Optimal Secondary frequency control

connecting AGC with economic dispatch

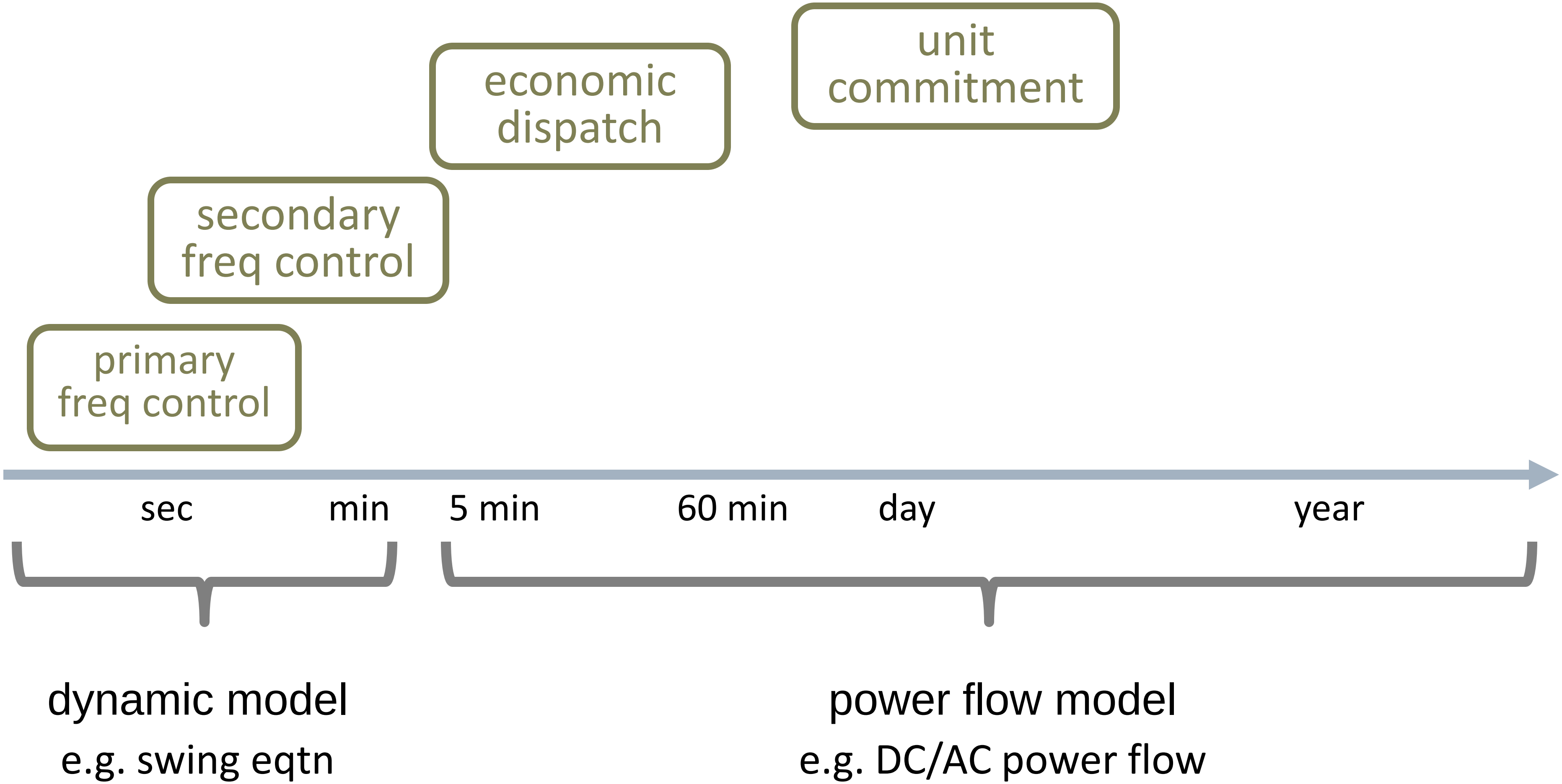
Na Li
LIDS, MIT

Lijun Chen
Telecommunication
CU-Boulder

Changhong Zhao
Steven Low
CMS
Caltech

CMU
02/04/2014

Frequency control



Frequency control

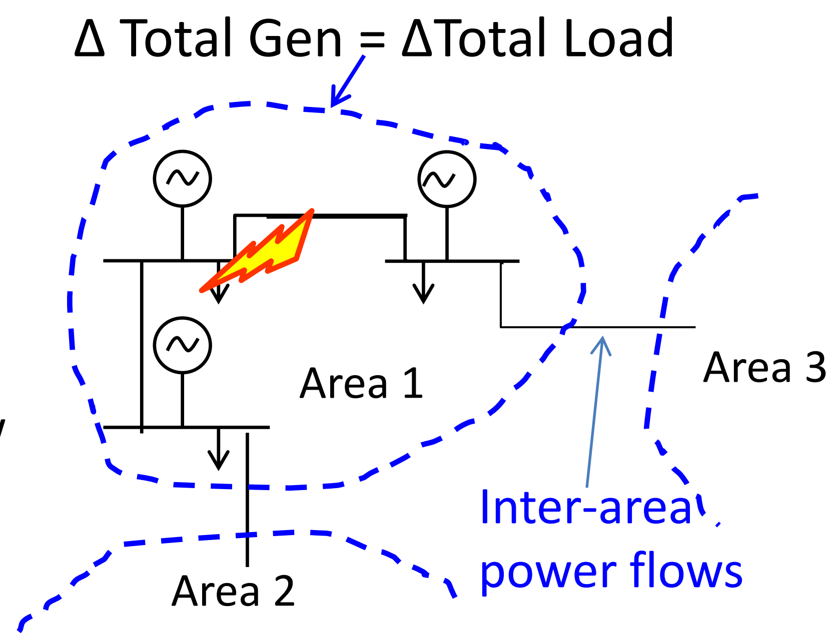
economic dispatch → **Optimally schedule** gens/loads, and the inter-area power flows

secondary freq control → **Maintain** the nominal freq and the inter-area power flows

sec | min | 5 min

dynamic model
e.g. swing eqtn

power flow model
e.g. DC/AC power flow



Frequency control

To Improve

secondary freq control

economic dispatch

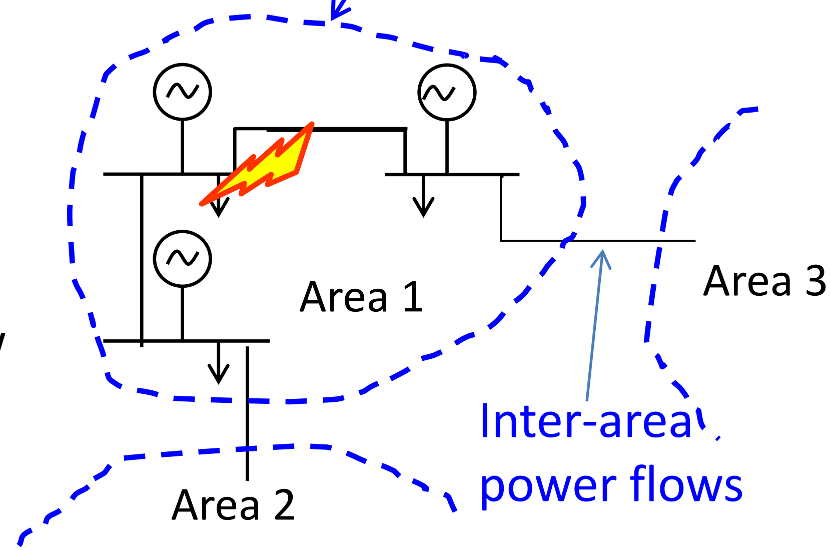
1. Optimally schedule gens/loads within a area

\+

Maintain the nominal freq and ~~the inter-area power flows~~

2. Optimally (re)schedule gens/loads ~~among areas (if it is permitted)~~

~~ΔTotal Gen = ΔTotalLoad~~



sec | min | 5 min

dynamic model
e.g. swing eqtn

power flow model
e.g. DC/AC power flow

Frequency control

To Improve

secondary freq control

economic dispatch

1. Optimally schedule gens/loads within a area

Maintain the nominal freq and ~~the inter-area power flows~~

2. Optimally (re)schedule gens/loads among areas (if it is permitted)

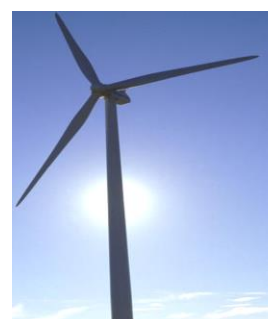
sec min 5 min

dynamic model
e.g. swing eqtn

power flow model
e.g. DC/AC power flow

Motivation:

Power system efficiency



Frequency control

To Improve

secondary freq control

economic dispatch

1. Optimally schedule gens/loads within a area

+

Maintain the nominal freq and ~~the inter-area power flows~~

2. Optimally (re)schedule gens/loads among areas (if it is permitted)

sec min 5 min

dynamic model
e.g. swing eqtn

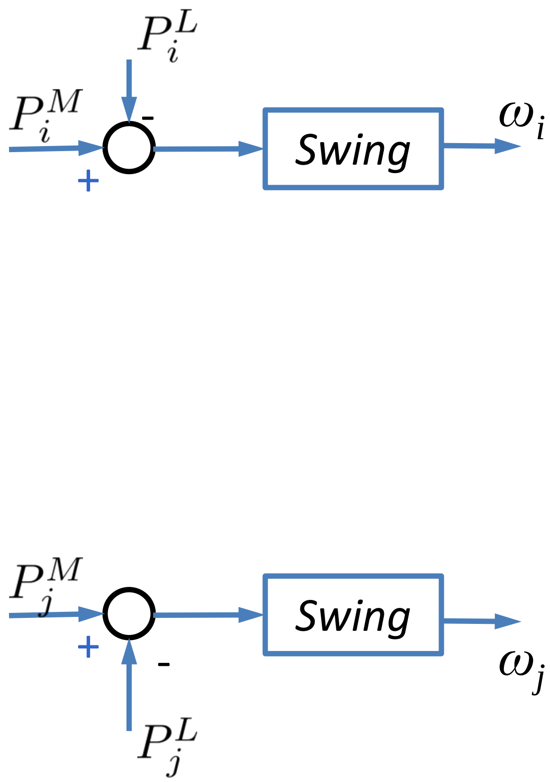
power flow model
e.g. DC/AC power flow

Challenge:

Physical dynamics + existing control mechanisms (AGC)

Physical dynamics: Swing Dynamics

$$\dot{\omega}_i \quad = \quad -\frac{1}{M_i}(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\rightarrow j} P_{ij} - \sum_{k:k\rightarrow i} P_{ki})$$



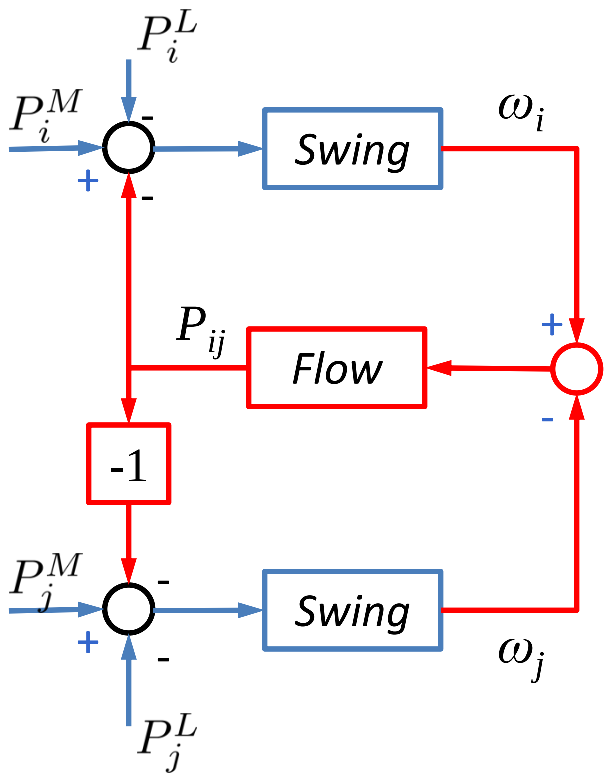
P_i^M: Mechanical power
P_i^L: Load
ω_i : Frequency
P_{ij} : Power Flow
M_i, D_i: Constant parameters

Bus *i:* control area/ balance authority

Variables: the deviations from reference (steady state) values

Power Flow dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

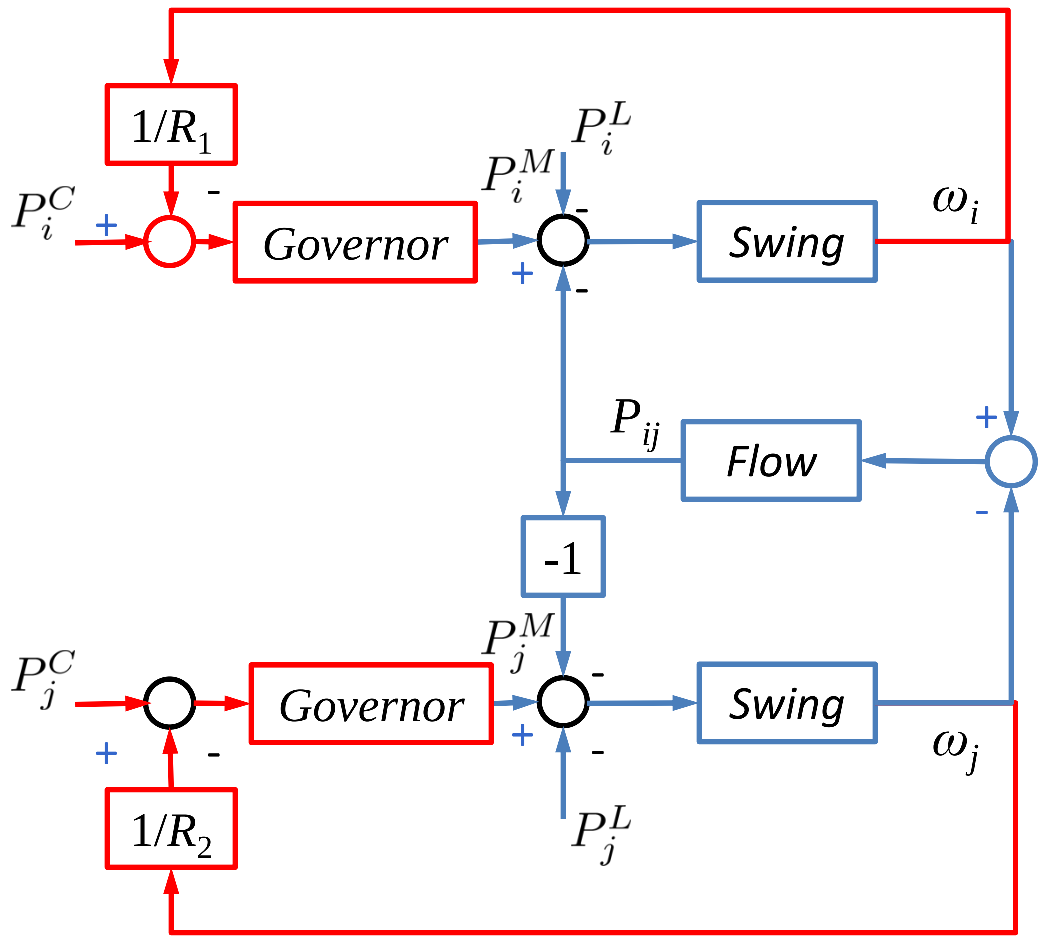


P_i^M: Mechanical power
P_i^L: Load
ω_i : Frequency
P_{ij} : Power Flow
T_{ij}: Constant parameter

Assumptions:
Lossless (resistance=0)
Fixed voltage magnitudes
Small deviation of angles

Turbine-Governor Control

$$\dot{P}_i^M = -\frac{1}{T_j}(P_i^M - P_i^C + \frac{1}{R_i}\omega_i)$$



P_i^M: Mechanical power

P_i^L: Load

ω_i : Frequency

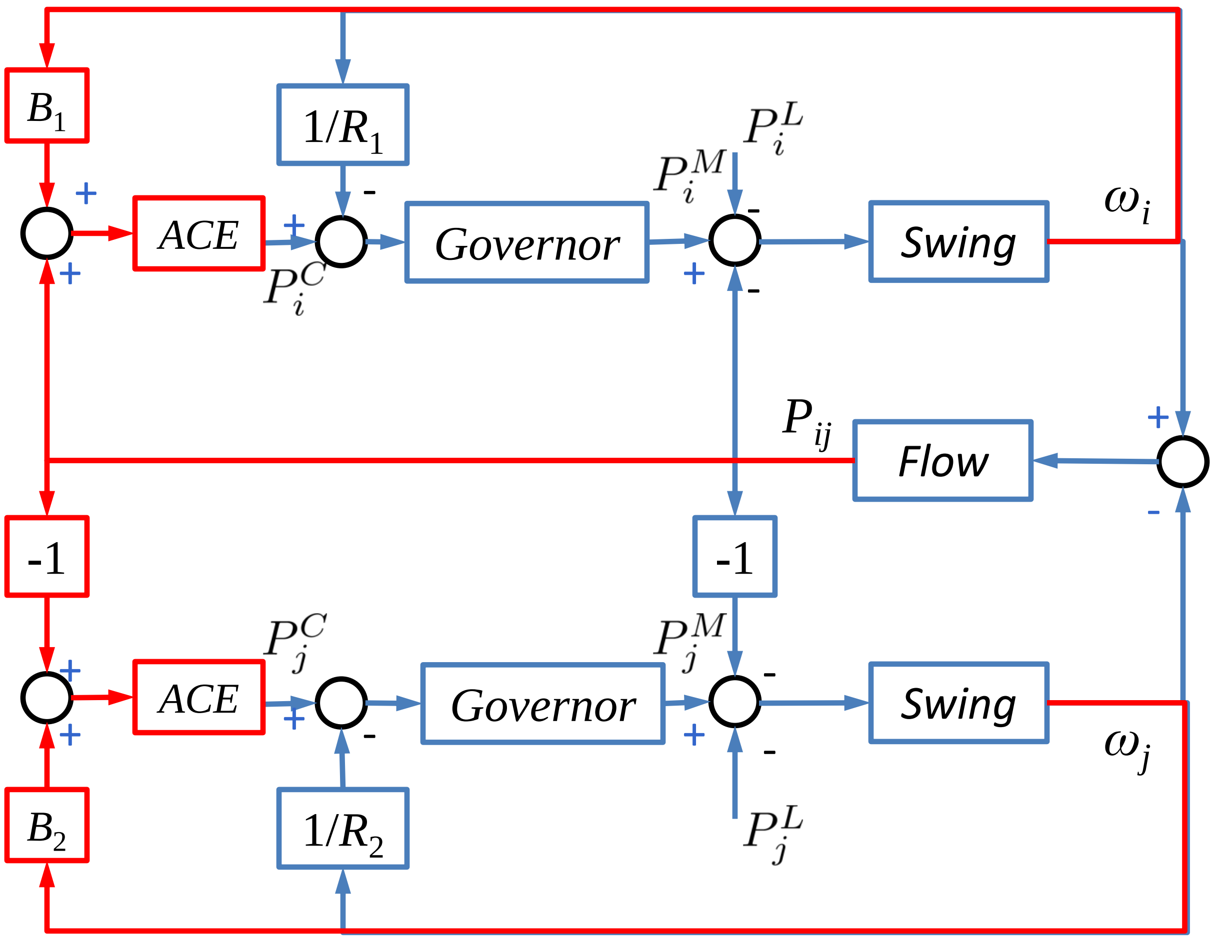
P_{ij} : Power Flow

P_i^C: Power command input

T_i, R_i: Constant parameters

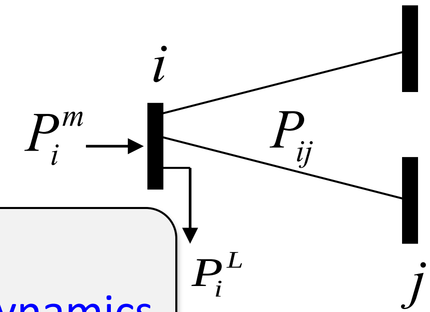
(Area Control Error) ACE-based AGC

$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$



P_i^M: Mechanical power
P_i^L: Load
ω_i : Frequency
P_{ij} : Power Flow
P_i^C: Power command input
K_i, B_i: Constant parameters

Recap: System Dynamics with AGC



$$
\begin{aligned}
\dot{\omega}_i &= -\frac{1}{M_i}\left(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\rightarrow j} P_{ij} - \sum_{k:k\rightarrow i} P_{ki}\right) && \leftarrow \text{Swing dynamics} \\
\dot{P}_{ij} &= T_{ij}(\omega_i - \omega_j) && \leftarrow \text{Power Flow Dynamics} \\
\dot{P}_i^M &= -\frac{1}{T_j}\left(P_i^M - P_i^C + \frac{1}{R}\omega_i\right) && \leftarrow \text{Turbine-Governor Control} \\
\dot{P}_i^C &= -K_i\left(B_i\omega_i + \sum_{j:i\rightarrow j} P_{ij} - \sum_{k:k\rightarrow i} P_{ki}\right) && \leftarrow \text{ACE-based AGC}
\end{aligned}
$$

Suppose the system is in steady state (all variables = 0)

Disturbance, e.g. P_i^L

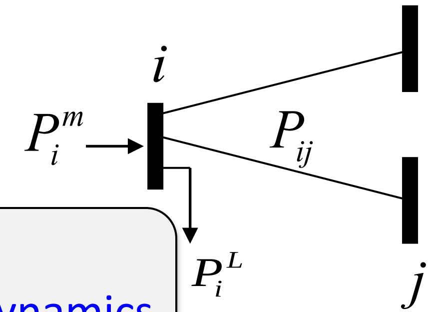
Drive the system to a new steady state

(i) $\omega_i = 0$

(ii) P_i^M ~~$=$~~ P_i^L → **Economically Dispatch** P_i^M to minimize generation cost

Our objective

Recap: System Dynamics with AGC



$$\dot{\omega}_i = -\frac{1}{M_i}\Big(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\Big)$$ ← Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$ ← Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}\Big(P_i^M - P_i^C + \frac{1}{R}\omega_i\Big)$$ ← Turbine-Governor Control

$$\dot{P}_i^C = -K_i\Big(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\Big)$$ ← ACE-based AGC

Suppose the system is in steady state (all variables = 0)

Disturbance, e.g. P_i^L

Drive the system to a new steady state

(i) $\omega_i = 0$

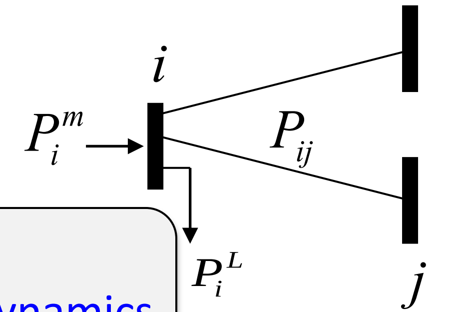
(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$ ← Generation cost

$$\text{s.t. } P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

↑ Power flow balance

Economic AGC

Recap: System Dynamics with AGC



$$\dot{\omega}_i = -\frac{1}{M_i}\left(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\right)$$ ← Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$ ← Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}\left(P_i^M - P_i^C + \frac{1}{R}\omega_i\right)$$ ← Turbine-Governor Control

$$\dot{P}_i^C = -K_i\left(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\right)$$ ← ACE-based AGC

Question: How to modify AGC to be economic AGC?

Drive the system to a new steady state

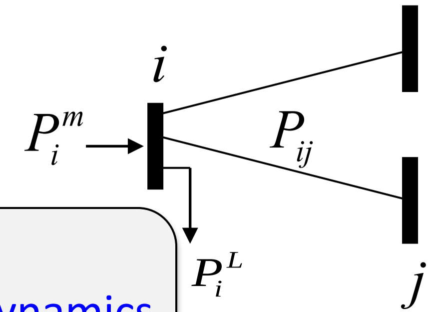
(i) $\omega_i = 0$

(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

} **Economic AGC**

Results: Economic AGC



$$\dot{\omega}_i = -\frac{1}{M_i}\left(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\right)$$

← Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

← Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}\left(\frac{1 - R_i K_i M_i}{R_i} C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i\right)$$

$$\dot{P}_i^C = -K_i\left(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki})\right)$$

$$\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j$$

Drive the system to a new steady state

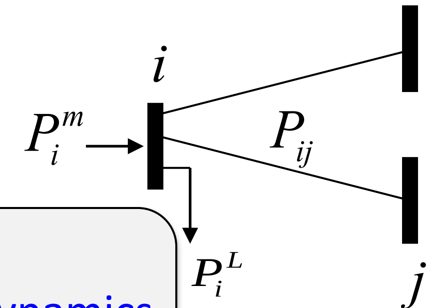
(i) $\omega_i = 0$

(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

Economic AGC

Results: Economic AGC



$$\dot{\omega}_i = -\frac{1}{M_i}\left(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\right)$$

← Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

← Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}\left(\frac{1 - R_i K_i M_i}{R_i} C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i\right)$$

$$\dot{P}_i^C = -K_i\left(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki})\right)$$

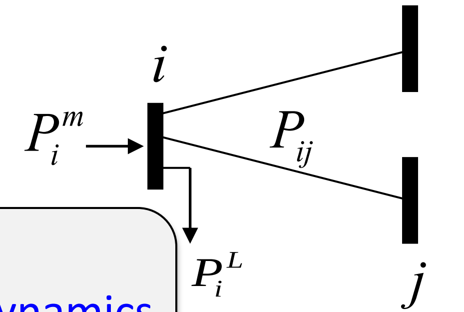
$$\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j$$

***Theorem*(Li, Chen, Zhao, Low 2013):**

Any trajectory $\left(P^M(t), \omega(t), P(t)\right)$ converges to $\left(P^{M*}, \omega^*, P^*\right)$

- P^{M*} is optimal to economic dispatch
- $\omega^* = 0$
- P^* is a feasible power flow

Results: Economic AGC



$$\dot{\omega}_i = -\frac{1}{M_i}(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}(\frac{1 - R_iK_iM_i}{R_i}C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i)$$

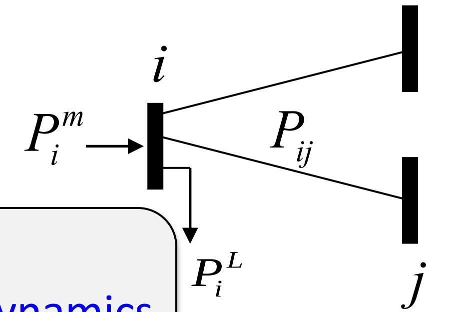
$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki}))$$

$$\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j$$

Local computation ***Local communication***

- ***Additional local variable***
- ***Frequency, Power flow, Power Command: local measurable signals***
- ***A decentralized algorithm; Local information and communications***
- ***Not just local…***

Not Just Local...



$$
\begin{aligned}
\dot{\omega}_i &= -\frac{1}{M_i}\Big(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\rightarrow j} P_{ij} - \sum_{k:k\rightarrow i} P_{ki}\Big) \\
\dot{P}_{ij} &= T_{ij}(\omega_i - \omega_j) \\
\dot{P}_i^M &= -\frac{1}{T_j}\Big(\frac{1 - R_iK_iM_i}{R_i} C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i\Big) \\
\dot{P}_i^C &= -K_i\Big(B_i\omega_i + \sum_{j:i\rightarrow j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\rightarrow i}(P_{ki} - \gamma_{ki})\Big)
\end{aligned}
$$

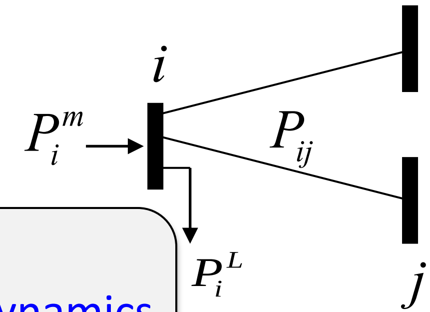
Swing dynamics

Power Flow Dynamics

$$
\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j
$$

Only use information/signals that are easy to measure or calculate

Extension to load freq control



$$\dot{\omega}_i = -\frac{1}{M_i}\Big(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}\Big)$$

← Swing dynamics

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

← Power Flow Dynamics

$$\dot{P}_i^M = -\frac{1}{T_j}\Big(\frac{1 - R_iK_iM_i}{R_i}C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i\Big)$$

$$\dot{P}_i^C = -K_i\Big(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki})\Big)$$

$$\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j$$

Similar mechanisms apply to *load control*

Drive the system to a new steady state

(i) $\omega_i = 0$

(ii) $$\min_{P_i^M} \sum_i C_i(P_i^M) + \sum_i D_i(P_i^L)$$

← **Load disutility**

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

} **Optimal gens/loads control**

Tool: reverse/forward engineering

System Dynamics & Existing Control

$$
\begin{aligned}
\dot{\omega}_i &= -\frac{1}{M_i}(D_i\omega_i - P_i^m + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}) \\
\dot{P}_{ij} &= T_{ij}(\omega_i - \omega_j) \\
\dot{P}_i^M &= -\frac{1}{T_j}(P_i^M - P_i^C + \frac{1}{R}\omega_i) \\
\dot{P}_i^C &= -K_i(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})
\end{aligned}
$$

Economic Generation Control

(i) $\omega_i = 0$

(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

solve

Analogy ?

Optimization Problem

Reverse

Tool: reverse/forward engineering

System Dynamics & Existing Control

$$\dot{\omega}_i = -\frac{1}{M_i}(D_i\omega_i - P_i^m + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

$$\dot{P}_i^M = -\frac{1}{T_j}(P_i^M - P_i^C + \frac{1}{R}\omega_i)$$

$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

Economic Generation Control

(i) $\omega_i = 0$

(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

Equivalent

Modified

Optimization Problem

Forward

Tool: reverse/forward engineering

System Dynamics & Modified Control

$$\dot{\omega}_i = -\frac{1}{M_i}(D_i\omega_i - P_i^M + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

$$\dot{P}_i^M = -\frac{1}{T_j}(\frac{1-R_iK_iM_i}{R_i}C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i)$$

$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki}))$$

$$\dot{\gamma}_i = \epsilon_i\left(M_i\omega_i - P_i^C/K_i\right); \quad \gamma_{i,j} = \gamma_i - \gamma_j$$

solve

Economic Generation Control

(i) $\omega_i = 0$

(ii) $\min_{P_i^M} \sum_i C_i(P_i^M)$

$$\text{s.t.} \quad P_i^M = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

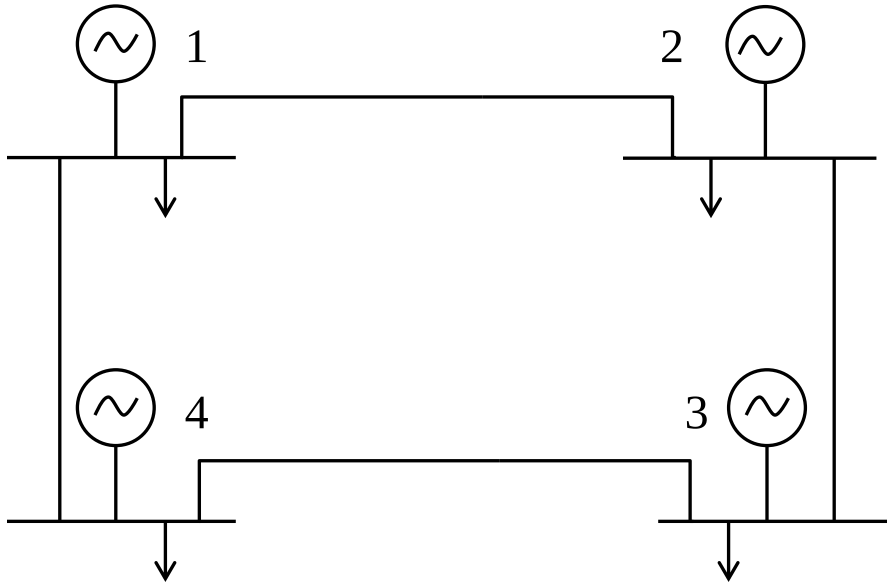
Equivalent

Modified

Optimization Problem

Forward

Case Study

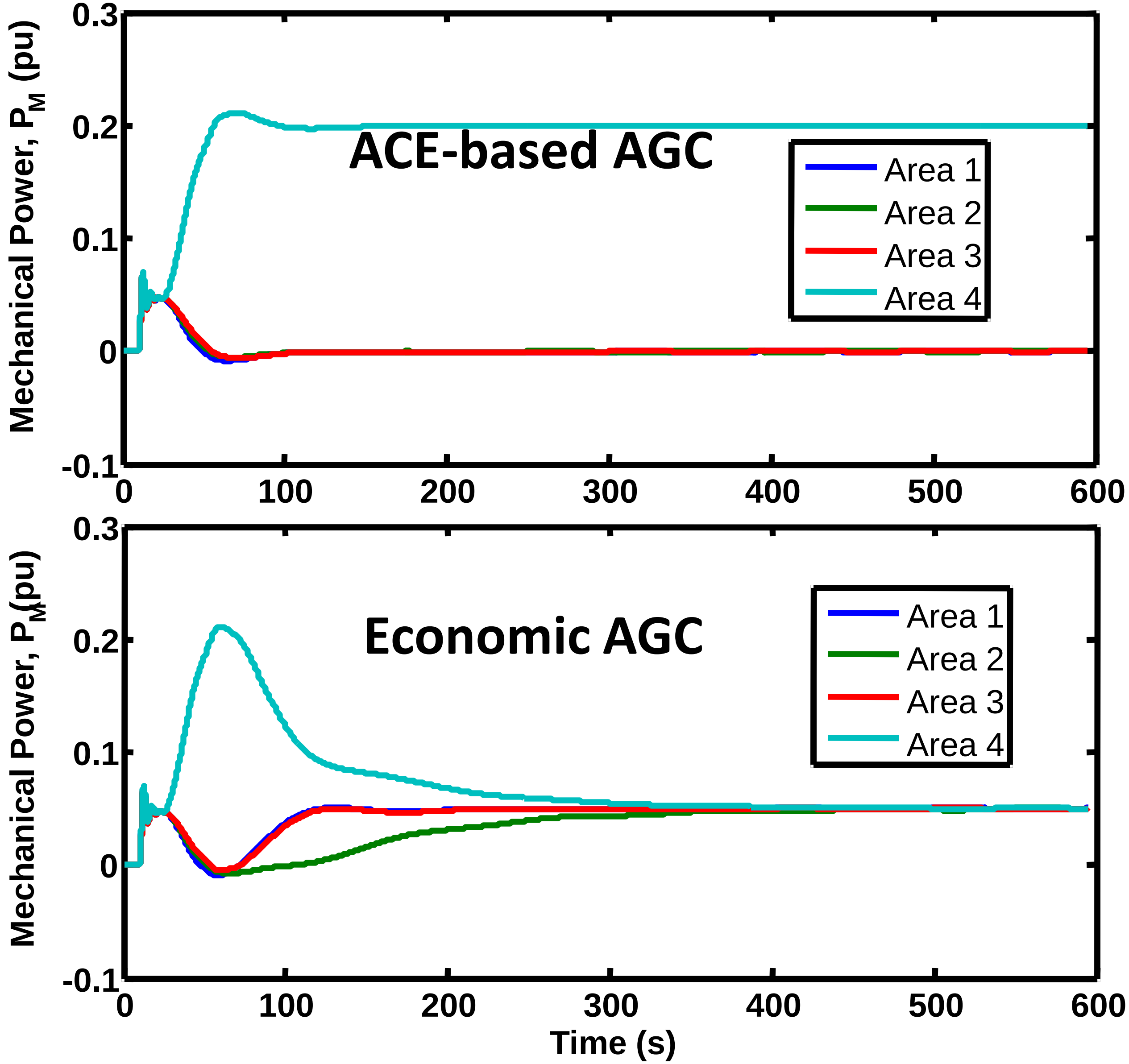


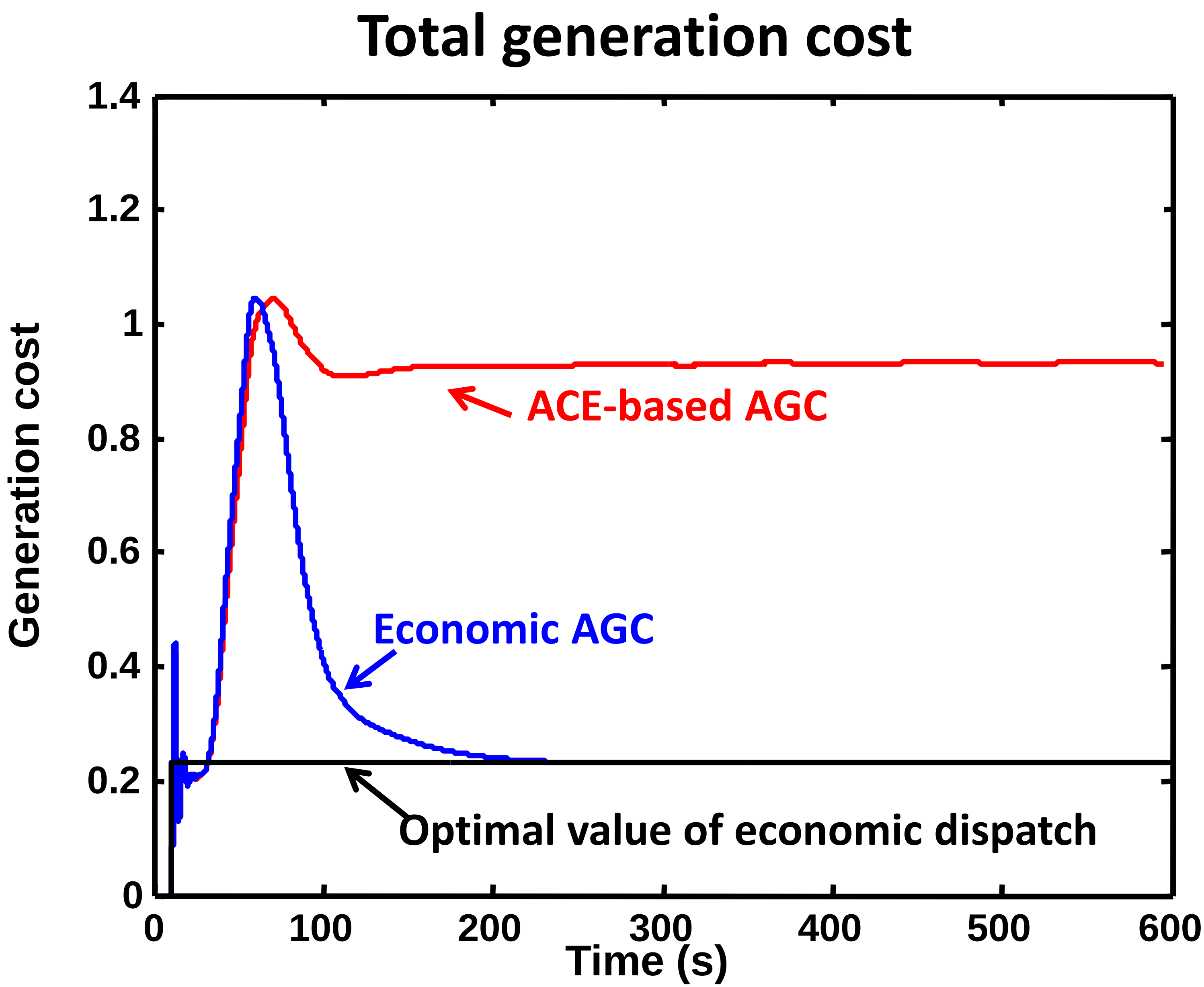
4 control areas: a 39-bus New England Transmission Network

Source: Ilic, et.al. IEEE TPS, vol. 8, no. 1, 1993

- At time t = 10s, load increase 0.2 pu at area 4
- (Nonlinear) power flow model; Nonzero resistance
- Sample rate =15s (e.g., ACE is reset for every 15s)
- Each area has a same cost function (optimal value should be that each area increase generation by 0.05 pu)

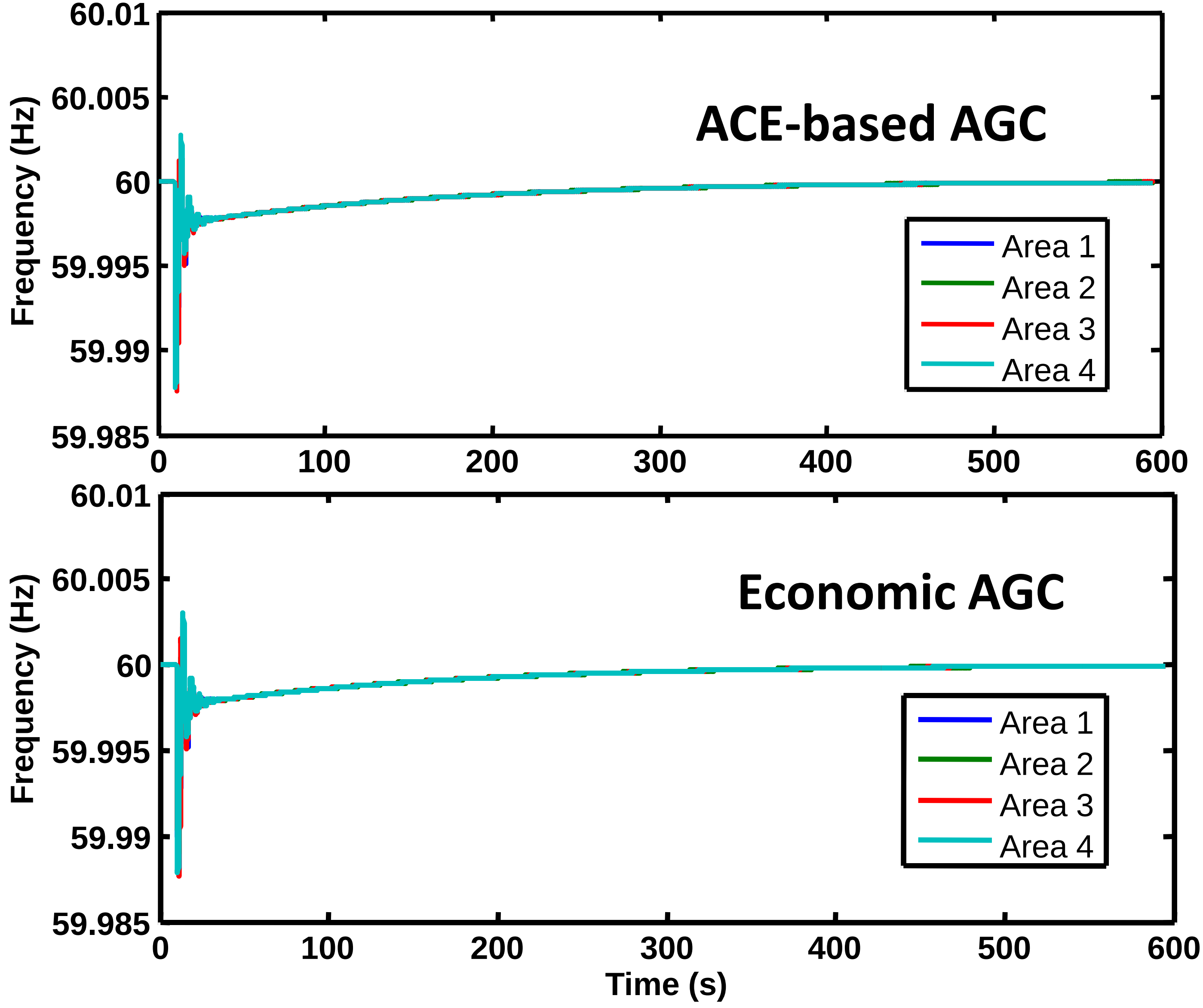
Mechanical Power Generation





Total generation cost
Generation cost
Time (s)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
ACE-based AGC
Economic AGC
Optimal value of economic dispatch

Frequency


ACE-based AGC
Frequency (Hz)
60.01
60.005
60
59.995
59.99
59.985
0
100
200
300
400
500
600
Area 1
Area 2
Area 3
Area 4
Economic AGC
Frequency (Hz)
60.01
60.005
60
59.995
59.99
59.985
0
100
200
300
400
500
600
Time (s)
Area 1
Area 2
Area 3
Area 4

Conclusion

economic dispatch

secondary freq control

primary freq control

1. Optimally schedule gens/loads within a area

+

Maintain the nominal frequency ~~and the inter-area power flows~~

2. Optimally (re)schedule gens/loads among areas (if it is permitted)

Minor Modification

(additional local computation and communication based on local measurable signals)

THANK YOU

Back Up

System dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}(D_i\omega_i - P_i^m + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

$$\dot{P}_i^M = -\frac{1}{T_j}(P_i^M - P_i^C + \frac{1}{R}\omega_i)$$

$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

Economic Energy Control

$$\min \quad \sum_i \left(c_i(P_i^m)\right)$$

$$\text{over} \quad P_i^m, \quad \omega$$

$$\text{s. t.} \quad P_i^m = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

$$\omega_i = 0$$

Partial primal-dual gradient algorithm

Reverse Engineering

$$\min \quad \sum_i \left(\tfrac{1}{2}(P_i^m)^2 + \tfrac{D_i}{2}\omega_i^2\right)$$

$$\text{over} \quad P_i^M, \omega$$

$$\text{s. t.} \quad P_i^M = P_i^L + D_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

$$P_i^M = P_i^L$$

?

Economic AGC

$$\dot{\omega}_i = -\frac{1}{M_i}(D_i\omega_i - P_i^m + P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki})$$

$$\dot{P}_{ij} = T_{ij}(\omega_i - \omega_j)$$

$$\dot{P}_i^M = -\frac{1}{T_j}(\frac{1 - R_iK_iM_i}{R_i}C_i'(P_i^M) - P_i^C + \frac{1}{R}\omega_i)$$

$$\dot{P}_i^C = -K_i(B_i\omega_i + \sum_{j:i\to j}(P_{ij} - \gamma_{i,j}) - \sum_{k:k\to i}(P_{ki} - \gamma_{ki}))$$

$$\dot{\gamma}_{ij} = \epsilon_{ij}\left(\left(M_i\omega_i - P_i^C/K_i\right)\epsilon_i - \left(M_j\omega_j - P_j^C/K_j\right)\epsilon_j\right)$$

Partial primal-dual gradient algorithm

Economic Energy Control

$$\min \quad \sum_i \left(c_i(P_i^m)\right)$$

$$\text{over} \quad P_i^m, \quad \omega$$

$$\text{s. t.} \quad P_i^m = P_i^L + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

$$\omega_i = 0$$

Forward Engineering

min ~~$\sum_i \left(\frac{1}{2}(P_i^m)^2 + \frac{D_i}{2}\omega_i^2\right)$~~ $\sum_i \left(c_i(P_i^m) + \frac{D_i}{2}\omega_i^2\right)$

$$\text{over} \quad P_i^M, \omega$$

$$\text{s. t.} \quad P_i^M = P_i^L + D_i\omega_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}$$

~~$P_i^M = P_i^L$~~ $P_i^M = P_i^L + \sum_{j:i\to j} \gamma_{ij} - \sum_{k:k\to i} \gamma_{ki}$