PMU Placement to Ensure Observable Frequency and Voltage Dynamics:
A Structured System Approach

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(Joint work with Sergio Pequito, Prof. Soummya Kar and Prof. Marija Ilic)

Electrical and Computer Engineering
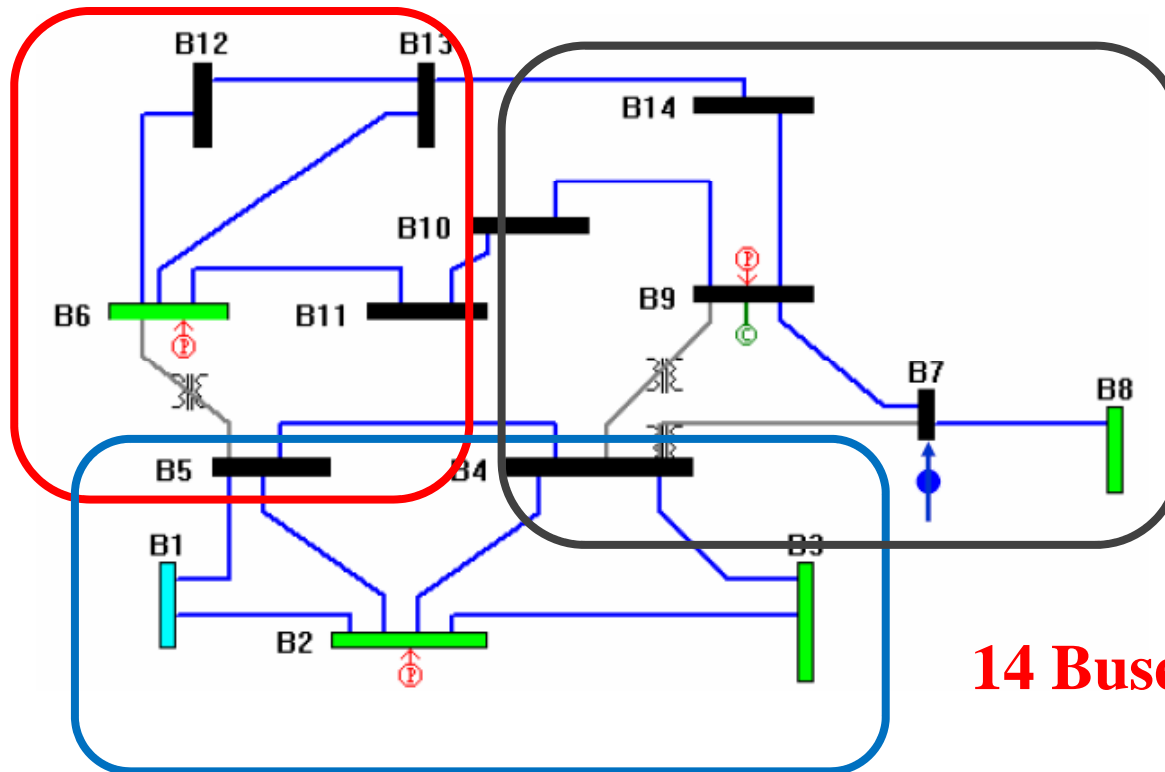Carnegie Mellon University
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Outline

◆ **Static Observability vs Dynamic Observability**

◆ **The key role of Phasor Measurement Units**

◆ **Coupled Frequency-Voltage Dynamic Model**

◆ **Structural Systems**

◆ **PMU Placement**

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Static Observability vs Dynamic Observability

Topological Static Observability [*Abur 1995*]

Covering problem (IEEE 14 Example)

14 Buses, 3 PMUs

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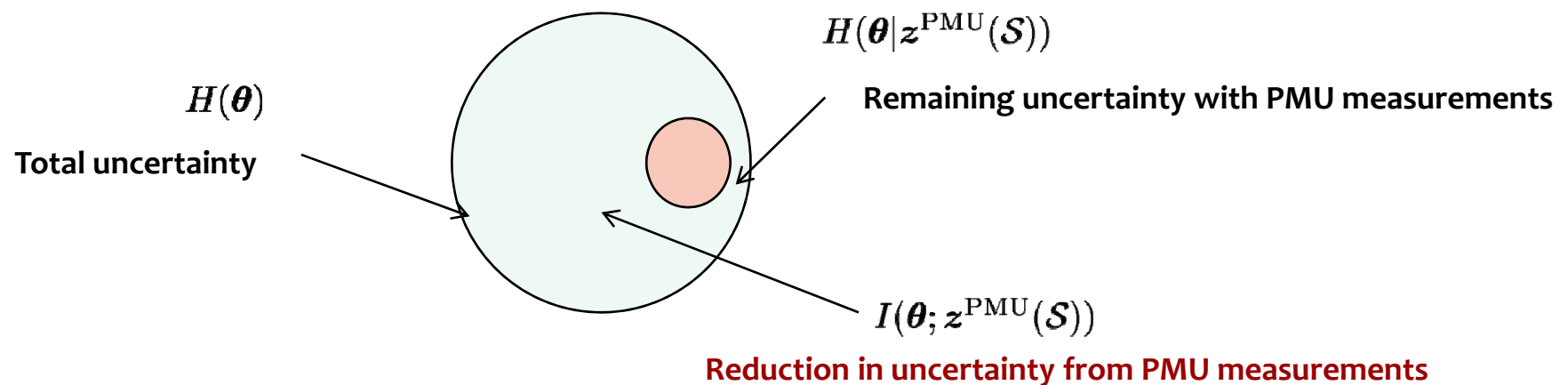Carnegie Mellon | PORTUGAL
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Static Observability vs Dynamic Observability

**Mutual Information to Improve Static Observability [*Li 2011*]**

Mutual information (MI): **the amount of information that PMU measurement carries about the system state**

Placement Objective: **Maximize mutual information**

$H(\boldsymbol{\theta}|\boldsymbol{z}^{\mathrm{PMU}}(\mathcal{S}))$

Remaining uncertainty with PMU measurements

$H(\boldsymbol{\theta})$

Total uncertainty

$I(\boldsymbol{\theta}; \boldsymbol{z}^{\mathrm{PMU}}(\mathcal{S}))$

Reduction in uncertainty from PMU measurements

IEEE 14-Bus: 3 PMUs

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Static Observability vs Dynamic Observability

Static Observability $\left\{\begin{array}{l} \\ \\ \end{array}\right.$ **Topological Observability**

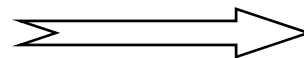**Mutual information**
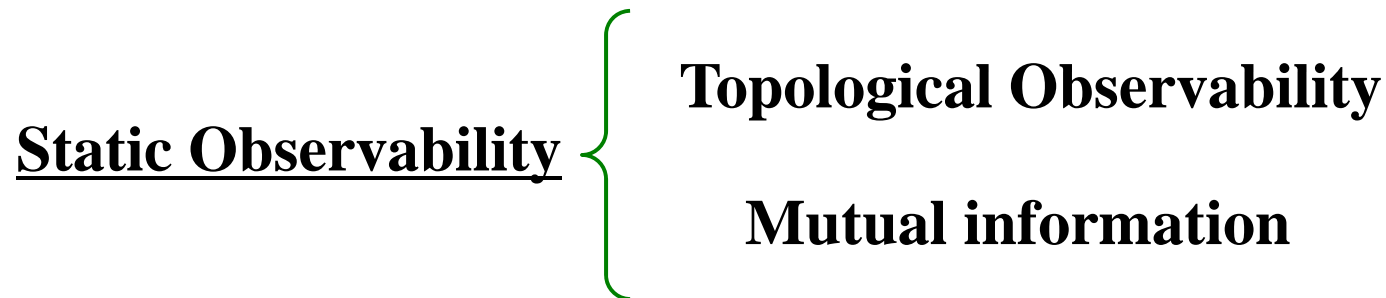
Dynamic Observability

A system is said to be observable if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the outputs.

(Descriptor form)

$$E\dot{x} = Ax$$
$$y = Cx$$

DAEs vs ODEs $\Longrightarrow$

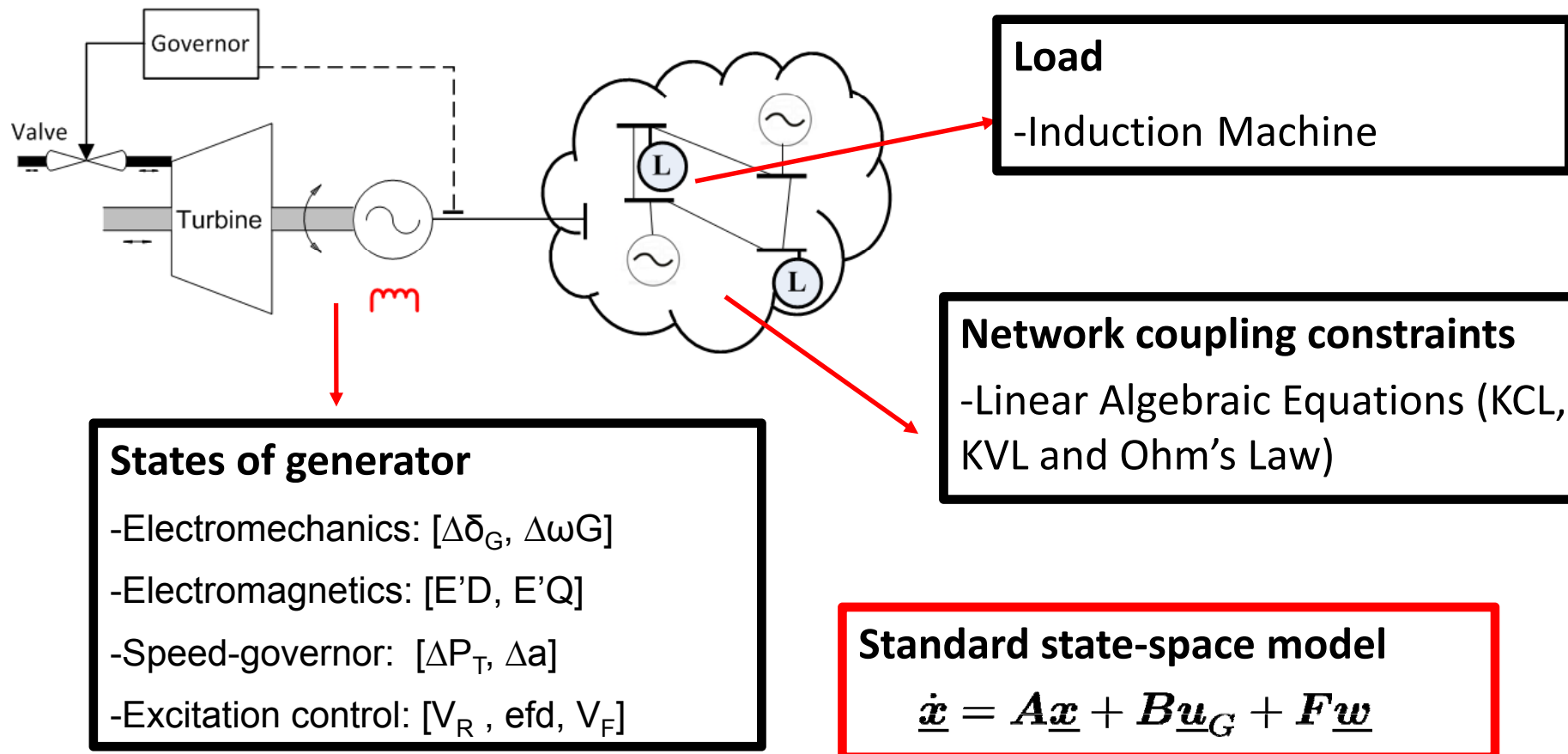Our model

$$\dot{x} = Ax$$
$$y = Cx$$

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The Key Role of Phasor Measurement Units

◆ **Real-time synchronized measurements for geographically large-scale systems**

◆ **Enable systematically designed dynamic observer**

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Modeling Power Systems
Coupled frequency-voltage dynamics

Load

-Induction Machine

Network coupling constraints

-Linear Algebraic Equations (KCL, KVL and Ohm's Law)

States of generator

-Electromechanics: [$\Delta\delta_G$, $\Delta\omega G$]

-Electromagnetics: [E'D, E'Q]

-Speed-governor: [ΔP_T, Δa]

-Excitation control: [V_R , efd, V_F]

Standard state-space model

$$\underline{\dot{x}} = A\underline{x} + B\underline{u}_G + F\underline{w}$$

Given <u>only</u> the dynamics/structure

$$\dot{x} = Ax$$

How to design

$$y = Cx$$

Such that (A,C) is observable, I.e,

$$rank \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}}_{\text{Observability Matrix}} = n$$

PMU Placement

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Structural Systems

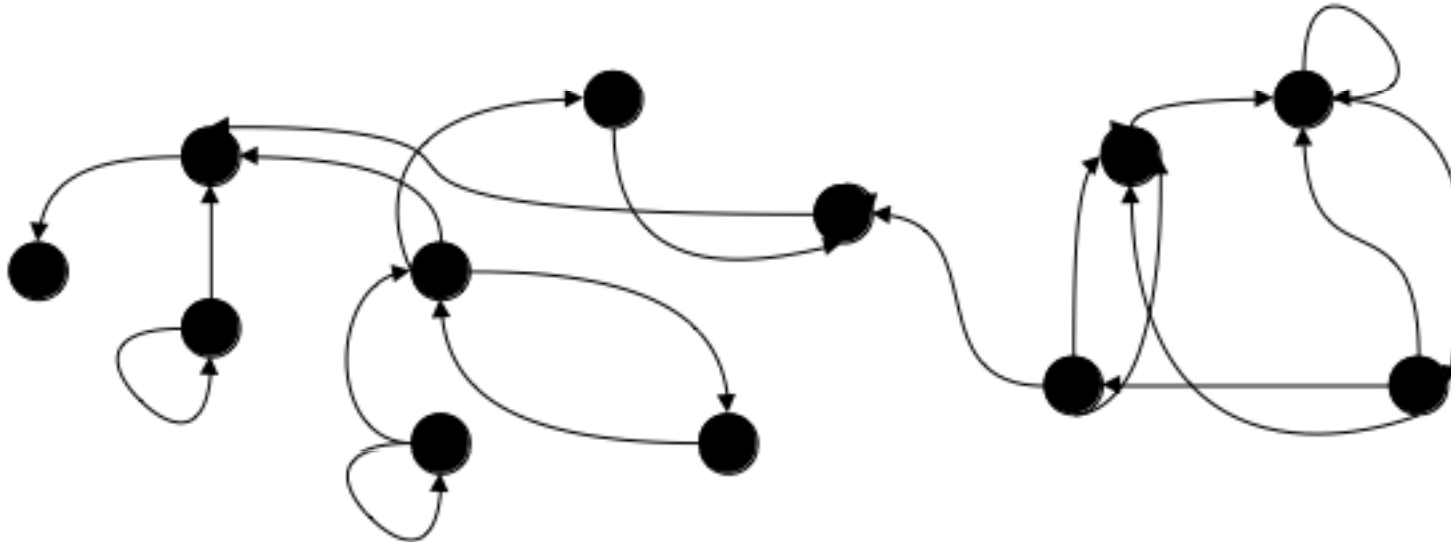## Representation of the dynamics as digraph

From

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$
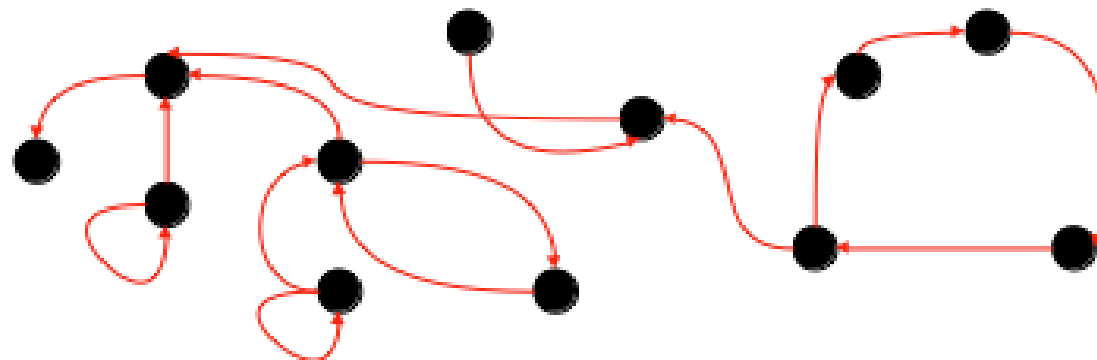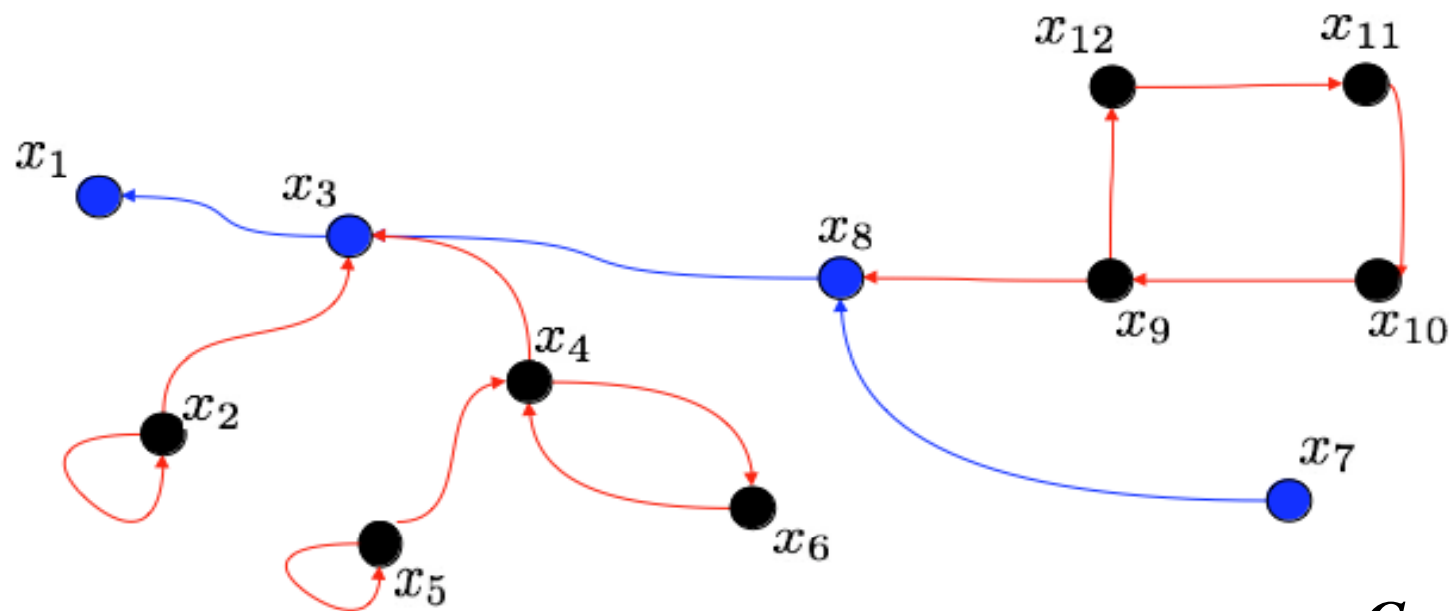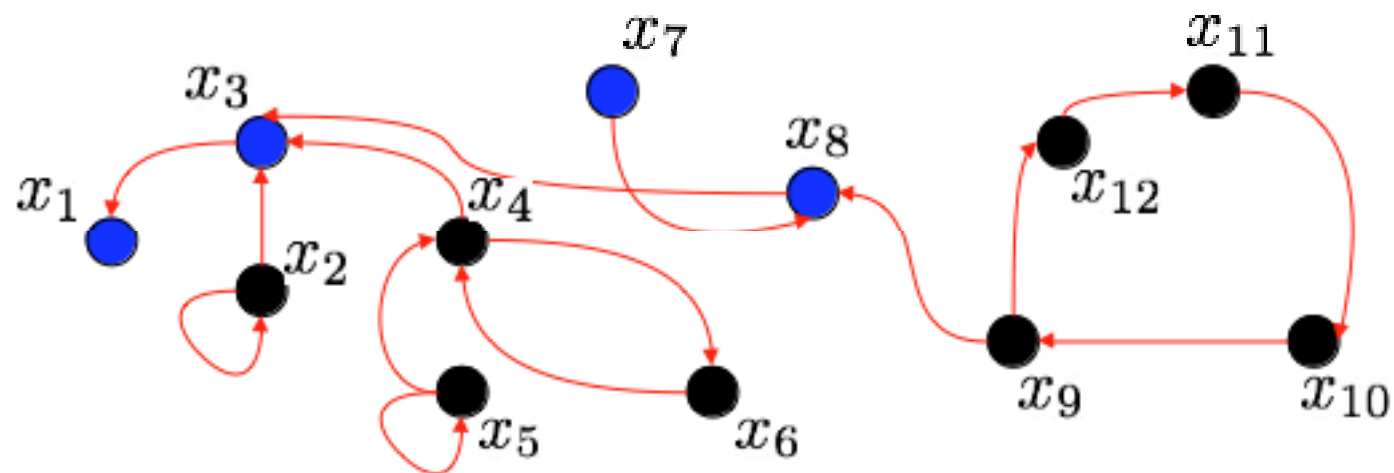
To

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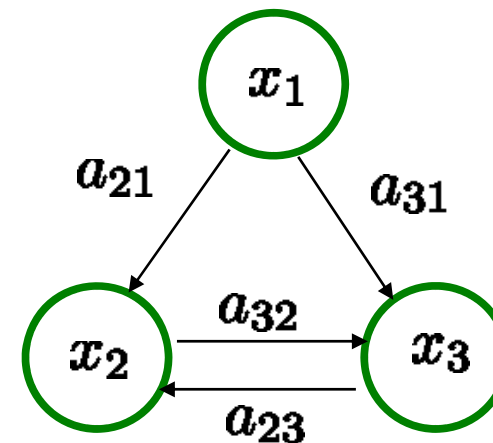
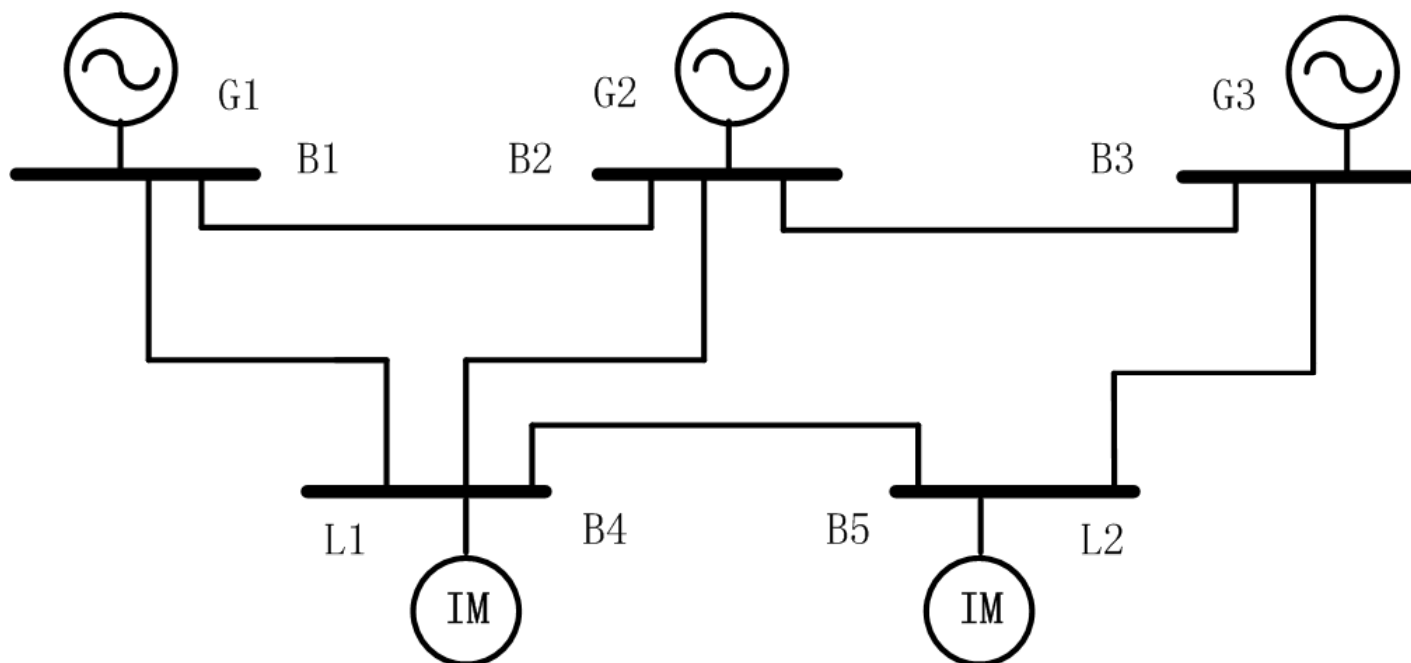# Given the system Digraph

Does it have a substructure as follows

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Cactus

PMU

Cactus

PMU = # Cacti

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Case Study: 5 bus

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Is this system dynamically observable?

Number of states: 37

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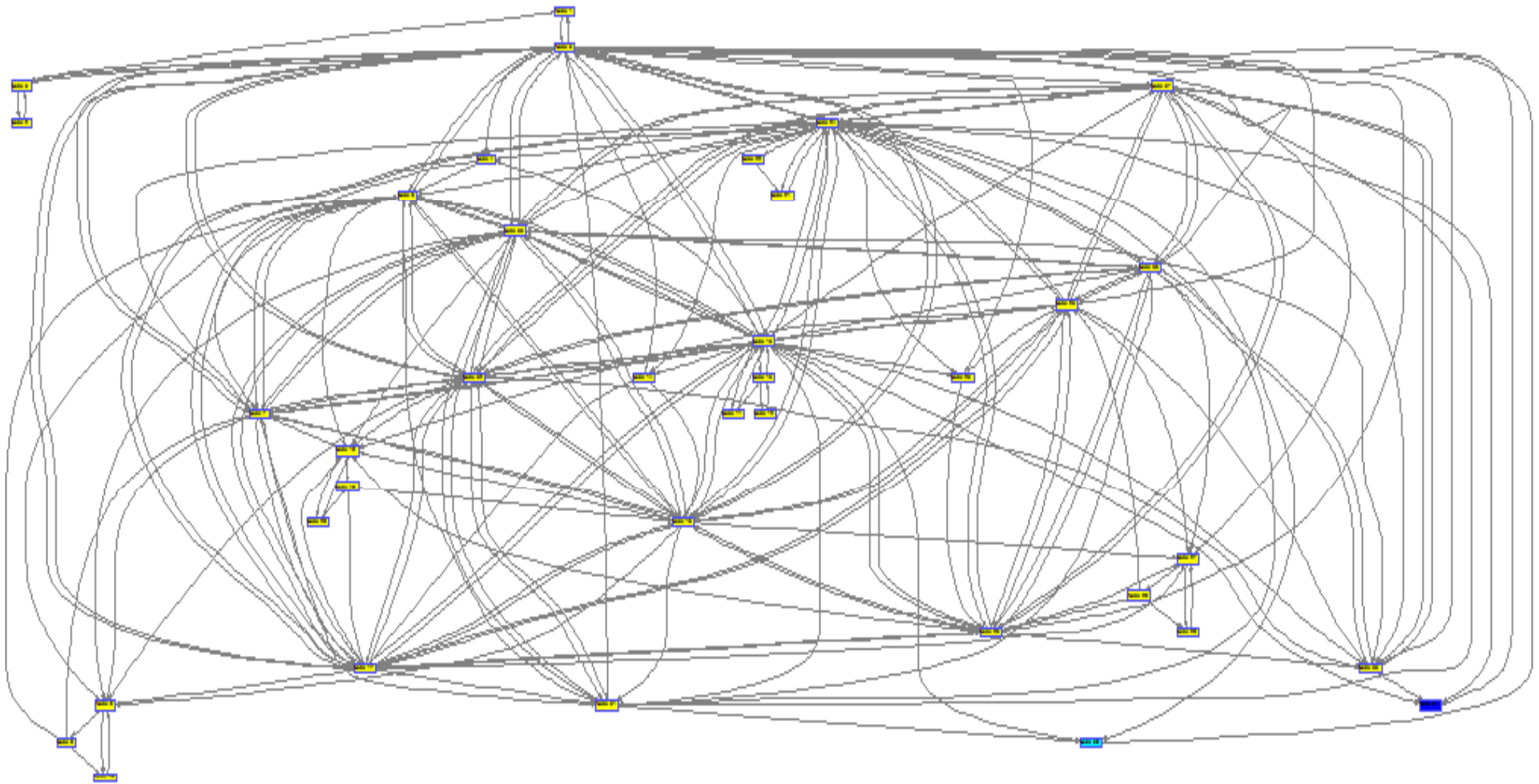
Compute one tree [Node 1-Tree (Path 1-37)]
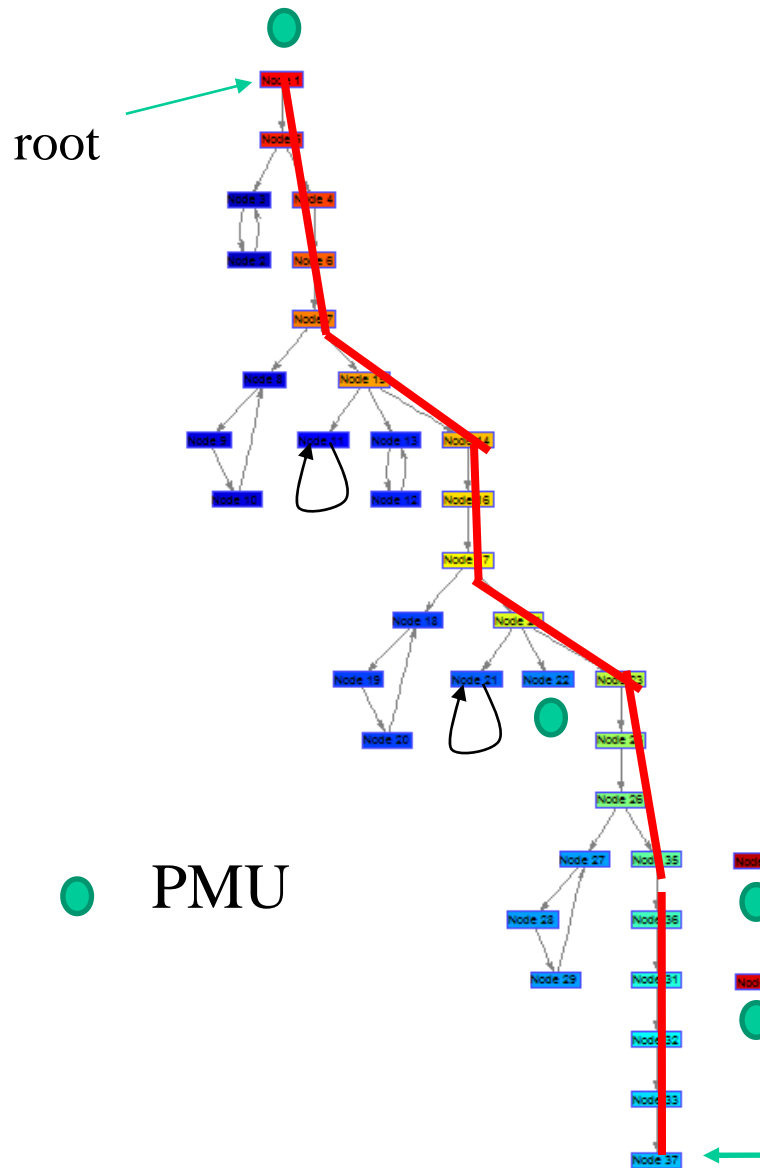
Measured States	1, 22, 30, 34
Interpretation	**Governor Hydro (1)** **Governor Diesel (22)** **Load 1 Angle (30)** **Load 2 Angle (34)**
Buses	**1, 3, 4, 5**
# PMUs	**4**

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Compute one tree [Node 4-Tree (Path 4-22)]
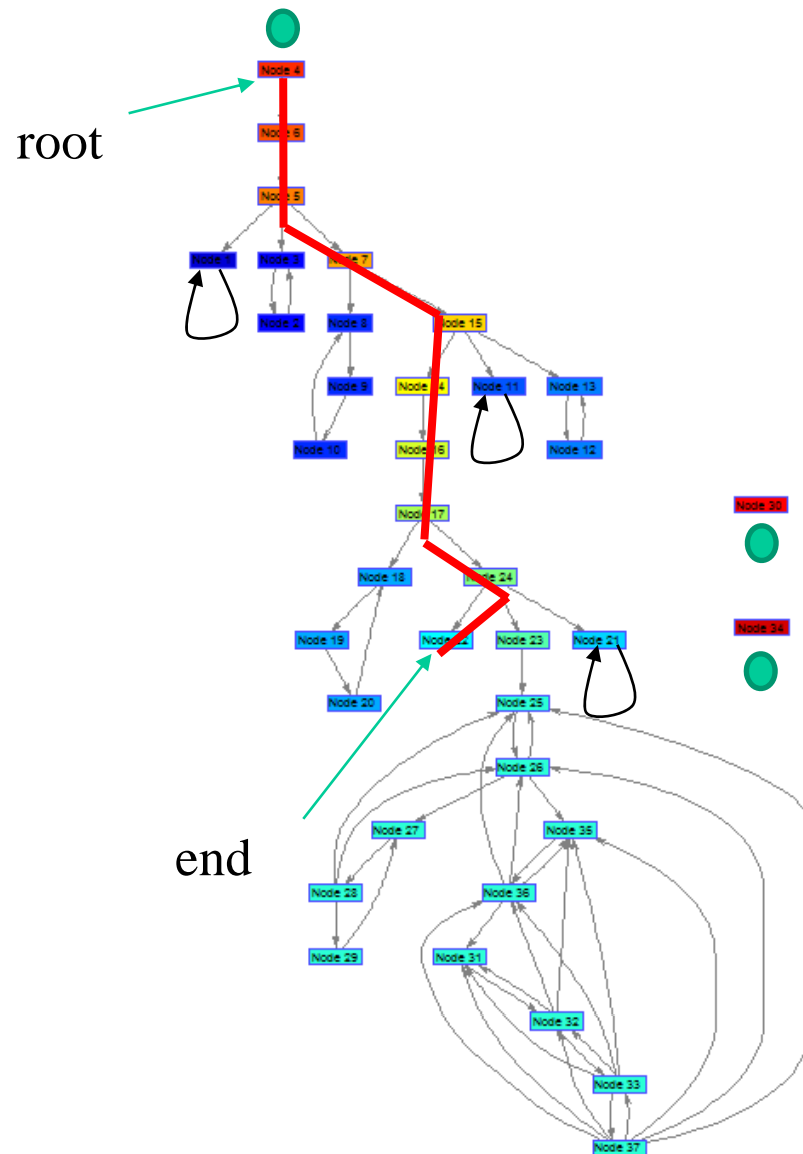
root

end

Measured States	4, 30, 34
Interpretation	**Hydro G1 Angle (4)** **Load 1 Angle (30)** **Load 2 Angle (34)**
Buses	1, 4, 5
# PMUs	3

Final combinations with minimum number of sensors

Measured States	4, 30, 34
Interpretation	**Hydro G1 Angle (4)** **Load 1 Angle (30)** **Load 2 Angle (34)**
Buses	1, 4, 5
# PMUs	3

Conclusions and Further Research

- ◆   Implement this framework in IEEE 14/20/30 bus

- ◆   Robustness analysis and quantitative analysis

- ◆   *User friendly toolbox*

References

[1] Ali Abur, Optimal Plancement of Phasor Measurement Units for State Estimation. PSERC Report, 2005

[2] Qiao Li, Tao Cui, Yang Weng, Rohit Negi, Franz Franchetti, Marija Ilic, An Information-Theoretic Approach to PMU Placement in Electric Power Systems

[3] PMU Placement to Ensure Observable Frequency and Voltage Dynamics: A Structured System Approach, *8th CMU Electricity Conference. http://www.ece.cmu.edu/~electriconf/index.html*

[4] Sergio Pequito, S. Kar, A. Pedro Aguiar, Actuator Placement Design Strategies for Controllability of Large Scale Systems: Minimality through Structured Systems, *To be submitted to Transactions of Automatic Control*

Questions?

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Thank you!