Demand Response with Stochastic Renewables and Inertial Thermal Loads: A Study of Some Fundamental Issues in Modeling and Optimization

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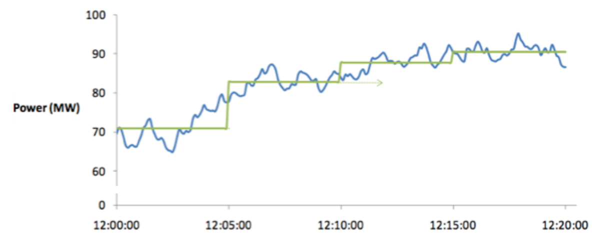
NINTH ANNUAL CARNEGIE MELLON CONFERENCE
ON THE ELECTRICITY INDUSTRY
CMU
February 4, 2014

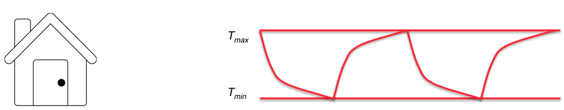
Variability of Wind

- Goal: Use renewable energy – wind
- Problem: Highly variable – "stochastic"



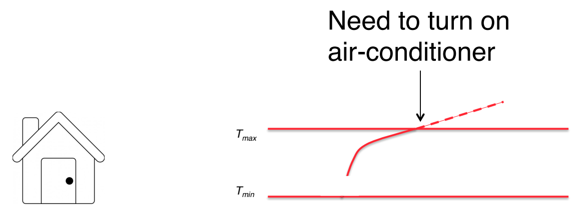
Demand Response

- Adjust demand to match supply
- Inertial thermal loads – building air conditioners
  - Air conditioner can be switched off for a short while without loss of comfort
  - Traditionally under thermostatic control



Limited capability of demand response

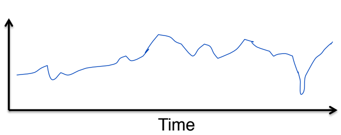
- Renewable energy may not be enough to satisfy load requirements



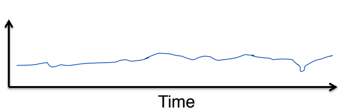
- So there are limitations to demand response

Variability of power demand not met by renewables

- Residual power demand not met by renewables



- Prefer less variability so that operating reserve requirements are less



Several Questions – 1

- To what extent can demand of inertial loads be met by renewable sources?
- How does flexibility of load requirements, such as comfort level settings, influence how much renewable power can be used?
- How much flexibility can be extracted from thermal inertial loads for maximum utilization of variable generation such as wind?

Several Questions – 2

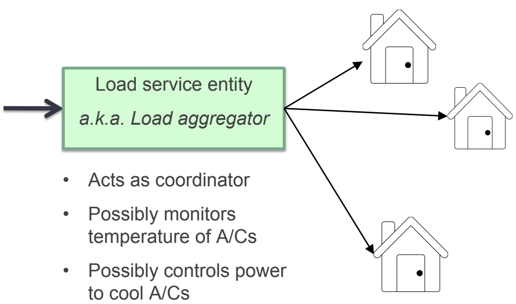
- To what extent can operating reserve required be minimized?
- How beneficial is “demand pooling”?
- Can we come up with quantitative answers?
- How can demand “pooling” be done?
- What are the communication requirements?
- How much information exchange is needed between suppliers and consumers?

Several Questions – 3

- What are the privacy implications?
- Does it require intrusive sensing?
- How distributed can the solution be?
- How tractable (computational complexity) is the solution?
- How robust is the solution?
- How implementable is it?

Role of model features, cost functions, stochasticity assumptions, convexity, asymptotics, etc

Load Aggregator

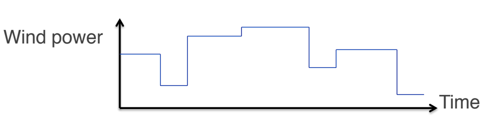


- Acts as coordinator
- Possibly monitors temperature of A/Cs
- Possibly controls power to cool A/Cs
- Appears as aggregated load

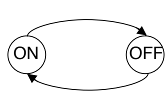
Features of problem

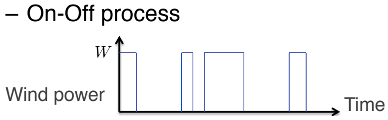
Wind model

- Model wind as a finite state Markov process

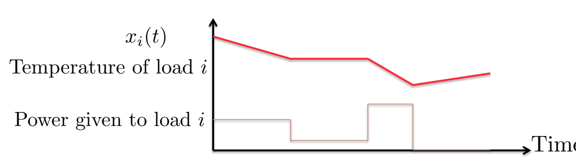


- Even simpler model for illustration
  - On-Off process





Temperature dynamics



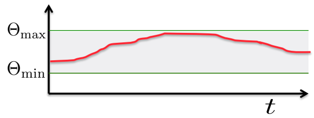
- Inertial thermal load (A/C) dynamics

$$\dot{x}_i(t) = h_i(t) - P_i(t)$$

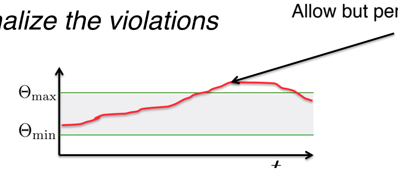
  - Rate of change in temperature = Ambient heating – Power for cooling.

User specified comfort range

- Range of comfortable temperature $[\Theta_{\min}, \Theta_{\max}]$
- Either: *Enforce hard constraint*



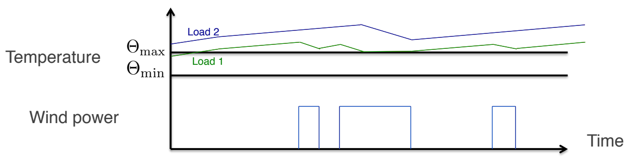
- Or: *Penalize the violations*



Stochastic control problem: Comfort violation probability

- Minimize the probability of leaving a user specified comfort range $[\Theta_{\min}, \Theta_{\max}]$
- Wind process $\sum P_i^w(t) \sim \text{Markov process}$
- Temperature dynamics $\dot{x}_i(t) = h_i - P_i^w(t)$
- Cost function $\lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_i \mathbb{I}(x_i(t) > \Theta_{max}) dt$

Optimal policy in comfort violation probability model

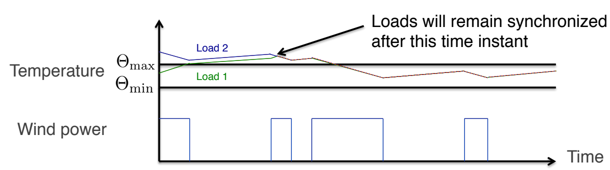


Theorem: Provide power to the coolest load that is above the temperature range.

- Issue: Unfair, temperatures of some loads will remain higher than others
- Possible solution: Minimize the *variance* of comfort violation

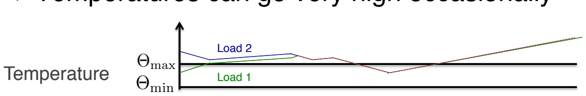
Stochastic control problem: Variance minimization

- Stochastic control problem:
  - Cost function $\lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_i [(x_i(t) - \Theta_{max})^+]^2 dt$
- **Theorem**: Optimal policy "synchronizes" loads

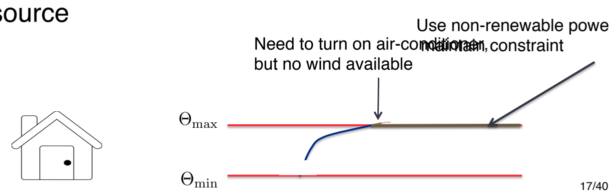


Requirement for reserves (of non-renewable power)

- Temperatures can go very high occasionally

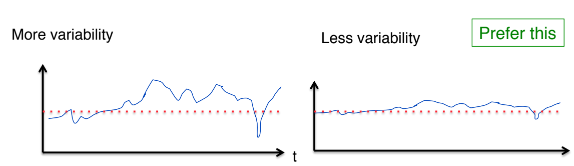


- Hard constraints require reliable non-renewable source



Cost function for reducing operating reserves

- Desire low operating reserve requirements



- Impose a quadratic cost on non-renewable power usage

$$\int (\sum P_i^n(t))^2 dt$$

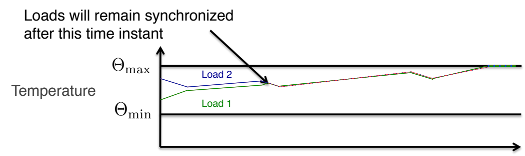
Stochastic control problem: Reduction of variability with temperature constraint

Stochastic control Problem:

- Wind process $\sum P_i^w(t) \sim \text{Markov process}$
- Temperature dynamics $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- Non-renewable power $P_i^n(t) \geq 0$
- Temperature constraint $x_i(t) \in [\Theta_{\min}, \Theta_{\max}], \forall i$
- Quadratic cost to reduce variability $\lim_{T\to\infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$

Optimal solution: Reduction of variability with temperature constraint

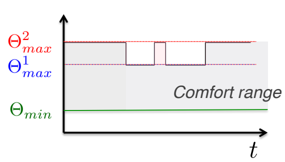
- **Theorem**: Optimal policy still synchronizes loads!

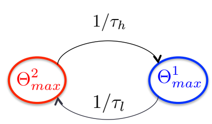


- Counter-intuitive??
- Question: Is there some modification in the model or cost function which leads to de-synchronization ?

How to induce desynchronization: Markov model for changes in $\Theta_{\max}$

- Suppose users occasionally change Θ_{max} setting at the same time
  - E.g. Super Bowl Sundays @ game time.
- E.g. Θ_{max} is a two state Markov process



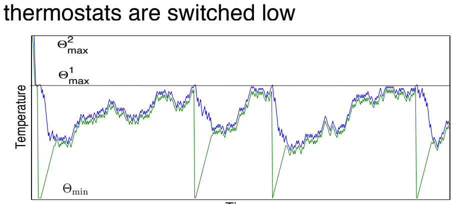


Stochastic control problem: Stochastic variation of temperature constraints

- Wind process: $\sum P_i^w(t) \sim \text{Markov process}$
- Temperature dynamics: $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- Non-renewable power $P_i^n(t) \geq 0$
- Stochastic comfort level $\Theta_{max}(t) \sim \text{Markov process}$, $\Theta_{max}(t) \in \{\Theta^1_{max}, \Theta^2_{max}\}$
- Temperature constraint: $x_i(t) \in [\Theta_{min}, \Theta^2_{max}], \forall i$
- Maximum cooling rate: $P_i^n(t) = M$ If $x_i(t) > \Theta_{max}(t)$
- Quadratic cost: $\lim_{T\to\infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$

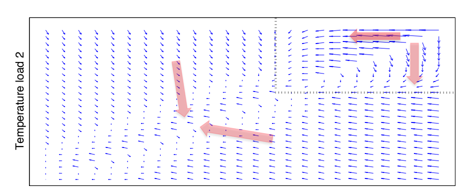
Optimal de-synchronization and re-synchronization

- It is optimal to break symmetry at high temperatures
  - Hedges against the future eventuality that the thermostats are switched low



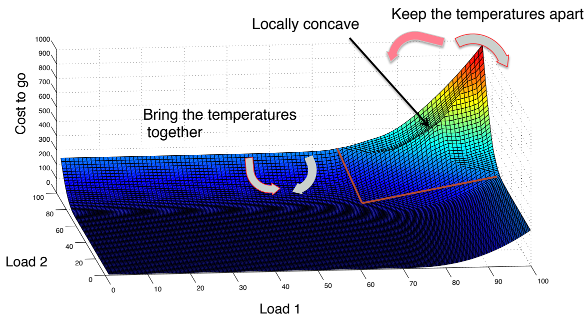
Vector field of optimal solution

- Nature of optimal solution
  - De-synchronization at high temperatures
  - Re-synchronization at low temperatures



Vector field of temperature changes

Local concavity/convexity of optimal cost-to-go resulting from HJB equation

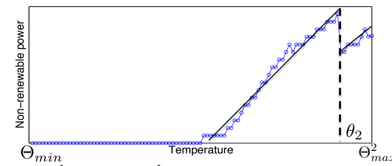


- But optimal policy is difficult to compute

A heuristic

- Approximate optimal policy
  - De-synchronization above temperature
    - » Provide power to load with minimum temperature amongst all loads with temperature higher than θ_2
    - » Bring the temperatures together for loads in $[\Theta_{min}, \theta_2]$
  - Power is assumed affine in $[\Theta_{min}, \theta_2]$ and $[\theta_2, \Theta_{max}]$
  - Policy is a function of a few parameters, optimize iteratively

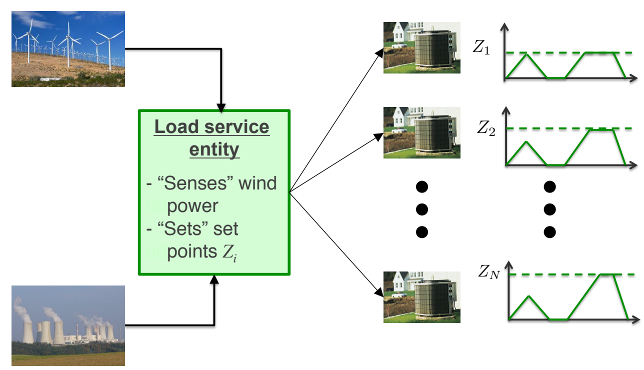
Two loads optimal solution along $x_1 = x_2$



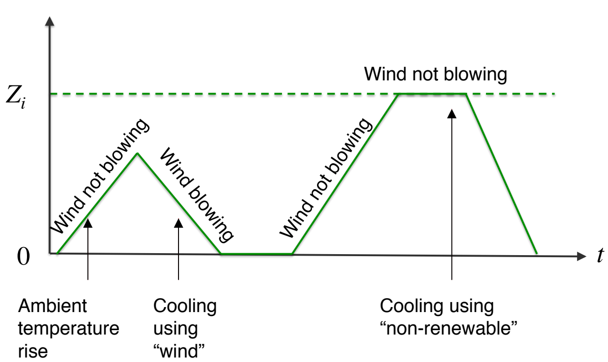
- But requires intrusive sensing

A possibly implementable architecture of a solution

Thermostatic control with set points



Control policy



Information flow in architecture

- Wind blowing or not = "Price signal"
  - Information from LSE to consumer
  - Minimal information needed to be responsive?
- LSE need not "set" thermostat set-points
  - Only needs to set empirical distribution of set-points
  - Not detailed actuation
- No flow of state information from home to LSE
- Information and communication requirements
  - Price signal to consumers
  - Infrequent distribution signaling to consumers
- (LSE also monitors total power usage by consumers)

Overall optimization problem

- Stochastic Wind process: $\sum_{1}^{N} P_i^w \sim \mathcal{W}_t$
- Temperature dynamics: $\dot{x}_i(t) = f(P_i^w(t) + P_i^g(t), x_i(t))$
- Cost: $\text{Min}\left[\lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\sum_1^N P_i^g(t)\right)^2 dt\right]$

Grid power variation cost

Overall optimization problem

- Stochastic Wind process: $\sum_{1}^{N} P_i^w \sim \mathcal{W}_t$
- Temperature dynamics: $\dot{x}_i(t) = f(P_i^w(t) + P_i^g(t), x_i(t))$
- Comfort setting dynamics: $\Theta_i^M(t) \sim \text{Stochastic process}$
- Cost: $\text{Min}\left[\lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\sum_1^N P_i^g(t)\right)^2 + \gamma\sum_1^N[(x_i(t) - \Theta_i^M(t))^+]^2 dt\right]$

Grid power variation cost — *User discomfort cost*

Overall optimization problem

- How to choose $\{Z_1, Z_2, \ldots, Z_N\}$ so as to minimize:

$$\text{Min}\left[\lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\sum_1^N P_i^g(t)\right)^2 + \gamma\sum_1^N[(x_i(t) - \Theta_i^M(t))^+]^2 dt\right]$$

- Difficult:
  - Complex as N is large, high dimensional.
  - Need to solve different problem for different N
- Solution:
  - Study asymptotic limit as $N \to \infty$.
  - Solution becomes explicit!
  - And asymptotic solution is also nearly optimal even for small N

Continuum limit of Z-policy

- Continuum of loads in $[0,1]$
- $u(x)$= fraction of loads with set-points less than x,
  = empirical distribution of set-points
- Cost function

$$C^{[0,1]}(u) = \gamma\int_0^{\Theta_2}\Phi(z)u'(z)dz + (h)^2(\delta_{\Theta_2}$$

$$+ \int_0^{\Theta_2} u^2(z)\mathbb{P}(\{X_z = z\}\cap\{X_{z+dz} < z+dz\}))$$

  - Masses at set points

Continuum limit optimization problem

- Resulting variational problem:

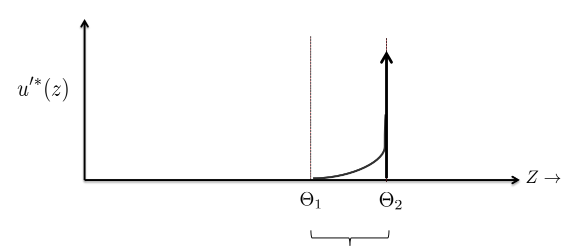
$$\begin{aligned} &\text{Minimize} && J[u] = \int_0^{\Theta_2} F(z,u,u')dz \\ &\text{s.t.} && u \in \mathcal{U} \\ & && u(0) = 0,\, u(\Theta_2) = 1. \end{aligned}$$

- Solution to variational problem (Euler-Lagrange):

$$\frac{\partial F}{\partial u} - \frac{d}{dx}\frac{\partial F}{\partial u'} = 0 \Rightarrow 2(h)^2u(x)D(x) - \frac{d}{dx}\gamma\Phi(x) = 0 \Rightarrow u(x) = \frac{\gamma\Phi'(x)}{2(h)^2D(x)}$$

- Not so fast, singularity: $\frac{\partial^2 F}{\partial u'^2} = 0$
- Solution is given by: $u^*(z) = \begin{cases}\min\left(1, \frac{\gamma\Phi'(z)}{2(h)^2D(z)}\right) & \text{If } z < \Theta_2 \\ 1 & \text{If } z = \Theta_2.\end{cases}$

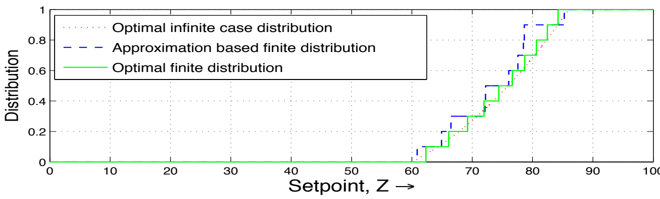
Optimal solution of continuum limit



Optimal desynchronization of demand response

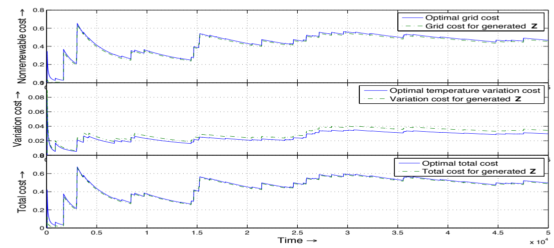
Z-policy: Finite population approximation from continuum limit asymptotic

- Generate $\{Z_i\}_1^N$ to approximate continuum limit



Some simulation results

- This appears to work very well even when N is small
- Even $N = 5$



Concluding remarks

- Attempt to develop an architecture and tractable solution for demand response

- Many extensions needed and feasible
  - Response to comfort variations
  - Availability of wind power
  - Generalize wind model, temperature dynamics, etc.

Thank you