppe et al (1979, 1980)

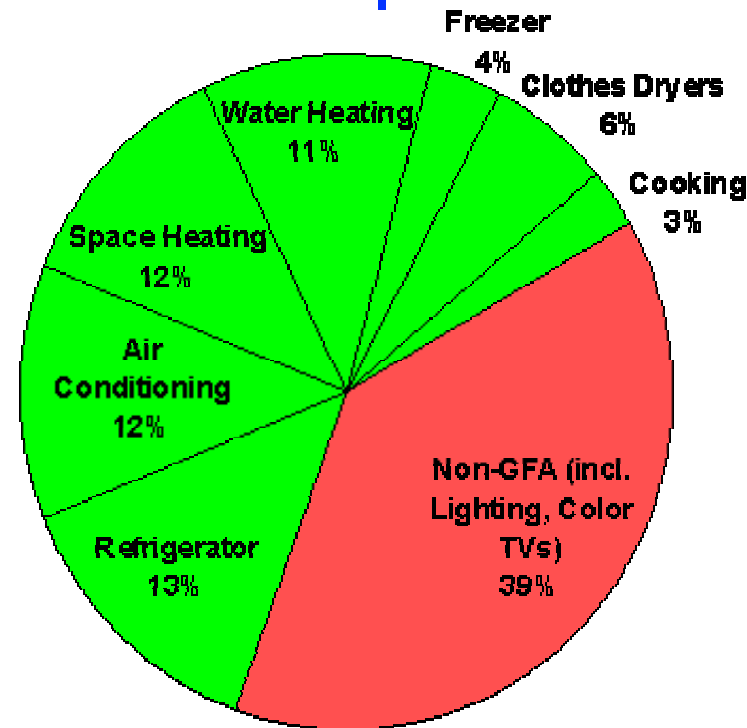


# Potential benefit



- Residential load accounts for ~1/3 of peak demand
- 61% residential appliances are Grid Friendly

US:  
operating reserve: 13% of peak  
total GFA capacity: 18%





# Small demo: PNNL

## PNNL Grid Friendly Appliance Demo Project (early 2006 – March 2007)

- 150 clothes dryers, 50 water heaters
- Under-frequency threshold: 59.95 Hz (0.08% dev)
- 358 under-freq events during project, lasting secs – 10 mins
- All GFA detected events correctly and loads shedded as designed, despite wide geographical distribution
- Survey reported no customer inconvenience

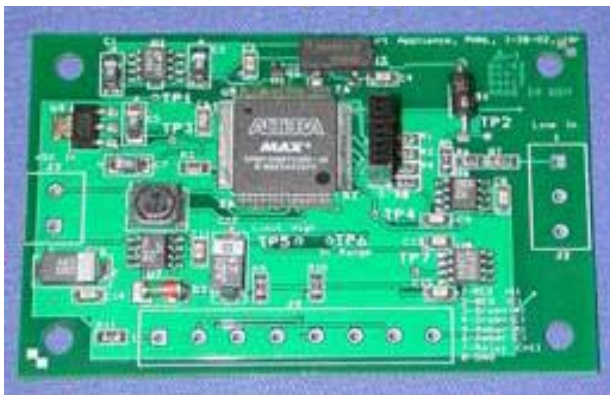
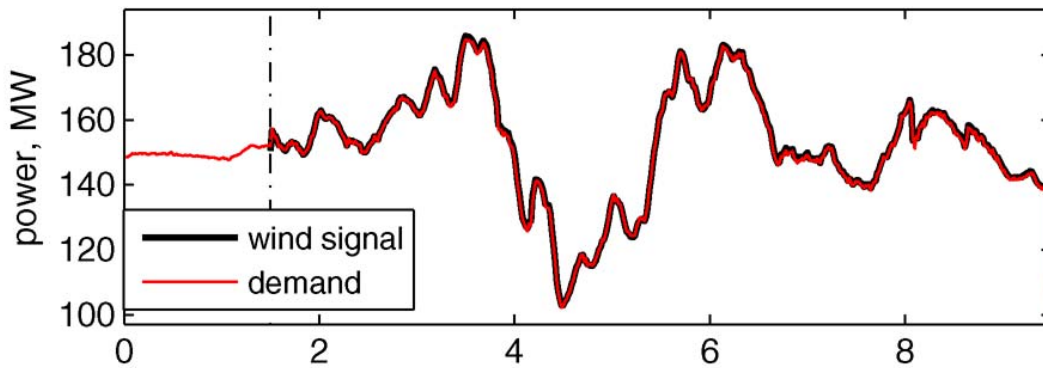


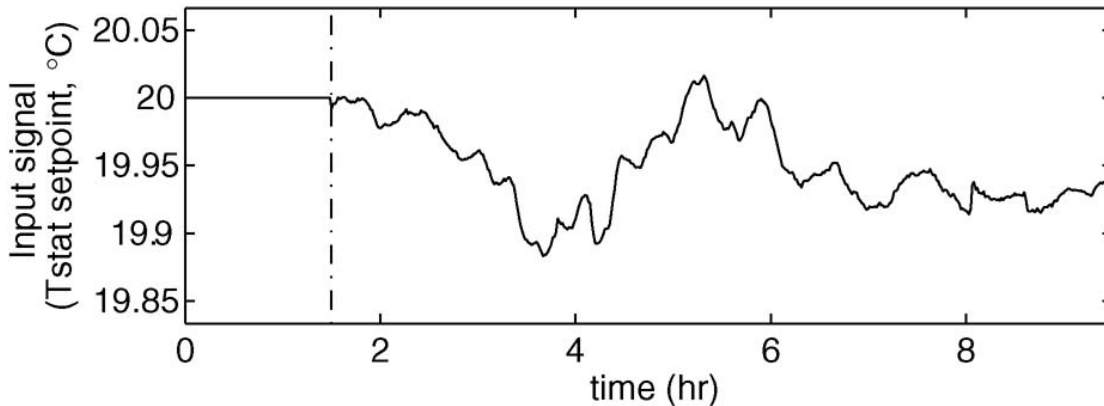
Figure 1.3. GFA Controller Board used in the Grid Friendly Appliance Project





Can household Grid Friendly appliances follow its own PV production?

- 60,000 AC
- avg demand ~ 140 MW
- wind var: +/- 40MW
- temp var: 0.15 degC



Dynamically adjust thermostat setpoint

**Fig. 7.** Load control example for balancing variability from intermittent renewable generators, where the end-use function—in this case, thermostat setpoint—is used as the input signal.



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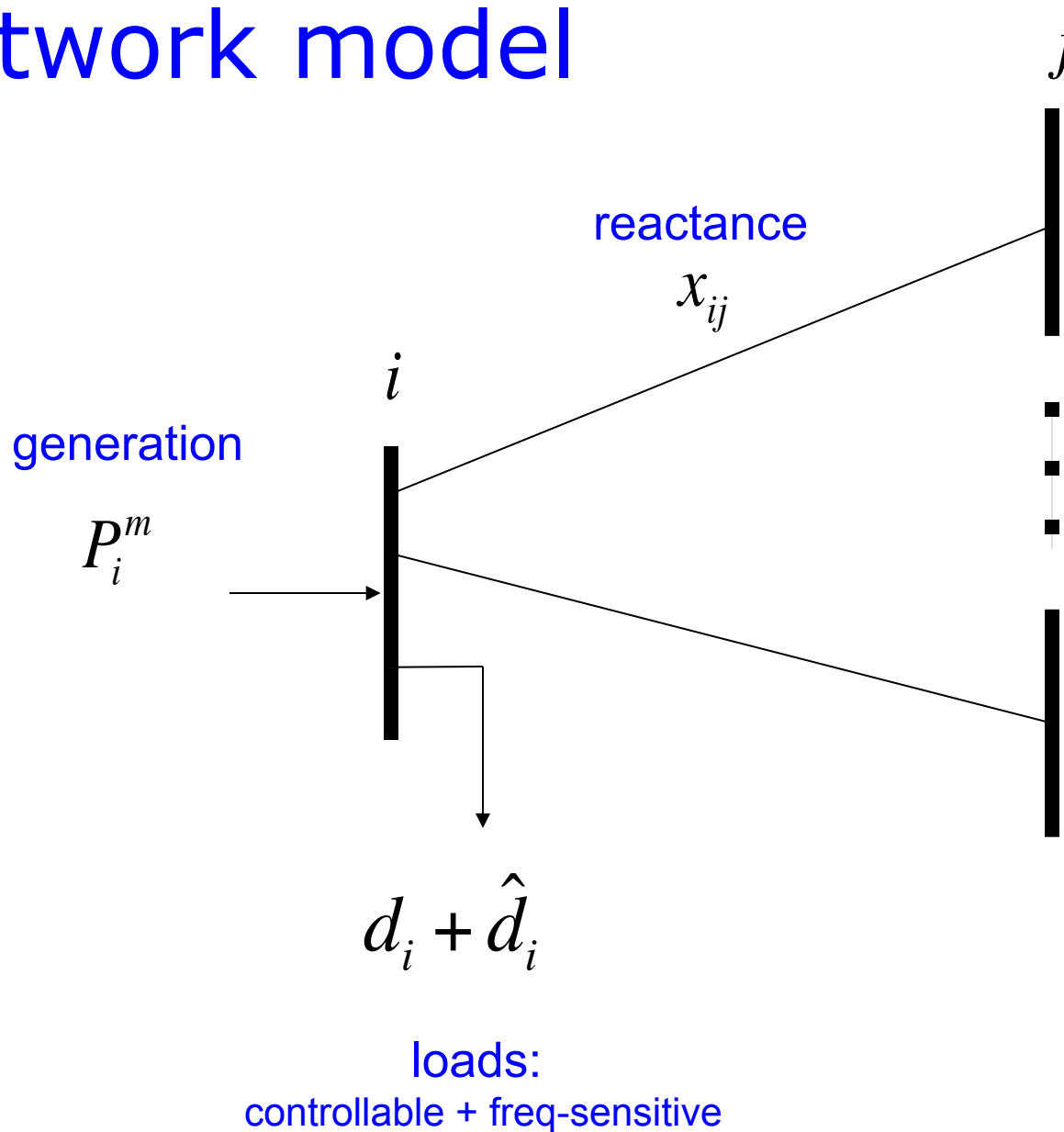
Simulations

Zhao, Topcu, Li, Low, TAC 2014  
Mallada, Low, 2013





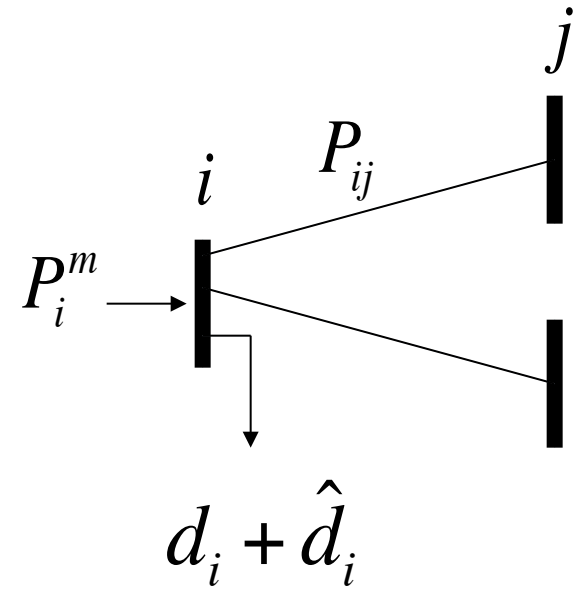
# Network model



$i$  : bus/control area/balancing authority



# Network model



## DC approximation

- Lossless network ( $r=0$ )
- Fixed voltage magnitudes
- Reactive power ignored
- Do *not* assume small angle difference



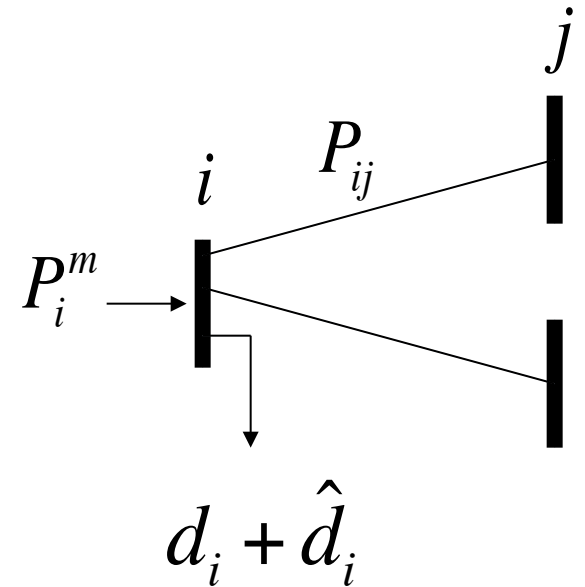
# Dynamic model

## Swing equation on bus $i$

frequency                      mechanical power                      electrical power

↓                                      ↓                                      ↓

$$M_i \dot{\omega}_i = P_i^m - P_i^e$$

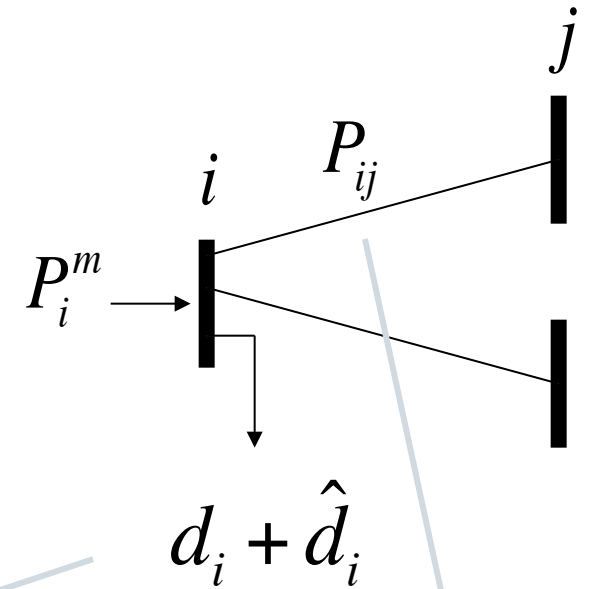


- Newton's 2<sup>nd</sup> law
- Variables: **deviations** from nominal values



# Dynamic model

Swing equation on bus  $i$



$$M_i \dot{\omega}_i = P_i^m - P_i^e$$

$$P_i^e := d_i + D_i \omega_i + \sum_{i \sim j} P_{ij}$$

controllable  
loads

freq-sens  
loads

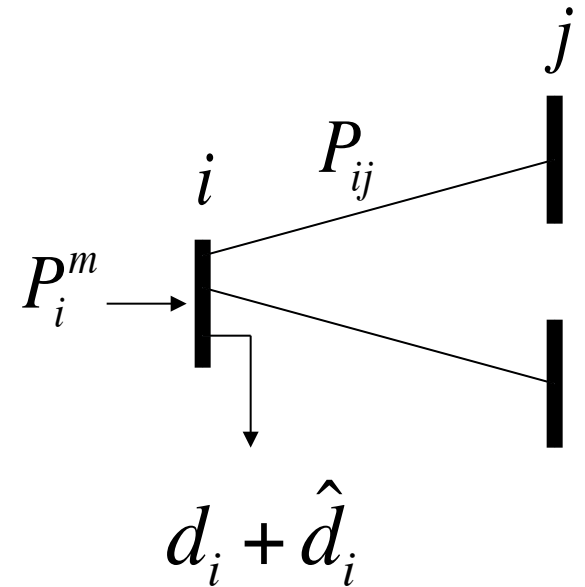
branch  
power flow



# Dynamic model

Swing equation on bus  $i$

$$M_i \dot{\omega}_i = P_i^m - P_i^e$$



$$\dot{P}_{ij}^e = b_{ij} (\omega_i - \omega_j) + \sum_{i \sim j} P_{ij}$$

$$b_{ij} = 3 \frac{|V_i| |V_j|}{x_{ij}} \cos(\theta_i^0 - \theta_j^0)$$

linearization around nominal



# Network model

Generator bus (may contain load):

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

Load bus (no generator):

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

Real branch power flow:

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

**swing dynamics**





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Mallada, Low, 2013





# Frequency control

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \quad \dot{P}_{ij} = 0$$

and suddenly ...



# Frequency control

Given: disturbance in gens/loads

Current: adapt remaining generators  $P_i^m$

- to re-balance power
- (and restore nominal freq, zero ACE)

Our goal: adapt controllable loads  $d_i$

- to re-balance power
- while minimizing disutility of load control



# Frequency control

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

proposed  
approach

current  
approach

this talk: ignores generator-side control



# Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

How to design feedback control law

$$d_i = F_i (\omega(t), P(t))$$



# Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

## Control goals

Zhao, Topcu, Li, Low  
TAC 2014

Mallada, Low 2013

- Rebalance power
- Resynchronize/stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows



# Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

Desirable properties of  $d_i = F_i(\omega(t), P(t))$

- simple, scalable
- decentralized/distributed



# Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji} \right)$$

$$0 = d_i + D_i \omega_i - P_i^m + \sum_{i \rightarrow j} P_{ij} - \sum_{j \rightarrow i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j) \quad \forall i \rightarrow j$$

Proposed approach: forward engineering

- formalize control goals into OLC **objective**
- derive **local** control as distributed solution





# Outline

Motivation

Dynamic network model

## Load-side frequency control

- Primary control
- Secondary control

Zhao, Topcu, Li, Low, TAC 2014

## Simulations





# Optimal load control (OLC)

controllable  
load



uncontrollable  
load



$$\min \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right)$$

over

s. t.



# Optimal load control (OLC)

controllable  
load



uncontrollable  
load



$$\begin{aligned} \min \quad & \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right) \\ \text{over} \quad & \text{loads } d_l \in [\underline{d}_l, \bar{d}_l], \quad \hat{d}_i \\ \text{s. t} \quad & \end{aligned}$$



# Optimal load control (OLC)

controllable  
load



uncontrollable  
load



$$\min \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right)$$

$$\text{over loads } d_l \in [\underline{d}_l, \bar{d}_l], \hat{d}_i$$

$$\text{s. t. } \sum_i (d_i + \hat{d}_i) = \sum_i P_i^m$$

demand = supply  
across network



disturbances



# Punchline

## Theorem

swing dynamics  
+ frequency-based load control  
= primal-dual algorithm that solves OLC

- Completely decentralized
- Not need explicit communication
- Not need detailed network data
- Exploit free global control signal

... reverse engineering swing dynamics



# Recall OLC

controllable  
load



freq-sens  
load



$$\min \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right)$$

$$\text{over loads } d_l \in [\underline{d}_l, \bar{d}_l], \hat{d}_i$$

$$\text{s. t. } \sum_i (d_i + \hat{d}_i) = \sum_i P_i^m$$

demand = supply  
across network



# Punchline

swing dynamics (recap)

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{i \rightarrow j} P_{ij}(t) - \sum_{j \rightarrow i} P_{ji}(t) \right)$$

$$\dot{P}_{ij} = b_{ij} (\omega_i(t) - \omega_j(t))$$

implicit

load control

$$d_i(t) := \left[ c_i'^{-1} (\omega_i(t)) \right]_{\underline{d}_i}^{\bar{d}_i}$$

active control



# Punchline

## Theorem

system trajectory  $(d(t), \hat{d}(t), \omega(t), P(t))$

converges to  $(d^*, \hat{d}^*, \omega^*, P^*)$  as  $t \rightarrow \infty$





# Punchline

## Theorem

system trajectory  $(d(t), \hat{d}(t), \omega(t), P(t))$

converges to  $(d^*, \hat{d}^*, \omega^*, P^*)$  as  $t \rightarrow \infty$

- $(d^*, \hat{d}^*)$  is unique optimal load control
- $\omega^*$  is unique optimal for DOLC
- $P^*$  is optimal for dual of DOLC

Load-side primary frequency control works !



# Implications

- Freq deviations contains right info on **global** power imbalance for **local** decision
- Decentralized load participation in primary freq control is stable
- $\omega^*$  : Lagrange multiplier of OLC  
info on power imbalance
- $P^*$  : Lagrange multiplier of DOLC  
info on freq asynchronism



# Recap: control goals

Yes ■ Rebalance power

Yes ■ Resynchronize/stabilize frequency

No ■ Restore nominal frequency ( $\omega^* \neq 0$ )

No ■ Restore scheduled inter-area flows

Proposed approach: forward engineering

- formalize control goals into OLC **objective**
- derive **local** control as distributed solution



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Motivation

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## Load-side frequency control

- Primary control
- Secondary control

Mallada, Low, 2013

## Simulations





# Freq preserving OLC

$$\min \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right)$$

$$\text{s. t.} \quad \sum_i (d_i + \hat{d}_i) = \sum_i P_i^m$$

demand = supply  
across network

$$\min \sum_i \left( c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \right)$$

$$\text{s. t.} \quad d_i + \hat{d}_i = P_i^m - \sum_{e \in E} C_{ie} P_e$$

demand = supply  
per bus

$$d_i = P_i^m - \sum_{e \in E} C_{ie} R_e$$

to restore nominal  
frequency



# Recall primary control for OLC

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{e \in E} C_{ie} P_e(t) \right)$$

$$\dot{P}_{ij} = b_{ij} (\omega_i(t) - \omega_j(t))$$

← implicit

load control:  $d_i(t) := \left[ c_i'^{-1} (\omega_i(t)) \right]_{\underline{d}_i}^{\bar{d}_i}$  ← active control



# Recall primary control for OLC

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i} \left( d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{e \in E} C_{ie} P_e(t) \right)$$
$$\dot{P}_{ij} = b_{ij} (\omega_i(t) - \omega_j(t)) \quad \leftarrow \text{implicit}$$

---

load control:  $d_i(t) := \left[ c_i^{-1} (\omega_i(t) + \lambda(t)_i) \right]_{\underline{d}_i}^{\bar{d}_i}$

computation & communication:

$$\dot{\lambda}_i = -\gamma_i \left( d_i(t) - P_i^m + \sum_{e \in E} C_{ie} R_e(t) \right), \quad \dot{R}_{ij} = a_{ij} (\lambda_i(t) - \lambda_j(t))$$



# Punchline

## Theorem

system trajectory  $(d(t), \hat{d}(t), \omega(t), P(t))$

converges to  $(d^*, \hat{d}^*, \omega^*, P^*)$  as  $t \rightarrow \infty$

- $(d^*, \hat{d}^*)$  is unique optimal load control
- $\omega^* = 0$

Load-side secondary frequency control works !





# Recap: control goals

Yes ■ Rebalance power

Yes ■ Resynchronize/stabilize frequency

Yes ~~No~~ ■ Restore nominal frequency ( $\omega^* \neq 0$ )

No ■ Restore scheduled inter-area flows

Secondary control restores nominal frequency but **requires communication with neighbors**



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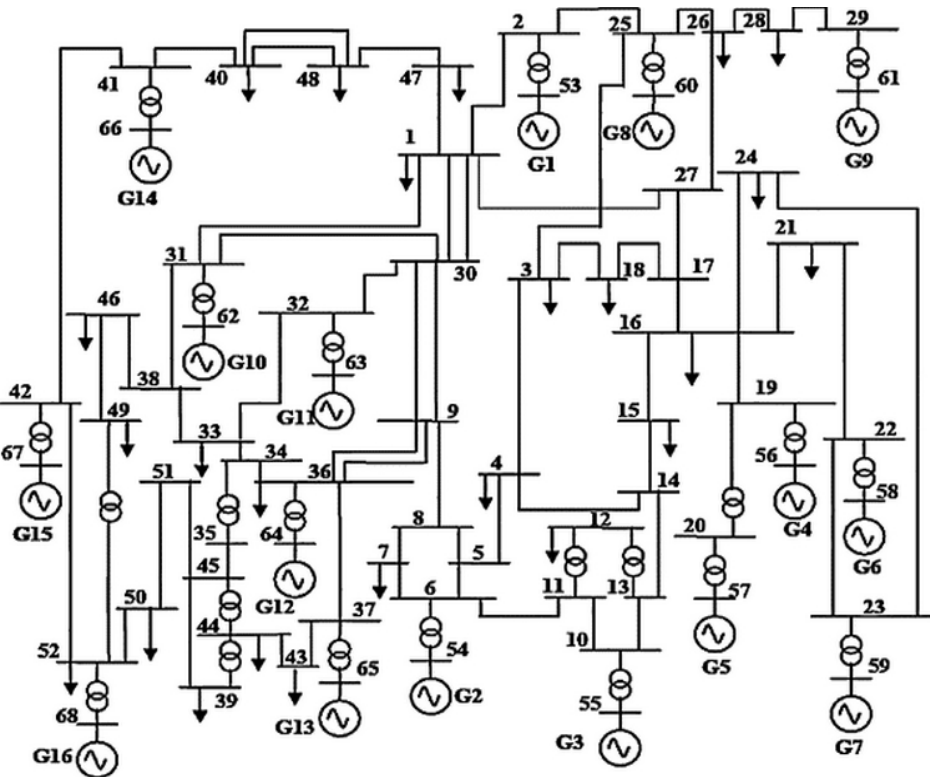
Zhao, Topcu, Li, Low, TAC 2014  
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# Simulations

## Dynamic simulation of IEEE 68-bus system

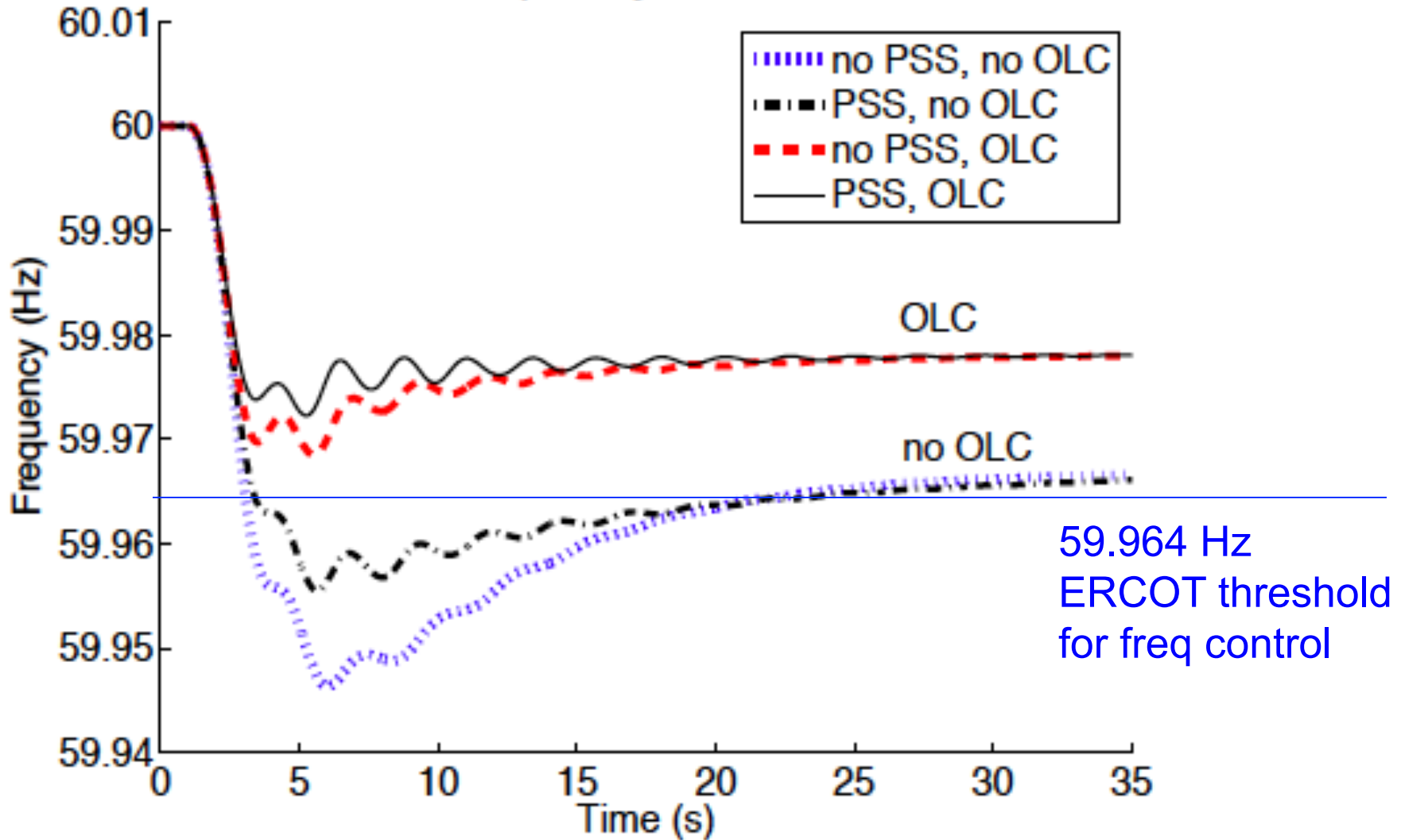


- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines



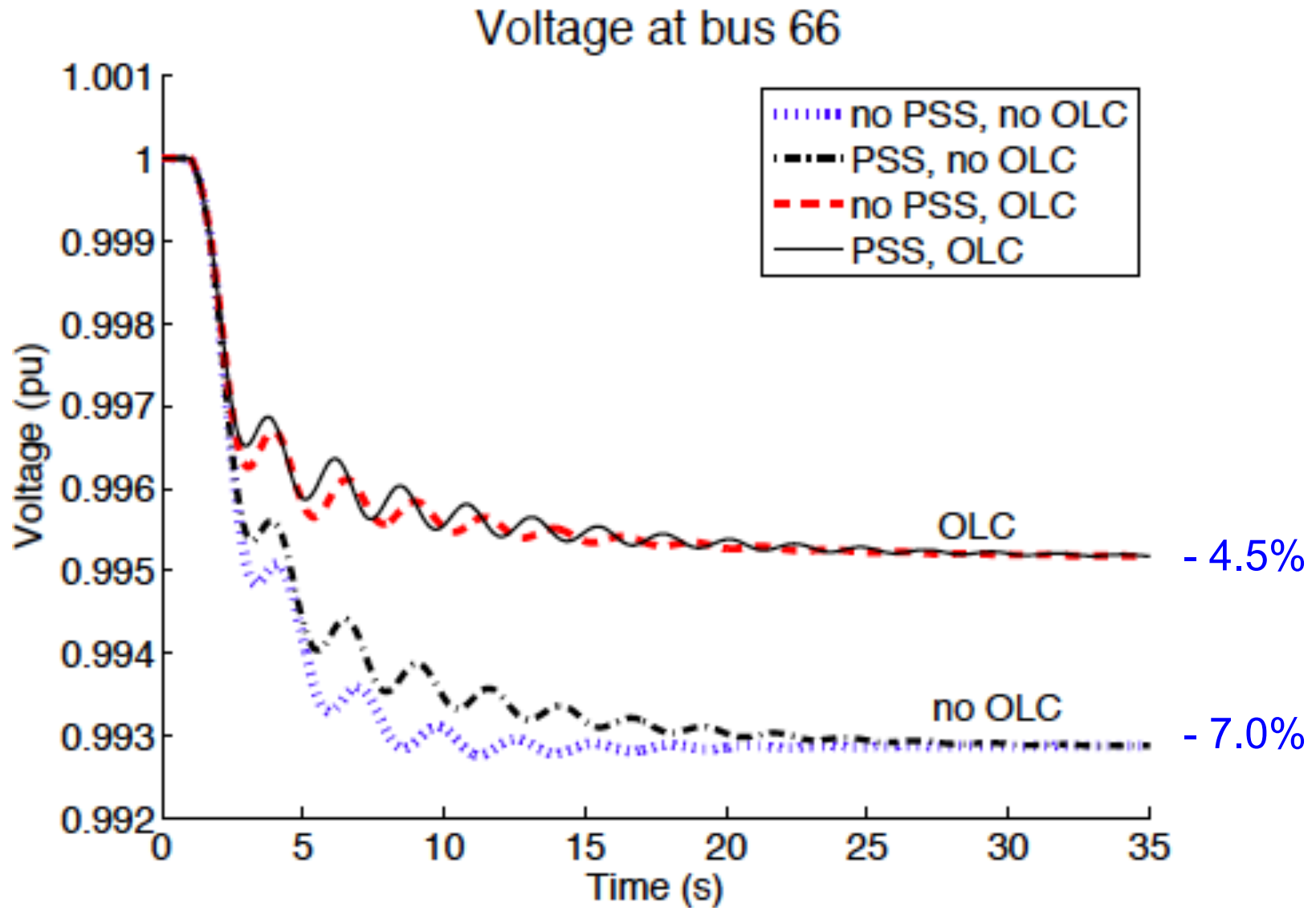
# Simulations

Frequency at bus 66





# Simulations





# Simulations

